

# Preface

Nonlinear wave equations belong to a typical category of nonlinear evolutionary equations that is of great theoretical significance and practical value. Research on the global existence and the blow-up phenomenon of classical solutions to the Cauchy problem with small initial data for nonlinear wave equations relates to the asymptotic stability of the null solution or the stabilization of the corresponding control system of such equations, and is a highly meaningful and challenging subject of study. The research in this field was initiated by Prof. F. John in the late 1970s and early 1980s when he gave some examples to reveal the blow-up phenomenon of solutions to nonlinear wave equations. Later on, Profs. S. Klainerman, D. Christodoulou, and L. Hörmander together with Prof. F. John, as well as some other Profs. like M. Kovalyov, H. Lindblad, G. Ponce, J. Shatah, T.C. Sideris, gave various results about the global existence and the lower bound estimates of life-span of classical solutions in different space dimensions and with different powers of nonlinear terms on the right-hand side, which formed a frontier research direction of great significance and attraction. Although the results obtained by these mathematicians were profound, they did not cover all the possible important situations at the time, and there remained a lack about the sharpness of the established lower bound estimates of life-span for classical solutions. The whole research is still, so to speak, in the initial stages of development. On the other hand, there is a great deal of diversity in the methods adopted by these mathematicians, each having its own characteristics and some being rather complex. So there does not seem to be a unified and easy approach to tackling such type of problems.

During my stay in France, I visited Heriot-Watt University of Great Britain in 1980 and encountered Prof. F. John who at the time happened to visit that university, too. So I got the chance to consult him face to face, which turned out to be a most instructive experience. At the beginning of 1981 when I paid a visit to Courant Institute of Mathematical Science in the USA, I met Prof. S. Klainerman and had careful discussions with him. He presented me with the preprint of his long article close to 60 pages. All these aroused my concern and interest in nonlinear wave equations and prompted our studies in this field. Some of my doctor students in the earlier years, including Yunmei Chen, Xin Yu, and Yi Zhou, chose this as the

subject of their doctoral dissertations and made valuable contributions. Thanks to their participation and efforts, especially Yi Zhou's long persistent hard work, we have managed to carry on with this research subject in Fudan University up to now and obtained fruitful results. Our limited accomplishment in this area can be generally summarized into two aspects. One is to have established the complete lower bound estimates of life-span (including the result of global existence) for classical solutions to the Cauchy problem of nonlinear wave equations with small initial data in all possible space dimensions and with all possible powers of nonlinear terms on the right-hand side, and the estimates are the best ones that are unlikely to be improved, that is to say, we in principle draw the conclusion for research in this regard. The other is to have proposed the unified and straightforward approach to handling such problems, that is, the global iteration method, which applies the simple contraction mapping principle and requires roughly the same amount of work done to prove the local existence of classical solutions.

In the book *Nonlinear Evolution Equations* (in Chinese) coauthored by Yunmei Chen and me and published by Science Press (Beijing) in 1989, we proved the global existence of classical solutions to the Cauchy problem with small initial data for nonlinear wave equations by the global iteration method. Later, in another book *Global Classical Solutions for Nonlinear Evolution Equations* coauthored also by Yunmei Chen and me and published by Longman Scientific & Technical Press in 1992, the method was further employed to make some lower bound estimates of life-span for classical solutions. However, restricted by the progress of scientific research at that time, we did not touch upon or get the best results about the important situations of space dimensions  $n = 2$  and  $n = 4$ , etc. Besides, we failed to deal with theories related to the null condition and the sharpness of some lower bound estimates of life-span. Since the two books both mentioned nonlinear evolution equations including also nonlinear heat equations and nonlinear wave equations are only a part of them, the length of the discussion was inevitably limited, which to a certain extent caused the deficiency mentioned above. Around 1995, we basically finished work on the global existence and the lower bound estimates of life-span of classical solutions to the Cauchy problem with small initial data for nonlinear wave equations, so Yi Zhou and I began to think about a monograph on nonlinear wave equations. In fact, the Shanghai Scientific and Technical Publishers had invited us to write on this long before, but with too many errands to go, we wrote on and off or sometimes even put off for quite a long period. Another important reason that hindered us from finishing the book soon was that some of the lower bound estimates of life-span we got then had not been proved to be the best ones immune to further improvement, so if we wrapped up the book in haste and delivered it for publication, it might never be consummate, which is a fact we would not be content with. In the recent years, the sharpness of all the lower bound estimates of the life-span has been obtained, and thus we felt the urgency of finishing the book as quickly as possible. Meanwhile, throughout the years we have found that some of the previous proofs can be simplified or improved so as to be presented in a relatively new form, which can be seen as an additional achievement. Although we were determined to pull our forces together and start afresh, it took yet

another two to three years for us to complete the final version of the book in 2014, as many proofs needed to be rewritten. Seeing the book finally come out after such a long track of time, the gratification of the authors can well be imagined.

The whole book has got fifteen chapters in all. The first seven serve as a prelude for later discussions, but still have their own meanings and values. Among the later eight chapters, five discuss the global existence and the lower bound estimates of life-span for classical solutions in all possible situations by adopting the global iteration method, including the proof of global existence of classical solutions under the hypothesis of the null condition; two focus on demonstrating the sharpness of the obtained lower bound estimates of life-span; and the last chapter entails relevant applications and extensions. Most of the references listed in the bibliography are cited in the body part of the book, while a few of them, though not formally quoted, are more or less related to the content, from which we hope readers can get some necessary information. Dr. Ke Wang has been responsible for the typewriting and typesetting of the book in Chinese, while the English version is to be translated by Prof. Yachun Li and published by the Springer-Verlag. The authors would like to express their deep-felt gratitude to them all for their hearty devotion, earnest support, and strong help.

Owing to the limitation of the authors' knowledge, there must be mistakes and careless omissions in the book, so the authors hereby sincerely invite readers to make frank comments and criticism in any respect.

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