

Contents

1	Quantum Field Theory (QFT) on the Lattice	1
1.1	A Brief History of Quarks and Gluons	1
1.2	Classical Fields and Gauge Invariance	2
1.3	Hamiltonian and Path Integral Formulations of Quantum Mechanics	3
1.3.1	One Bosonic Oscillator	4
1.3.2	One Fermionic Oscillator	6
1.4	Quantum Fields on a Lattice	7
1.4.1	From One Oscillator to a Field	7
1.4.2	Correlation Functions	10
1.4.3	Analytic Continuation to Minkowski Space	11
1.4.4	Gauge Invariance on the Lattice	13
1.4.5	The Wilson Gauge Action	14
1.4.6	Strong Coupling Expansions	15
1.4.7	The Wilson Fermion Action	17
1.4.8	Chiral Symmetry and the Nielsen–Ninomiya No-Go Theorem	18
1.4.9	Other Fermion Formulations	21
1.5	Recovering Continuum QCD	23
1.5.1	Classical Continuum Limit	24
1.5.2	Renormalisation Group Equation	25
1.5.3	Isospin Symmetry and Ward Identities	27
1.5.4	Improvement of the Continuum Limit	28
1.5.5	Improvement of Wilson Fermions	30
1.5.6	Twisted Mass Fermions	31
1.6	Further Reading	32
	References	33
2	Monte Carlo Methods	35
2.1	Markov Chain Monte Carlo	35
2.2	Sampling Yang–Mills Gauge Fields	37

2.2.1	Random Numbers and the Rejection Method	38
2.2.2	Heatbath Algorithms	39
2.2.3	Overrelaxation Algorithms	42
2.3	Hybrid Monte Carlo	43
2.3.1	Detailed Balance Condition.	44
2.3.2	Hamilton's Equation of Motion.	46
2.4	Symplectic Integration Schemes	48
2.5	Summary and Further Reading	52
	References.	53
3	Handling Fermions on the Lattice	55
3.1	Wick Contractions	55
3.2	Sparse Linear Algebra	58
3.2.1	Krylov Subspace Techniques	59
3.2.2	Iterative Solvers for Linear Systems	61
3.2.3	Algebraic Multigrid Methods	67
3.2.4	Other Uses of Krylov Subspaces.	68
3.3	Fermion Determinant	73
3.3.1	Pseudofermions.	73
3.3.2	Factorisations	75
3.4	HMC with Fermions Revisited.	77
3.4.1	Equations of Motion	77
3.4.2	Multi-rate Integration Schemes	80
3.4.3	A Single Dynamical Quark Flavour: The RHMC Algorithm	81
3.5	The Quark Propagator from a Point Source	84
3.5.1	Quark Observables from a Single Point Source	84
3.5.2	Reducing Variance in Point Propagator Calculations	86
3.6	All-to-All Quark Propagators	87
3.6.1	Stochastic Estimators.	87
3.6.2	Exploiting Low Eigenmodes of the Dirac Operator.	92
3.7	Summary and Further Reading	93
	References.	94
4	Calculating Observables of Quantum Fields	97
4.1	Symmetry Properties of Creation and Annihilation Operators.	97
4.1.1	Gauge-Invariant Observables.	98
4.1.2	Charge Conjugation, Isopin and Flavour Symmetry	98
4.1.3	Spin and Parity	100
4.1.4	Translational Invariance and Momentum	102
4.1.5	Reducing Representations of Symmetries	102
4.2	Techniques for Hadron Spectroscopy	103
4.2.1	Variational Methods	104
4.2.2	Scale Setting	105
4.2.3	The Wilson Flow	106

4.2.4	Scattering and the Lüscher Method	108
4.3	Gluons, Wilson Loops and Glueballs	111
4.3.1	Gauge Smearing	111
4.3.2	Glueballs	113
4.3.3	The Static Potential and Strong Coupling Constant.	114
4.3.4	Topological Charge	117
4.4	Quarks and Hadron Physics	118
4.4.1	Quark Smearing	119
4.4.2	Hadron Physics with Quarks	120
4.4.3	Hadron Scattering in Lattice Monte Carlo Calculations.	123
4.4.4	The Static Potential with Light Quarks: String Breaking	125
4.5	Statistical Data Analysis	126
4.5.1	Controlling Bias, Covariance and Autocorrelations in Data from a Markov Chain	127
4.5.2	Comparing Monte Carlo Data to a Model	130
4.6	Summary	131
	References	131
	Appendix A: Notational Conventions	135

Lattice Quantum Chromodynamics

Practical Essentials

Knechtli, F.; Günther, M.; Peardon, M.

2017, XIII, 140 p. 12 illus., 1 illus. in color., Softcover

ISBN: 978-94-024-0997-0