Contents

1 Overall Introduction 1
1.1 What Is Probability? 1
1.1.1 Knowledge and States of the World 1
1.1.2 Information and Events 2
1.1.3 Frequentist Approach 4
1.1.4 Subjectivist Approach 5
1.1.5 Axiomatic Approach 6
1.1.6 Objective Probability and Randomizing Device 7
1.1.7 Subjective Probability and Ir/rationality 8
1.2 Risk and Uncertainty 9
1.2.1 Frank Knight and Leonard Savage 9
1.2.2 Maxmin Expected Utility (MEU) and Choquet Expected Utility (CEU) 10
1.2.3 Pessimism and Optimism 12
1.3 What Is This Book Going to Achieve? 13
1.3.1 Indeterminacy of Equilibrium Prices 13
1.3.2 Increase in Risk Versus Increase in Uncertainty 14
1.3.3 The ε-Contamination: Analytic Workhorse of Knightian Uncertainty 17
1.3.4 Concept of "Expectation" Extended: Ellsberg Paradox and Persistent Bias 18
1.3.5 Learning Under Knightian Uncertainty 20
1.4 Organization of This Book 21
1.4.1 Mathematics: Chap. 2 21
1.4.2 Decision Theory: Chap. 3 22
1.4.3 Statics: Chaps. 4, 5, 6 and 12 23
1.4.4 Dynamics: Chaps. 7, 8, 9, 10 and 11 24
1.4.5 Two-Period Models: Chaps. 13 and 14 and Future Research: Chap. 15 26
References 27

2 Mathematics for Reading Later Chapters 29
2.1 Introduction and Summary 29
2.2 Probability Charges and Probability Measures 29
2.2.1 Algebra, σ-Algebra, and Measurable Spaces 29
2.2.2 Probability Charge and Finite Additivity 30
2.2.3 Dunford-Schwartz Integral with Respect to Charge 31
2.2.4 Probability Measure and σ-Additivity 32
2.3 Probability Capacity 33
2.3.1 Basic Definitions 33
2.3.2 Decomposability 34
2.3.3 The ε-Contamination 35
2.3.4 The Core 36
2.3.5 Updating Probability Capacity 37
2.4 Choquet Integral 38
2.4.1 Definition 38
2.4.2 Properties of Choquet Integral 40
2.5 Capacitary Kernel 44
2.6 Remarks: Upper Quasi/Semi-Continuity of a Stochastic Kernel 48
References 49

3 Decision-Theoretic Foundations of Knightian Uncertainty 51
3.1 Introduction and Summary 51
3.2 Preference Order 52
3.3 Expected Utility Theory Under Risk à la von Neumann–Morgenstern 53
3.4 Mixture Spaces and Representation Theorem à la Herstein–Milnor 54
3.5 Subjective Expected Utility Theory Under Uncertainty à la Savage 55
3.5.1 Existence of Subjective Probability 55
3.5.2 Act 56
3.5.3 Savage's Axioms and Representation by Subjective Expected Utility 56
3.6 The Sure-Thing Principle and Probabilistic Sophistication 57
3.7 Ellsberg's Paradox 59
3.8 An Alternative Framework à la Anscombe–Aumann 60
3.8.1 Lottery Act 60
3.8.2 Anscombe–Aumann's Axioms and Representation by Subjective Expected Utility 61
3.9 Choquet Expected Utility Theory à la Schmeidler 62
3.9.1 Representation by Choquet Expected Utility 62
3.9.2 Rank-Dependent Subjective Expected Utility 62
3.9.3 Uncertainty Aversion 63
3.9.4 Ellsberg's Paradox Reconsidered 63

3.10 Maxmin Expected Utility Theory à la Gilboa-Schmeidler 64
3.10.1 Representation with Multiple Priors 64
3.10.2 Ellsberg's Paradox Re-Reconsidered 64
3.11 A Relation Between CEU and MEU 65
3.12 Epstein's Definition of Uncertainty Aversion 66
3.13 Dynamic Preferences 66
3.13.1 Unconditional and Conditional Preferences 67
3.13.2 Axioms and Representation 67
3.13.3 Discussion of Each Axiom 69
3.13.4 Dynamical Consistency and Its Implication 70
3.13.5 Two Updating Rules 72
3.14 Remarks 73
References 74

4 Portfolio Inertia 77
4.1 Introduction and Summary 77
4.2 Non-differentiability of Choquet Integrals 78
4.3 Portfolio Choice Model à la Dow–Werlang 80
References 82

5 Equilibrium Indeterminacy in Arrow–Debreu Economy with Knightian Uncertainty 83
5.1 Introduction and Summary 83
5.2 Arrow–Debreu–Dana Economy 84
5.2.1 Initial Endowments 84
5.2.2 Consumption and Preferences 84
5.2.3 Arrow–Debreu Equilibrium 84
5.3 Indeterminacy of Equilibria 85
5.3.1 Preliminary Results 85
5.3.2 Indeterminacy Theorem 86
5.4 Two-Consumer Two-State Case: The Edgeworth Box 88
5.5 Nonrobustness of Indeterminacy 89
References 90

6 Monetary Equilibria and Knightian Uncertainty 91
6.1 Introduction and Summary 91
6.2 Related Literature 93
6.3 The Model 95
6.3.1 Stochastic Environment 95
6.3.2 Preferences 95
6.3.3 Stationary Monetary Equilibrium 97
6.4 Two-State Illustrative Examples 98
6.5 Robust Indeterminacy 102
6.5.1 Characterization of Stationary Monetary Equilibria 102
6.5.2 Existence and Robust Indeterminacy of Equilibria 104
6.6 Comparative Statics 107

6.7 Optimality . . . 108
References . . . 109

7 Dynamic Programming . . . 113
7.1 Introduction and Summary . . . 113
7.2 Recursive Utility Function . . . 114
7.2.1 Stochastic Environment . . . 114
7.2.2 Consumption Plan . . . 114
7.2.3 Return Function and Feasibility Correspondence . . . 114
7.2.4 Utility Function and Upper-Convergence . . . 115
7.2.5 Recursivity and Koopmans' Equation . . . 116
7.3 Dynamic Programming: A General Theory . . . 116
7.4 Non-negative Dynamic Programming . . . 118
7.4.1 A Special Case: Bounded Returns . . . 119
7.4.2 An Example: CRRA Utility and *AK* Production . . . 120
7.5 Non-positive Dynamic Programming . . . 120
References . . . 121

8 Dynamic Asset Pricing . . . 123
8.1 Introduction and Summary . . . 123
8.2 Lucas–Epstein–Wang Economy . . . 124
8.3 Principle of Optimality . . . 125
8.4 The Euler Inequalities . . . 126
8.5 In/determinacy of Equilibrium Prices . . . 130
8.5.1 Uniqueness of Equilibrium Prices . . . 130
8.5.2 Finite State Space . . . 131
8.6 Economy with Endowments . . . 133
8.6.1 Endowment Process . . . 133
8.6.2 Feasible Plans and Optimal Plans . . . 133
8.6.3 The Euler Inequalities and Partial Aggregate Shocks . . . 134
References . . . 136

9 Search and Knightian Uncertainty . . . 137
9.1 Introduction and Summary . . . 137
9.2 An Example: Risk Versus Knightian Uncertainty . . . 138
9.2.1 Increased Risk: Mean-Preserving Spread . . . 140
9.2.2 Increased Knightian Uncertainty: ε-Contamination . . . 141
9.2.3 Risk Aversion . . . 144
9.3 Some Technical Issues . . . 146
9.3.1 Representation by Convex CEU . . . 146
9.3.2 δ-Approximation of ε-Contamination . . . 147
9.4 The Formal Model . . . 150
9.4.1 Stochastic Environment . . . 150
9.4.2 Objective Function . . . 151
9.4.3 Stopping Rule and Optimization Problem . . . 153

9.4.4 Existence and Characterization of Optimal Stopping Rule . . . 154
9.4.5 An Increase in Uncertainty . . . 157
References . . . 158

10 Irreversible Investment and Knightian Uncertainty . . . 161
10.1 Introduction and Summary . . . 161
10.2 A Two-Period, Two-State Example . . . 164
10.3 Knightian Uncertainty in Continuous Time . . . 167
10.3.1 Density Generators . . . 168
10.3.2 The Set of Stochastic Differential Equations . . . 170
10.3.3 Strong Rectangularity . . . 170
10.3.4 I.I.D. Uncertainty and κ-Ignorance . . . 171
10.4 The Value of a Utilized Patent . . . 172
10.5 The Optimal Investment Decision . . . 173
10.6 The Value of an Unutilized Patent . . . 174
10.6.1 κ-Ignorance and Infinite Horizon . . . 174
10.6.2 The Hamilton-Jacobi-Bellman Equation . . . 175
10.6.3 The Optimal Strategy . . . 177
10.7 Sensitivity Analysis . . . 178
10.7.1 An Increase in Risk . . . 179
10.7.2 An Increase in Knightian Uncertainty . . . 179
10.7.3 Value of Waiting . . . 181
References . . . 183

11 Liquidity Preference and Knightian Uncertainty . . . 185
11.1 Introduction and Summary . . . 185
11.1.1 Money Offering Flexibility . . . 185
11.1.2 Increase in Risk Versus Increase in Uncertainty . . . 187
11.1.3 Some Technical Aspects . . . 189
11.2 The Model . . . 190
11.2.1 Stochastic Environment . . . 190
11.2.2 The Fund Manager's Problem . . . 190
11.3 The Exact Solution . . . 192
11.3.1 Dynamic Programming . . . 192
11.3.2 Finding the Exact Solution . . . 193
11.3.3 Reservation Property . . . 195
11.4 Sensitivity Analyses . . . 195
11.4.1 An Increase in Risk . . . 196
11.4.2 An Increase in Uncertainty . . . 196
References . . . 197

12 A Simple Characterization of Pessimism and Optimism: ε-Contamination Versus ε-Exuberance 199
12.1 Introduction and Summary 199
12.1.1 Pessimism: The ε-Contamination 199
12.1.2 Optimism: The ε-Exuberance 200
12.1.3 Pessimism and Optimism: A Symmetry 201
12.2 Preliminaries 201
12.3 Pessimistic Behavior: Axioms 202
12.4 Pessimistic Behavior: Main Results 203
12.5 Optimistic Behavior: Axioms and Main Results 205
12.6 Anscombe–Aumann's Theory: Revisited 207
12.7 The Distribution-Function Framework 207
12.7.1 Distribution Functions with Compact Supports 207
12.7.2 Density Functions 208
12.7.3 Distribution Functions with Unbounded Supports 209
References 210

13 Persistent Pessimism and Optimism in Forecasts: Implicit Means and Law of Iterated Integrals 211
13.1 Introduction and Summary 211
13.2 The Implicit Mean 214
13.2.1 Preliminaries 214
13.2.2 The Betweenness Function and Its Uniqueness 215
13.2.3 Alternative Characterization of Implicit Mean 216
13.3 The Conditional Implicit Mean 218
13.3.1 Information Structure 218
13.3.2 Defining Conditional Implicit Mean 218
13.3.3 The Law of Iterated Integrals and Characterization of Quasi-linear Mean 221
13.4 Examples of the Implicit Mean 222
13.4.1 Quasi-linear Mean 222
13.4.2 L^q-Error Function 223
13.4.3 Asymmetric Error Function 224
13.4.4 Persistent Bearishness and Bullishness 225
13.5 Applications: Information Aversion and Ellsberg's Paradox Revisited 226
13.5.1 Information Aversion 227
13.5.2 Ellsberg's Paradox Revisited 229
References 230

14 Learning Under Knightian Uncertainty 231
14.1 Introduction and Summary 231
14.2 An Example: Rothschild's Learning Model 233

14.3 The Two-Period Model of Knightian Uncertainty . . . 237
14.3.1 Information Structure . . . 237
14.3.2 Income Process . . . 238
14.3.3 Updating Rules . . . 238
14.3.4 Objective Function . . . 239
14.3.5 Dilation of Knightian Uncertainty . . . 239
14.4 The (Generalized) Bayesian and Maximum-Likelihood Rules . . . 240
14.5 The ε-Contamination and Dilation of Knightian Uncertainty . . . 242
References . . . 243

15 Areas of Further Research . . . 245
15.1 Error-Functional Approach and Multiple Updating Schemes . . . 245
15.1.1 Error Functionals that Generate the Choquet Integral . . . 246
15.1.2 Multiplicity of Updating Schemes . . . 247
15.1.3 Updating Probability Capacity Revisited . . . 248
15.1.4 General Updating Rule à la Denneberg . . . 249
15.2 Two Distinct Types of Learning . . . 250
15.2.1 Learning to Elaborate One's Belief . . . 250
15.2.2 Learning to "Chop" the State Space . . . 252
15.3 Other Ongoing Research . . . 252
References . . . 254

Appendix: Proofs of Lemmas and Theorems . . . 255

Index . . . 323

http://www.springer.com/978-4-431-55901-6

Economics of Pessimism and Optimism
Theory of Knightian Uncertainty and Its Applications
Nishimura, K.G.; Ozaki, H.
2017, XX, 326 p. 12 illus. in color., Hardcover
ISBN: 978-4-431-55901-6