Chapter 2
Some Continuous Distributions

In this chapter several basic properties of some useful univariate distributions will be discussed. These properties will be useful for our characterization problems.

2.1 Beta Distribution

A random variable X is said to have a BE(m, n) distribution if its pdf $f_{m,\,n}(x)$ is of the following form.

$$f_{m\cdot n}(X) = \frac{1}{B(m,n)} x^{m-1}(1-x)^{n-1}, 0 < x < 1, m > 0, n > 0. \tag{2.1.1}$$

Mean $= \frac{m}{m+n}$ and variance $= \frac{mn}{(m+n)^2(m+n+1)}$.
The moment generating function $M_{m,\,n}(t)$ is
$M_{m,\,n}(t) = F(m, m + n, t)$, where

$$F(a, b, x) = 1 + \frac{a}{b}x + \frac{a(a+1)}{b(b+1)}\frac{x^2}{2!} + \cdots$$

The characteristic function $\phi_{m.n}(t) = F(m, m+n, it)$
The pdfs of BE(3, 3), BE(4, 6) and BE(4, 9) are given in Fig. 2.1.
If m = 1/2 and n = 1/2, then BE(1/2, 1.2) is the arcsine distribution.
If X is distributed as BE(m, n), then 1-x is distributed as BE(n, m).

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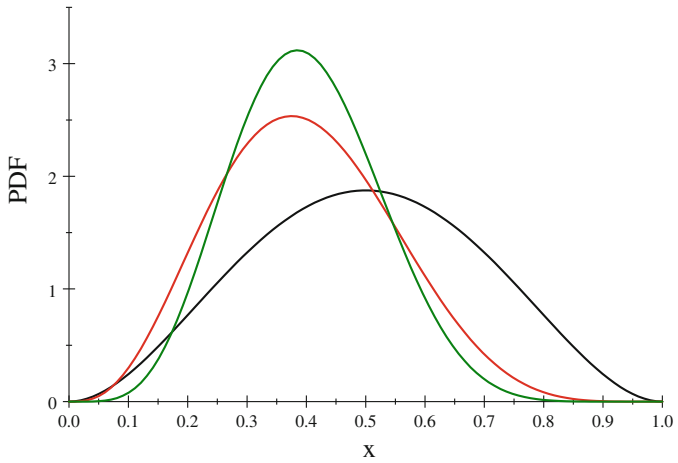


Fig. 2.1 BE(3, 3) *Black*, BE(4, 6) *Red* and BE(6, 9) *Green*

2.2 Cauchy Distribution

A random variable X is said to have a Cauchy $(\mathrm{CA}(\mu, \sigma))$ distribution with location parameter μ and scale parameter σ if the pdf $(\mathrm{f_c}(\mathrm{x}, \mu, \sigma))$ is of the following form.

$$\mathrm{f_c}(\mathrm{x}, \mu, \sigma) = \frac{1}{\pi\sigma\left(1 + \left(\frac{x-\mu}{\sigma}\right)^2\right)}, \; -\infty < x < \mu < \infty, \sigma > 0. \tag{2.1.2}$$

The Fig. 2.2 gives the pdfs of CA(0, 1), CA(0, 2) and CA(0, 5).

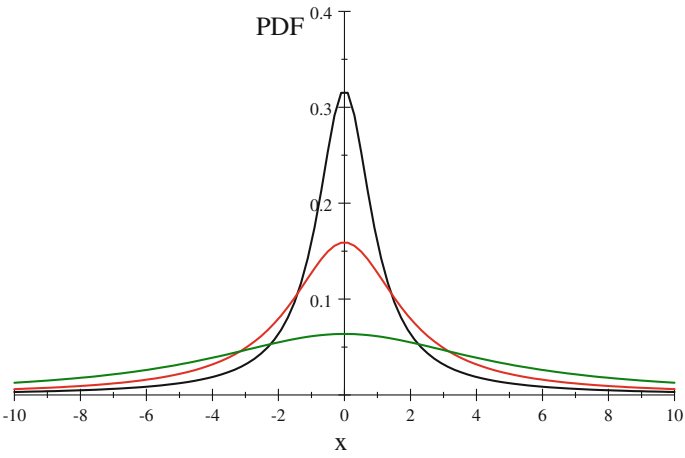


Fig. 2.2 CA(0, 1) *Black*, CA(0, 2) *Red* and CA(0, 5) *Green*

The mean of $CA(\mu, \sigma)$ does not exist. The median and the mode are equal to μ. The cdf $F_c(x, \mu, \sigma)$ is

$$F_c(x, \mu, \sigma) = \frac{1}{2} + tan^{-1}(\frac{x-\mu}{\sigma}) \tag{2.1.3}$$

If $X_1, X_2, \ldots, X_n$ are n independent $CA(\mu, \sigma)$, then $S_n = X_1 + X_2 + \cdots + X_n$ is distributed as $CA(n\mu, n\sigma)$..

If X_1 and X_2 are distributed as normal with mean = 0 and variance = 1, the X/Y is distributed as CA(0, 1).

If X is CA(0, 1), then $2X/(1-X^2)$ is distributed as CA(0, 1).

The pdf $f_{gc}(x, \mu, \sigma)$ of generalized Cauchy $(GCA(\mu, \sigma))$ is given by

$$f_{gc}(x, \mu, \sigma) = \frac{\Gamma(n)}{\sigma\sqrt{\pi}\Gamma(n-\frac{1}{2})} \frac{1}{(1+\left(\frac{x-\mu}{\sigma}\right)^2)^n}, n \geq 1, -\infty < \mu < x < \infty, \sigma > 0. \tag{2.1.4}$$

For n = 1, the mean does not exist.

For n > 1, the mean = μ, the median = μ and the odd moments are zero.

For n > 1,

$$E(X^m) = \frac{\Gamma(\frac{m+1}{2})\Gamma(n-\frac{m+1}{2})}{\Gamma(\frac{1}{m})\Gamma(n-\frac{1}{m})} \text{for m even, } m < 2n-1, m > 1.$$

2.3 Chi-Squared Distribution

A random variable X is said to have a Chi-squared $(CH(\mu, \sigma, n))$ distribution with location parameter μ and scale parameter σ if the pdf $(f_{ch}(x, \mu, \sigma, n))$ is of the following form.

$$f_{ch}(x, \mu, \sigma, n) = \frac{1}{2^{n/2}\Gamma(\frac{n}{2})} (\frac{x-\mu}{\sigma})^{\frac{n}{2}-1} e^{-(\frac{x-\mu}{2\sigma})}, n > 1, -\infty < \mu < x < \infty, \sigma > 0.$$

The parameter n is known as degrees of freedom.

For n > 1, Mean $= \mu + n\sigma$, *and variance* $= 2n\sigma^2$.

The moment generating function M_{CH} (t) is

$$M_{CH}(t) = e^{\mu t}(1-2\sigma t)^{-n/2}, t < \frac{1}{2\sigma}.$$

The Fig. 2.3 gives the pdfs of The CH(0, 1, 4), CH(0, 1, 10) and CH(0, 1, 20).

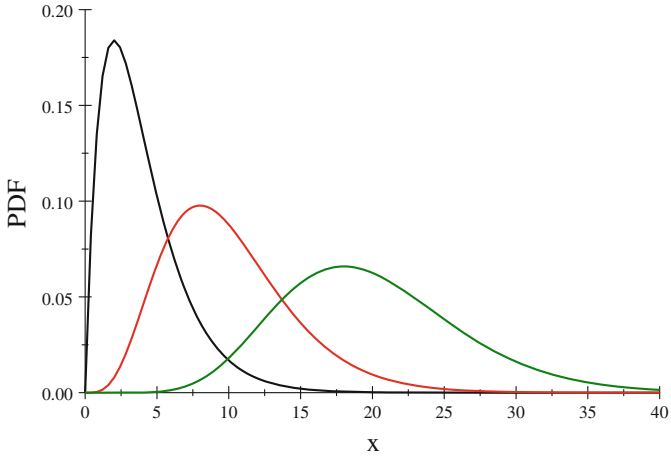


Fig. 2.3 The CH(0, 1, 4)-Black, CH(0, 1, 10)-red and CH(0, 1, 20)-green

If X_i, i = 1, 2, …, n are n independent CH(0, 1, n_i). i = 1, 2, …, n, random variables then $S_k = X_1 + X_2 + \cdots + X_k$, then S_k is distributed as CH(0, 1, m), where $m = n_1 + n_2 + \cdots + n_K$. If X is standard normal (N(0, 1)), then X^2 is distributed as CH(0, 1, 1).

2.4 Exponential Distribution

A random variable X is said to have a exponential $(E(\mu, \sigma))$ distribution with location parameter μ and scale parameter σ if the pdf $(f_e(x, \mu, \sigma))$ is of the following form.

$$f_e(x, \mu, \sigma) = \frac{1}{\sigma} e^{-\left(\frac{x-\mu}{\sigma}\right)}, \ -\infty < \mu < x < \infty.$$

The exponential distribution E(0, 1) is known as standard exponential.
The Fig. 2.4 gives the pdfs of E(0, 1), E(0, 2) and E(0, 5).
The cdf $F_e(x, \mu, \sigma)$ is given by

$$F_e(x, \mu, \sigma) = 1 - e^{-\left(\frac{x-\mu}{\sigma}\right)}, \ -\infty < \mu < x < \infty.$$

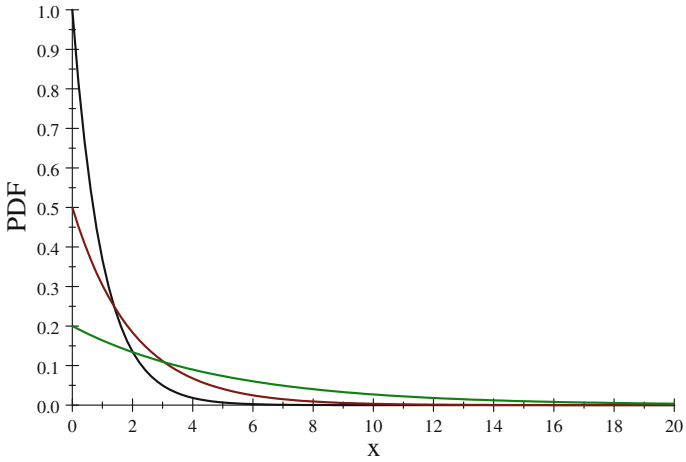


Fig. 2.4 E(0, 1) *Black*, E(0, 2) *Red* and E(0, 5) *Green*

The moment generating function $M_{ex}(t)$

$$M_{ex}(t) = (1 - \sigma t)^{-1} e^{-\mu t}$$

Mean $= \mu + \sigma$ and Variance $= \sigma^2$.

If X_i, i = 1, 2, …, n are i.i.d. exponential with $F(x) = 1 - e^{-x/\sigma}, x \geq 0, \sigma > 0$, and $S(n) = X_1 + X_2 + \ldots + X_n$, then pdf $f_{S(n)}(x)$ of S(n) is

$$f_{S(n)}(x) = \frac{1}{\sigma} e^{-x/\sigma} \frac{(x/\sigma)^{n-1}}{\Gamma(n)}, x \geq 0, \sigma > 0.$$

This is a gamma distribution with parameters n and σ.

If X_1 and X_2 are independent exponential random variables with scale parameters σ_1 and σ_2, then $P(X_1 < X_2) = \frac{\sigma_2}{\sigma_1 + \sigma_2}$.

If X_i, i = 1, 2, …, n are n independent exponential random variable with $F(x) = 1 - e^{-x/\sigma}, x \geq 0, \sigma > 0$. Let m(n) = min $\{X_1, \ldots, X_n\}$ and M(n) = max$\{X_1, \ldots, X_n\}$, $F_{(m)}$ be the cdf of m(n) and $F_{(M)}$ be the cdf of M(n), then

$$\begin{aligned} 1 - F_{(m)}(x) &= P(X_1 > x, X_2 > x, \ldots X_n > x) \\ &= e^{-nx/\sigma} \end{aligned}$$

$$F_{(M)}(x) = P(X_1 < x. X_2 < x, \ldots, X_n < x) = (1 - e^{-x/\sigma})^n.$$

Memoryless Property. P(X > s+t|X > t) P(X > s).

$$\begin{aligned} \mathrm{P}(\mathrm{X}>\mathrm{s}+\mathrm{t}|\mathrm{X}>\mathrm{t}) &= \frac{P(X>s+t, X>t}{P(X>t)}) \\ &= \frac{e^{-(s+t-2\mu)/\sigma}}{e^{-(t-\mu)/\sigma}} \\ &= e^{-(s-\mu)/\sigma} \\ &= P(X>s) \end{aligned}$$

2.5 F-Distribution

A random variable X is said to have F distribution F(m, n) with numerator degree of degrees of freedom m and numerator degrees of freedom n if its pdf f_F (x, m, n) is given by

$$\mathrm{f_F}(\mathrm{x},\mathrm{m},\mathrm{n}) = \frac{\Gamma(\frac{m+n}{2})(\frac{m}{n})^{\frac{m}{2}}x^{(m-2)/2}}{\Gamma(\frac{m}{2})\Gamma(\frac{n}{2})(1+\frac{mx}{n})^{(m+n)/2}}, \mathrm{x}>0, \mathrm{m}\ >0, \mathrm{n}>0.$$

The cdf F_F(x, m, m) is given by

$$\mathrm{F_F}(\mathrm{x},\mathrm{m},\mathrm{n}) = I_{\frac{mx}{m+n}}(\frac{m}{2},\frac{n}{2}),$$

where $= I_x(a,b) = \int_0^x u^{a-1}(1-u)^{b-1}du$ is the incomplete beta function.

The Fig. 2.5 gives the pdfs of F(5, 5), F(10, 1)) and F(10, 20).

Mean $= \frac{n}{n-2}$, n > 2 and variance $= \frac{2n^2(m+n-2)}{m(m-2)^2(n-4)}$, n > 4.

The characteristic function $\phi_F(\mathrm{m}\cdot\mathrm{n}) = \frac{\Gamma(\frac{m+n}{2})}{\Gamma(\frac{n}{2})}\mathrm{U}(\frac{m}{2}, 1-\frac{n}{2}, -\frac{m}{n}it)$, where U(a, b, x) is the confluent hypergeometric function of the second kind.

If U_1 and U_2 are independently distributed as chi-squared distribution with m and n degrees of freedom, then X = (n/m) (U_1/U_2) is distributed as F with cdf F_F (m, n).

If X is distributed as Beta (m/2, n/2), then $\frac{nX}{m(1-X)}$ is distributed as F with cdf F_F (m, n).

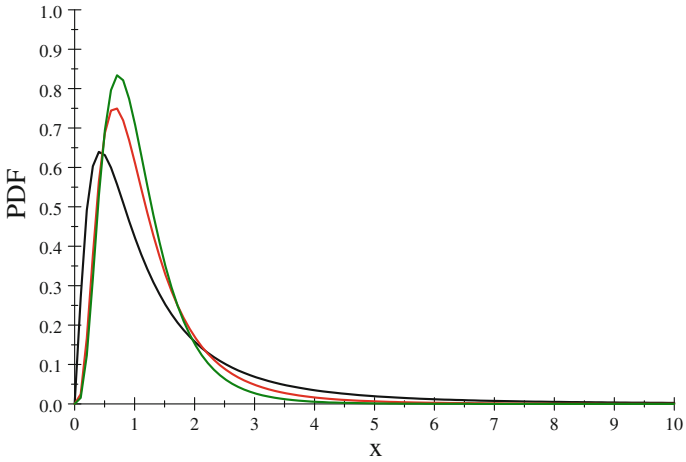


Fig. 2.5 F(5, 5) *Black*, F(10, 10) *Red* and F(10, 20) *Green*

2.6 Gamma Distribution

A random variable X is said to have gamma distribution GA (a, b) if its pdf $f_{ga}(a, b, x)$ is of the following form.

$$f_{ga}(a,b,x) = \frac{1}{\Gamma(a)b^a} x^{a-1} e^{-x/b}, \mathrm{x} \geq 0, a > 0, b > 0.$$

Mean $= ab$ and variance $= ab^2$.

The Fig. 2.6 give the pdfs of GA(2, 1), GA(5, 1) and GA(10, 1).

The moment generating function M(t) is

$$\mathrm{M(t)} = (1 - bt)^{-a}, \mathrm{t} < 1/\mathrm{b}.$$

The characteristic function $\phi_{ga}(t) is\ \phi_{ga}(t) = (1 - ibt)^{-a}$.

If a = 1, b = 1 then we GA(1, 1) is an exponential distribution and if a is a positive integer, then GA(a, b) is an Erlang distribution.

If b = 1, then we call GA (a, b) as the standard gamma distribution.

If X_1 and X_2 are independent gamma random variables then the random variables $X_1 + X_2$ and $\frac{X_1}{X_1+X_2}$ are mutually independent.

If X_1, X_2, .., X_n are n independently distributed as GA(a, b), then $S(n) = X_1 + X_2 + \cdots + X_n$ is distributed as GA(na, b).

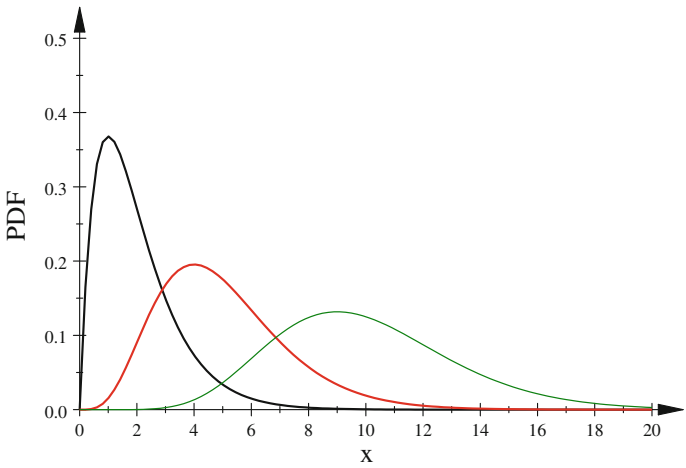


Fig. 2.6 GA(2, 1) *Black*, GA(5, 1) *Red* and GA(10, 1) *Green*

2.7 Gumbel Distribution

A random variable X is said to have Gumbel $(\mathrm{GU}(\mu, \sigma))$ distribution with location parameter μ and scale parameter σ if its pdf, $\mathrm{f_{ig}}\ (\mathrm{x}, \mu, \sigma)$ is of the following form

$$\mathrm{f_{gu}}(\mathrm{x}, \mu, \sigma) = \frac{1}{\sigma} e^{-\frac{x-\mu}{\sigma}} e^{-e^{-\frac{x-\mu}{\sigma}}},\ -\infty < \mu < x < \infty, \sigma > 0.$$

Gumbel distribution is also known as Type I extreme (maximum) value distribution. The cdf $\mathrm{F_{gu}}(\mathrm{x}, \mu, \sigma)$ is of the following form

$$\mathrm{F_{gu}}(\mathrm{x}, \mu, \sigma) = e^{-e^{-\frac{x-\mu}{\sigma}}},\ -\infty < \mu < x < \infty, \sigma > 0.$$

Mean $= \mu + \gamma\sigma$, where γ is Euler's constant.
Median $= \mu - \ln(ln2)\sigma$.
Variance $= \frac{\pi^2\sigma^2}{6}$.
The Fig. 2.7 gives the pdfs of GU(0, 1/2), GU(0, 1) and GU(0, 2).
If X is distributed as E(0, 1), then $\mu - \sigma lnX$ is distributed as $\mathrm{GU}(\mu, \sigma))$.
If X is distributed as GU(0, 1), then $Y = e^{-X}$ is distributed as E(0, 1).

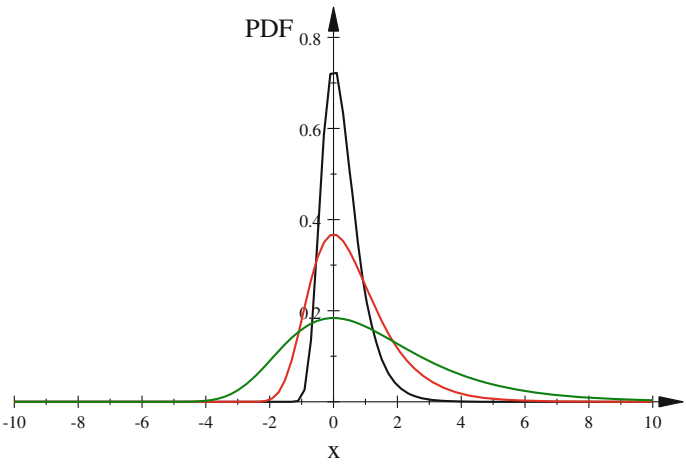


Fig. 2.7 GU(0, 1/2) *Black*, GU(0, 1) *Red* and GU(0, 2) *Green*

2.8 Inverse Gaussian (Wald) Distribution

A random variable X is said to have Inverse Gaussian $(\mathrm{IG}(\mu, \sigma))$ distribution with parameters μ and λ if its pdf

$f_{ig}(x, \mu, \lambda)$ is of the following form

$$f_{ig}(x, \mu, \lambda) = \left(\frac{\lambda}{2\lambda x^3}\right)^{\frac{1}{2}} e^{-\frac{\lambda}{2x}\left(\frac{x-\mu}{\mu}\right)^2}, 0 < \mu < x < \infty, \lambda > 0.$$

Mean $= \mu$ and variance $= \frac{\mu^3}{\lambda}$.

The Fig. 2.8 gives the pdfs of IG(1, 10), IG(1, 2) ans IG(1, 3).

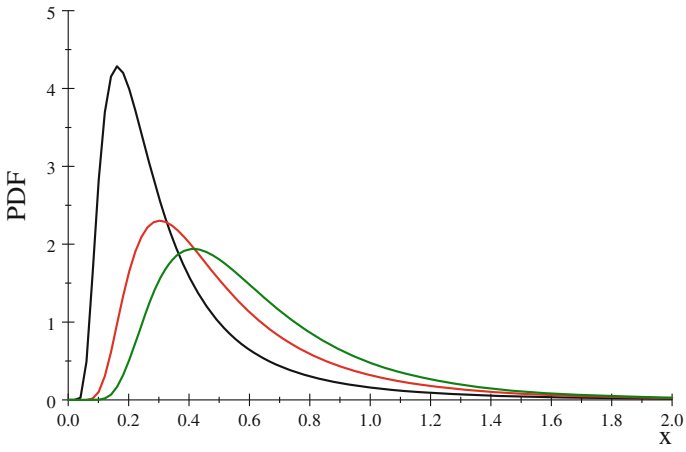


Fig. 2.8 IG(1, 1) *Black*, IG(1, 2) *Red* and IG(1, 3) *Green*

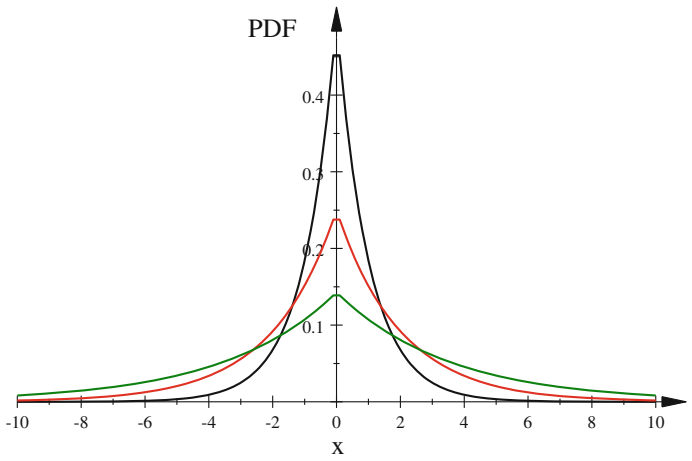


Fig. 2.9 LP(0, 1) *Black*, LP(0, 2) *Red* and LP(0, 3.5) *Green*

If X is distributed as IG (μ, λ) then αX is distributed as IG $(\alpha\mu.\alpha\lambda)$.
If X is distributed as IGv $(1, \lambda)$, then X is known as Wald distribution.

2.9 Laplace Distribution

A random variable X is said to have Laplace $(LP(\mu, \sigma))$ distribution with location parameters μ and scale parameter λ if its pdf

$f_{lp}(x, \mu, \lambda)$ is of the following form

$$f_{lp}(x, \mu, \lambda) = \frac{1}{2\sigma} e^{-\left|\frac{x-\mu}{\sigma}\right|}, \infty < x < \mu < \infty, \sigma > 0.$$

Mean $= \mu$ and variance $= 2\sigma^2$.
The Fig. 2.9 gives the pdfs of LP(0, 1), LP(0, 2) and LP(0, 3.5).
The moment generating function is $M_{lp}(t)$ is

$$M_{lp}(t) = \frac{e^{\mu t}}{1 - \sigma^2 t^2}.$$

The characteristic function $\phi_{lp}(t)$ is

$$\phi_{lp}(t) = \frac{e^{i\mu t}}{1+\sigma^2 t^2}.$$

If X and Y are independent E(0, 1), then X-Y is LP(0, 1).
If X is LP (μ, σ), then kX is LP $(k\mu, k\sigma)$.
If X is LP(0, 1), then |X| is E(0, 1).

2.10 Logistic Distribution

A random variable X is said to have Logistic $(\mathrm{LG}(\mu, \sigma))$ distribution with location parameters μ and scale parameter λ if its pdf

$f_{lg}(x, \mu, \sigma)$ is of the following form

$$\mathrm{f_{lg}}(x, \mu, \sigma) = \frac{1}{\sigma}\frac{e^{-\frac{x-\mu}{\sigma}}}{(1+e^{-\frac{x-\mu}{\sigma}})^2}, \; -\infty < \mu < x < \infty, \sigma > 0.$$

Mean = μ and variance = $\frac{\pi^2\sigma^2}{3}$.
Moment generating function $\mathrm{M_{lg\ (t)}}$ is

$$\mathrm{M_{lg}(t)} = e^{\mu t}\Gamma(1+\sigma t)\Gamma(1-\sigma t), t < \frac{1}{\sigma}.$$

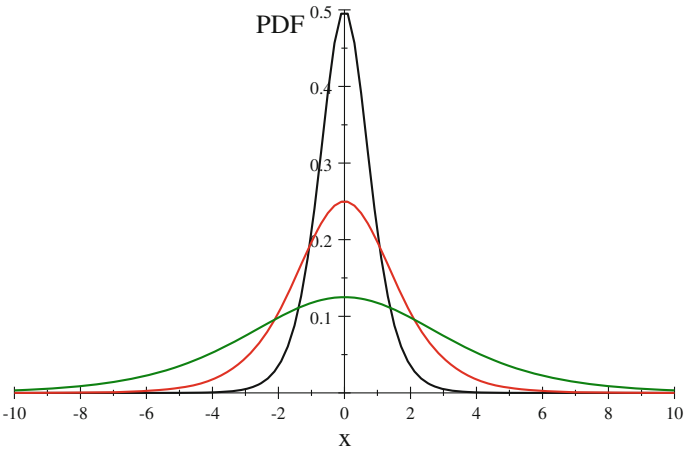


Fig. 2.10 LG(0, 1/2) *Black*, LG(0, 1) *Red* and LG(0, 2) *Green*

The characteristic function $\phi_{\lg}(t) is$

$$\phi_{\lg}(t) = e^{i\mu t}\Gamma(1+i\sigma t)\Gamma(1-i\sigma t).$$

The Fig. 2.10 gives the pdfs of LG(0, 1/2), LG(0, 1) and LB(0, 2).

Let $X_{i,}$ i = 1, 2, …, n are independent and identically distributed as LP(0, 1), then $Y = X_1X_2\ldots X_n$ is distributed as LG(0, 1).

If X and Y are independent GU (μ, σ), *then* $X - Y$ is LG(0, 1).

If X is LG$((\mu, \sigma)$ then kX is LG $(\mu k, k\sigma)$.

If X and Y are independent and E(0, 1), then $\mu - \sigma \ln(\frac{X}{Y})$ is LG (μ, σ).

2.11 Lognormal Distribution

A random variable X is said to have Lognormal $(LN(\mu, \sigma))$ distribution with location parameters μ and scale parameter σ if its pdf

$f_{ln}(x, \mu, \sigma)$ is of the following form

$$f_{ln}(x, \mu, \sigma) = \frac{1}{x\sigma\sqrt{(2\pi)}} e^{-\frac{1}{2}(\frac{\ln(x-\mu)}{\sigma})^2}, x > 0, > 0, \mu > 0, \sigma > 0.$$

Mean $= e^{\mu + \frac{\sigma^2}{2}}$

Variance $= (e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$.

Moment generating function $M_{ln}(t)$ is

$$M_{ln}(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} e^{n\mu + \frac{n^2\sigma^2}{2}}.$$

The characteristic function $\phi_{ln}(t) is$

$$\phi_{ln}(t) = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} e^{n\mu + \frac{n^2\sigma^2}{2}}$$

The Fig. 2.11 gives the pdfs of LN(0, 1/2), LN(0, 1) and LN(0, 2).

If X is distributed as normal with location parameter μ and scale parameter σ, *then* e^X is distributed as LN (μ, σ).

If X is distributed as LN (μ, σ), *then* $ln\, X$ *is distributed as* normal with location parameter μ and scale parameter σ.

If X_i, i = 1, 2, …, n are independent and identically distributed as LN (μ, σ), *then* $Y = X_1X_2\ldots X_n$ is distributed as LN $(n\mu, \sigma\sqrt{n})$.

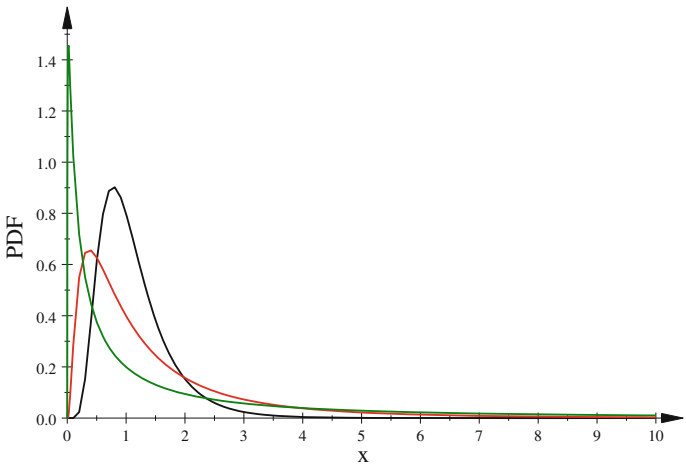


Fig. 2.11 LN(0, 1/2) *Black*, LN(0, 1) *Red* and LN(0, 2) *Green*

2.12 Normal Distribution

A random variable X is said to have normal $(N(\mu, \sigma))$ distribution with location parameters μ and scale parameter σ if its pdf

$f_n(x, \mu, \sigma)$ is of the following form

$$f_n(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \ -\infty < \mu < x < \infty, \sigma > 0.$$

$$Mean = \mu \ and \ variance = \sigma^2.$$

The moment generating function $M_n(t)$ *is*

$$M_n(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}.$$

The characteristic function $\phi_n(t)$ is

$$\phi_n(t) = e^{i\mu t - \frac{\sigma^2 t^2}{2}}.$$

The Fig. 2.12 gives the pdfs of N(0, 1/2), N(0, 1) and N(0, 2).

If X_i is N (μ_i, σ_i), i = 1, 2, …, n and X_i's are independent, then for any α_i, i = 1, 2, …, n, $\sum_{i=1}^{n} \alpha_i X_i$ is N($\sum_{i=1}^{n} \alpha_i \mu_1$, $\sqrt{(\sum_{i=1}^{n} \alpha_i^2 \sigma_i^2)}$).

If X is normal and $X = X_1 + X_2$, where X_1 and X_2 are independent, then both X_1 and X_2 are normal.

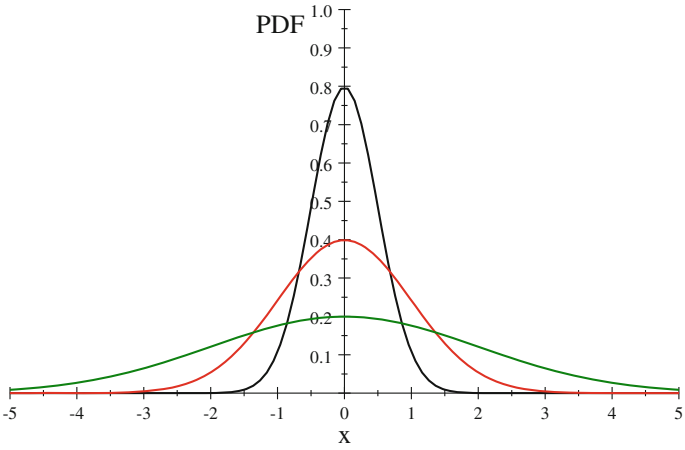


Fig. 2.12 N(0, 0.5) *Black*, N(0, 1) *Red* and N(0, 2) *Green*

2.13 Pareto Distribution

A random variable X is said to have Pareto (PA(α, β)) distribution with parameters α and β if its pdf $f_{pa}(x, \alpha, \beta)$ is of the following form

$$f_{pa}(x, \alpha, \beta) = \frac{\beta \alpha^{\beta}}{x^{\beta+1}}, x > \alpha > 0, \beta > 0.$$

Mean $= \frac{\alpha\beta}{\beta-1}, \beta > 1$ and variance $= \frac{\alpha^2\beta}{(\beta-1)^2(\beta-2)}. \beta > 2.$
The characteristic function $\phi_{pa}(t)$ is given by

$$\phi_{pa}(t) = \beta(-i\alpha t)^{\beta}\Gamma(-\beta, -i\alpha t).$$

The Fig. 2.13 gives the pdfs of PA(1, 1/2), PA(1, 1) and PA(1, 20).
If $X_1, X_2, \ldots, X_n$ are n independent PA(α, β), then

$2\beta \ln(\frac{\prod_{i=1}^{n} X_i}{\alpha^n})$ is distributed as CH(0, 1, n).

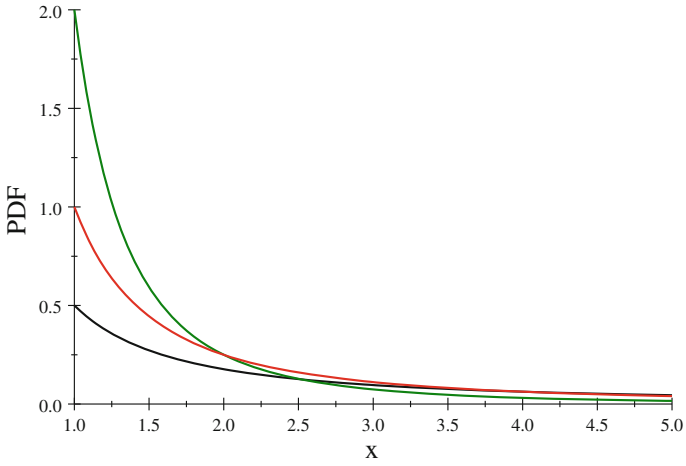


Fig. 2.13 PA(0, 1/2) *Black*, PA(1, 1) *Red* and PA(1, 2) *Green*

2.14 Power Function Distribution

A random variable X is said to have power function (Po(α, β, δ)) if its pdf $f_{po}(\alpha, \beta, \delta)$ is of the following form

$$f_{po}(\alpha,\beta,\delta) = \frac{\delta}{\beta-\alpha}\left(\frac{x-\alpha}{\beta-\alpha}\right)^{\delta-1},\ -\infty<\alpha<x<\beta<\infty,\delta>0$$

Mean $= \alpha + \frac{\delta}{\delta+1}(\beta-\alpha)$ and variance $= \frac{\delta(\beta-\alpha)^2}{(\delta+1)^2(\delta+2)}$.

The Fig. 2.14 gives the pdfs of Po(0, 1, 3), Po(0, 1, 4) and Po(0, 1, 4).

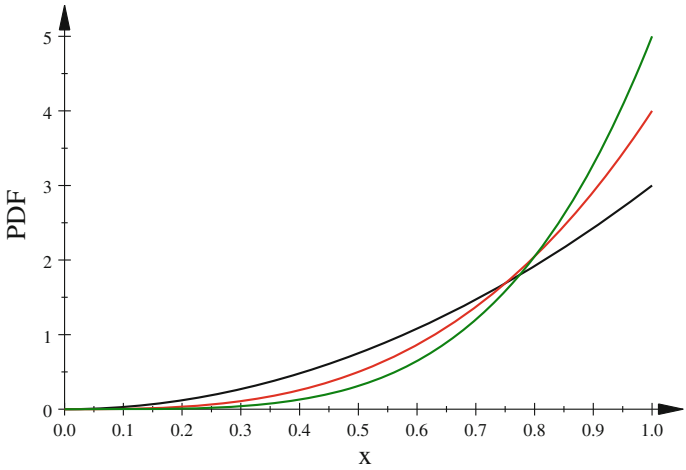


Fig. 2.14 Po(0, 1, 3) *black*, Po(0, 1, 4) *red* and Po(0, 1, 4) *green*

If $\delta = 1, then\ PO(\alpha, \beta, 1)$ becomes a uniform (U(α, β)) with pdf f_{un}(x, α, β) as

$$F_{un}(x, \alpha, \beta) = \frac{1}{\beta - \alpha}, -\infty < \alpha < \beta < \infty.$$

2.15 Rayleigh Distribution

A random variable X is said to have Rayleigh (RA(μ, σ)) with location parameter μ and scale parameter σ if its pdf f_{ra}(x, μ, σ) is of the following form

$$f_{ra}(x, \mu, \sigma) = \frac{x - \mu}{\sigma^2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < \mu < x < \infty, \sigma > 0.$$

Mean $= \mu + \sigma\sqrt{\frac{\pi}{2}}$ and variance $= \frac{4-\pi}{2}\sigma^2$;
Moment generating function M_{ra}(t) is

$$M_{ra}t = e^{\mu t}\left(1 + \sigma t e^{-\frac{\sigma^2 t^2}{2}}\sqrt{\frac{\pi}{2}}\left(\text{erf}\left(\frac{\sigma t}{\sqrt{2}}\right) + 1\right)\right)$$

The Fig. 2.15 gives the pdfs of RA(0, 1/2), RA(0, 1) and RA(0, 2).

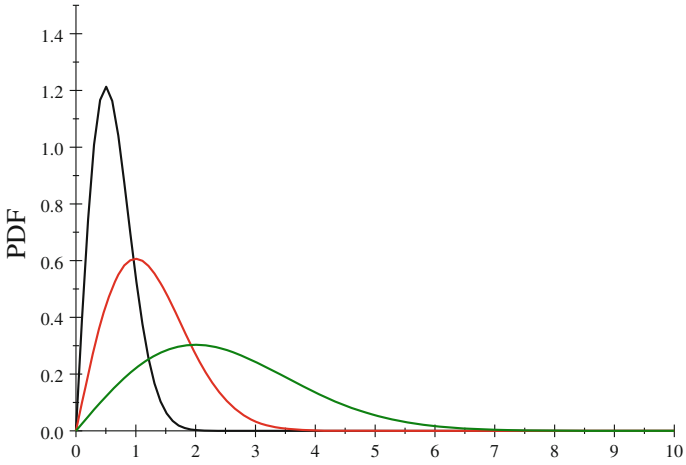


Fig. 2.15 RA(0, 1/2) *Black*, RA (0, 1) *Red* and RA(0, 2) *Green*

2.16 Student's t-Distribution

A random variable X is said to have Students t-distribution ST(n)) with n degrees of freedom, if its pdf $f_{st}\ (x,\ n)$ is as follows.

$$\mathrm{f_{st}(x,n)} = \frac{1}{\sqrt{n}}\frac{1}{B(n/2,1/2)}(1+\frac{x^2}{n})^{-(n+1)/2},\ -\infty < t < \infty, n \geq 1.$$

Mean = 0 if n > 1 and is not defined for n = 1.
Variance $= \frac{n}{n-2}, n > 2$.
The Fig. 2.16 gives the pdf of ST(!), ST(4) and ST(16) (Fig. 2.16).

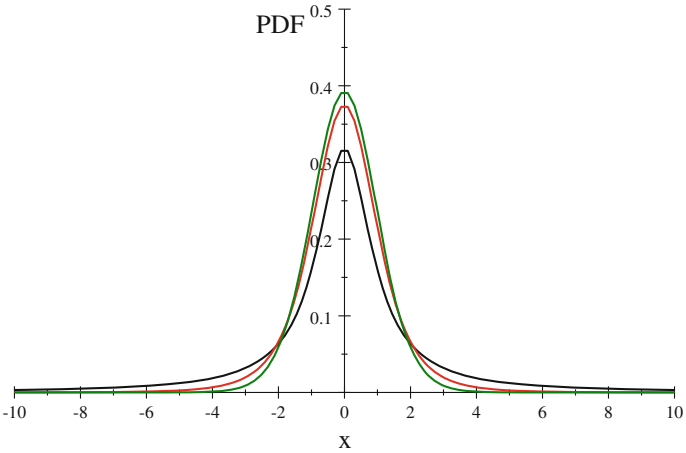


Fig. 2.16 ST(1) *Black*, ST(4) *Red* and ST(16) *Green*

2.17 Weibull Distribution

A random variable is said to have Weibull WB(x, μ, σ, δ) with location parameter μ, scale parameter σ and shape parameter δ if its pdf $\mathrm{f_{Wb}}$(x, μ, σ, δ) is of the following form.

$$\mathrm{f_{Wb}(x,\mu,\sigma,\delta)} = \frac{\delta}{\sigma}(\frac{x-\mu}{\sigma})^{\delta-1}e^{-(\frac{x-\mu}{\sigma})_{\delta}},\ -\infty < \mu < x < \infty, \sigma > 0, \delta > 0.$$

Mean $= \mu + \Gamma(1+\frac{1}{\delta})$ and variance $= \sigma^2((\Gamma 1+\frac{2}{\delta}) - (\Gamma(1+\frac{1}{\delta}))^2)$.

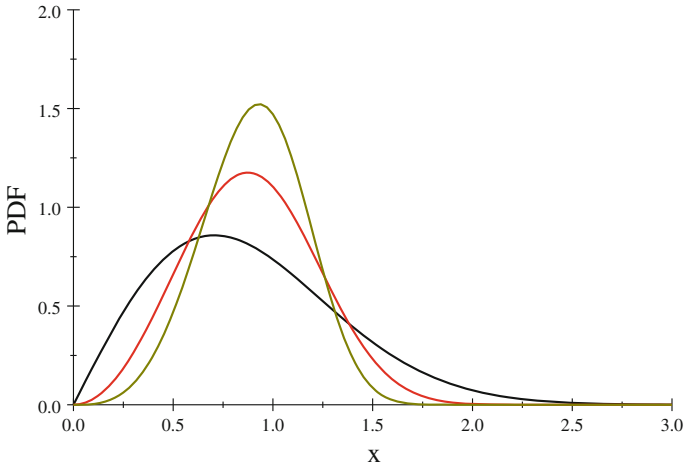


Fig. 2.17 WB(0, 1, 1) *Black*, WB(0, 1, 3) *red* and WB(0, 1, 4) *green*

The Fig. 2.17 gives the pdfs of B(0, 1, 1), WB(0, 1, 3) and B(0, 1, 4).

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