

## Chapter 2

# Strange Behaviour at Quantum Scale: The Full List

### 2.1 Introduction

In the preceding chapter a few examples already have been given to illustrate the strange behaviour of particles at atomic scale. It was announced that the full list of incomprehensible behaviour is much longer, and it is the task of the present chapter to sum it up completely. The proviso is, that we are talking only about a simple *oscillator*. The free electron, the electrons in atoms, or the double slit experiment mentioned in the introduction fall outside the scope of this chapter. Although a first understanding of the jumping up and down and back and forth of real electrons inside atoms may be obtained by studying the simple oscillator, it must always be kept in mind that the model of a simple oscillator only provides a *first*—although important—understanding, and is not the complete story. Nevertheless, the simple oscillator is so enlightening that all study books, both on classical mechanics as well as on quantum physics, start by considering and analysing it.

In order to make it a systematic story, we go back to the classical oscillator (such as the children's swing), and consider it in somewhat more detail than in the preceding chapter.

### 2.2 The Story of the Oscillator as Told in Classical Textbooks

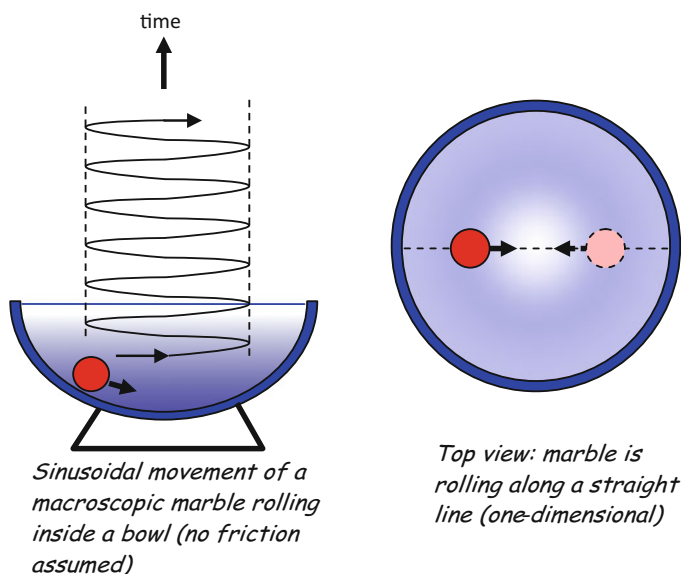
Imagine a bowl in which a marble is rolling back and forth. To keep it simple, invariably the situation is first assumed to be *one-dimensional*, i.e. looking on top of the bowl one sees the marble rolling in a straight line, going down from one side of the bowl through the lowest point in the centre, and up again to reach a position on the other wall diametrically opposed to its starting point. After this the motion is reversed and the marble rolls back along the same straight line (seen from above),

etc. Often one assumes furthermore that the excursions of the marble are not too large (the so-called “linearised one-dimensional oscillator”). That however is an assumption which is purely intended to simplify the mathematical analysis, and it does not concern us in this qualitative chapter. One could ask why this is called “one-dimensional”, because the position of the marble is given by *two* coordinates, its distance from the centre of the bowl and its height. The point here is, that the marble is not free in the choice of its height, it is constrained to move along the contour of the bowl. We therefore need only specify *one* coordinate, e.g. its off-centre position, and its height is then automatically known. Hence the terminology “one-dimensional” (Fig. 2.1).

Obviously, a perfectly analogous situation is a cyclist free-wheeling down the slope of a hill to cross a valley. After the lowest point the road curves upward again to climb a hill on the other side. It may seem trivial to point out that this is an analogous situation, but the funny thing is that exactly this cyclist has given a name to one of the magical phenomena occurring in quantum mechanics (so-called “tunnelling”). Curiously, we will meet the cyclist again in the part on atomic phenomena.

Another analogous situation is the pendulum of an old fashioned clock, as will be evident (*mathematically* analogous that is; a cyclist will be glad to hear that we do not confuse his/her actual appearance with a marble or a clock).

In these three examples the motion is driven by gravity. Gravity need not always be the driver of the oscillator. One might also imagine a body, having a certain mass, sliding over a horizontal surface, and performing its motion back and forth



**Fig. 2.1** One-dimensional oscillator driven by gravity: the marble and bowl

**Fig. 2.2** One-dimensional oscillator driven by mechanical springs: the mass-spring system



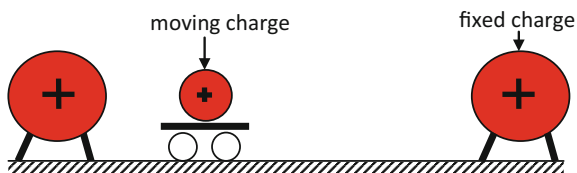
under the influence of a system of springs. These springs must be arranged such that they tend to drive the mass back to a central point, like the effect of gravity which tries to move the marble back to the centre of the bowl. Such a so-called “mass-spring system” is also a perfect analogy—in the mathematical sense—of the marble-bowl system. We repeat here the schematic shown earlier of the mass-spring system (Fig. 2.2).

The driving force could also be of an electrical nature, in which case the body is assumed to have an electrical charge. Stationary charges in the vicinity cause an electrical field (a so-called “electrical potential well”) such that the repulsive forces on the moving charge again have the same effect as gravity has on the rolling marble.

Note that we have drawn all the charges in the schematic below as positive ones, which seems to be illogical because an electron has a negative charge. This does not matter as far as the motion of the mobile charge is concerned, since like charges always repel each other, so that the oscillator is not affected by the sign of the charges. What is important to mention is, that everywhere in this book positive charges are assumed, because this choice has a few advantages. For instance, a flow of *positive* charges is called the electrical current, with the direction of flow and the current the same. It is an inconvenience dating back from the times that electrons were unknown yet, and one had to make a choice what to call a positive or negative charge. A choice that later appeared to be the “wrong”—or rather an inconvenient—one. Nowadays an electrical current is known to be most often be associated with a flow of electrons (negative charges), so we are stuck with the inconvenience that according to the historically chosen convention a current has a direction opposite to the velocity of electrons. Of course it is all a matter of definition and does not really make a difference (apart from the unexpected occurrence of minus signs in formulae), and it is reflected in schematics such as pictured below (Fig. 2.3).

The interesting thing is, that the actual form given to the experiment does not matter. The oscillator may be driven by gravity, mechanical springs or electrical

**Fig. 2.3** One-dimensional oscillator driven by electrical forces: the charge inside a potential well



forces, it turns out that the mathematical formulation in all these cases is identical, and that the motion in principle displays the same characteristics.

What this characteristic motion entails is of course familiar to anybody, even to people who have never opened a physics textbook. The result of the experiment will be a smooth, periodic motion back and forth, slowly dying out. This more or less slow subsidence of the motion is due to additional forces on the moving mass, occurring as soon as the mass has obtained some velocity. Obviously a marble, and even more obviously the cyclist, will experience air resistance opposing their motion. But there are more so-called *damping forces* of another kind as well: think of the rolling resistance of a bike, due to friction in the bearings and due to the constant deforming of the tires.

People who have “internalised” their lessons at school will intuitively interpret the motion of a marble as a continuous exchange of different forms of energy. This mental model says that the marble at rest in a high position has *potential* energy and no *kinetic* energy. Then, when the marble starts to roll downwards it acquires kinetic energy at the cost of its potential energy. You could say that potential energy is transformed into kinetic energy, until in the lowest point of its trajectory all its energy is in the form of kinetic energy.

Were there no frictional effects such as air resistance and rolling resistance, the motion would go on forever. This means that the total energy in the marble itself, i.e. the sum of its potential and kinetic energy, would be constant and would not change in time. However, the unavoidable resistance forces (or: “damping” forces) all the time tap some energy from the marble and transform its mechanical energy into other forms of energy such as heat. The air resistance experienced by a cyclist means that his energy is partially lost in the form of turbulence in his wake, which turbulence slowly decays by the friction of the air and is finally transformed into heat too. The result is in the end that the marble is left without mechanical energy and finds itself at rest in the bottom of the bowl, all the initially present potential energy having been lost, mostly in the form of heat. Now heat is another form of energy, and it turns out that what the marble or cyclist have lost in mechanical energy is completely found back in the form of heat, so that again the total amount of energy (potential-, kinetic- and heat energy) is the same. This is called the *law of conservation of energy*: energy cannot be “lost”, it is just transformed into a different form. Although it should be said that a cyclist will consider his loss of *mechanical* energy as a loss of *useful* energy, since the heating up of the atmosphere in his wake will not really interest him.

Apart from the production of heat, there are other interesting methods to lose energy. Think of the fact that the rolling motion of the marble is clearly accompanied by a distinct noise. An earthenware bowl will in this way produce even a rather annoying sound. The noise comes from the combination of the rolling marble and the bowl, the bowl vibrating as a consequence. The noise means that energy in the form of acoustic waves is radiated away, so that here is another source of energy “loss”. We should add that these acoustic waves also slowly die out or “dissipate” in the form of heat, although that may happen at a considerable distance from the bowl.

In the case of the rolling marble the energy loss by noise radiation is tiny compared with other energy losses such as the loss associated with air resistance. It is another matter when we consider the electrical oscillator.

Whenever a charged particle is in motion, in particular if it accelerates or decelerates, it sends out electromagnetic waves. This too is something known to everybody. Everybody knows what happens when electrons in large numbers are pumped up and down through a vertical copper rod. This arrangement is more usually called a radio antenna, and it emits radio waves. In all conductor wires such as power cables the same happens, so that an alternating current in them can cause a steady “hum” in a nearby radio receiver.

Likewise, a charged body moving back and forth within a potential well, i.e. the electric oscillator, will emit radio waves. Like sound waves, radio waves—or using the more general terminology: electromagnetic waves—represent an energy loss. A charged body moving back and forth will by this effect gradually lose its energy, and if we want to keep it moving, some external force has to be applied. The external driving force has to oppose the so-called “radiation resistance”, i.e. the recoil force on the electrons due to the emission of radio waves. The charges in a radio antenna have to be kept in motion by applying a varying electric potential to its ends, and by feeding energy into it in this way.

There is a close analogy here with the pendulum of a clock, which loses energy all the time by frictional effects, and must be kept in motion by the clock mechanism which gives it a kick during each swing, and replenishes the mechanical energy in the pendulum.

## 2.3 What Textbooks on Quantum Mechanics Tell About the Oscillator

And now the weird behaviour of a quantum oscillator! The oscillator may in principle have all of the above mentioned forms, be it on an extremely small scale, comparable with the size of atoms. Because in reality we are now probably dealing with charged particles such as electrons or protons instead of cyclists, the electrical model is the one closest to the physical reality. A single electron moving within a potential well is obviously not subjected to air resistance or other kinds of friction, so that radiation is the only mechanism by which it can lose energy.

What is observed is first of all that such a quantum oscillator does not always act as an antenna! In fact, most of the time it does not send out any electromagnetic waves, or so to speak: there is “radio silence” most of the time! This is a phenomenon that has no explanation in classical physical theory. Indeed, we would be astonished if an antenna in our human world would stay “silent” if we knew for certain that the masses of electrons within the antenna rod are moving in the proper way.

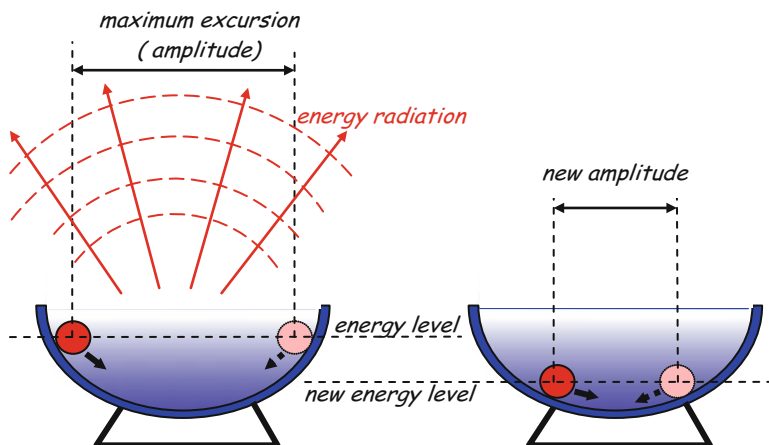
Less surprising is, that during these periods of “radio silence” the total energy (potential plus kinetic energy) present in the translation motion of the single electron or proton is constant in time. This is not so surprising, because without radiation there cannot be energy losses.

Sometimes however, at unpredictable times, the particle does shed some of its energy. It does not do so in a gradual way, but by giving off a sudden “burst” of energy. This is accompanied by a sudden break of the radio silence, and because of the suddenness of the event, the radiation is also in the form of a burst of electromagnetic waves. The frequencies we are talking about in the case of an electron are high, in the region of light frequencies. Such a burst of energy therefore takes the form of a flash of light. The flash of light emitted is so sudden and of such a short duration that onlookers experience it as a discrete “energy packet” shot at them by the moving electron. Accordingly, such an energy packet is known as a “photon”, or “a particle of light”.

One would expect that the frequency (in other words: the colour) of the emitted light would correspond to the frequency of the back and forth motion of the electron within the potential well. In the case of a macroscopic antenna that would be so. Not so in the quantum world: the colour of the emitted light is related in some way to the amount of energy that is being shed by the electron. Strangely, we thus find a definite frequency in the radiation, but there is nowhere any charge or part of a charge that is oscillating with this frequency. It is as if the radiation comes out of nothing, there is no obvious root cause for it. Apparently, in the realm of quantum mechanics almost nothing happens according to our expectations. And that same story goes on.

After having shed in this abrupt way some of its energy, the total energy in the translation of the electron of course has decreased. And here is again something strange: the new energy level is not arbitrary, but can have only certain fixed values. A marble in the *macroscopic* world can have *any* energy, which is evident from the fact that its excursions within the bowl very *gradually* become smaller when it loses energy by frictional losses. In contrast, the electron within its potential well cannot make excursions of any arbitrary magnitude. Only discrete values are permitted by nature. The magnitude of an electron’s excursions is restricted to certain discrete values with *no gradual transitions in between allowed*. An electron’s possible energy levels within a potential well are said to be “quantised”, and the permitted levels are in fact very precisely prescribed by the laws of quantum physics, laws that are completely unknown in the classical physics of every day.

If a marble rolling inside a bowl would display this kind of behaviour we would be puzzled, to say the least (Fig. 2.4). First of all, the marble would for a long time not lose its energy, which might be seen by observing the amplitude of its motion, i.e. the height to which it climbs up the wall of the bowl during every cycle of its motion, or to use the earlier terminology: by observing the magnitude of its excursions. It would seem as if there were no losses due to dissipation: no air resistance, no rolling resistance, not even a sound would be heard because even the emission of sound would be an energy loss.



**Fig. 2.4** Energy loss by emitting lumps of energy. Quantisation of the energy levels

And then suddenly, at an unpredictable moment, we would hear a thunderclap and at the same instant would observe that our marble no longer reaches the same height as before. And what is more: we could, using a felt tip, mark the new level reached after the thunderclap, and do it again after the next explosion of energy, etc. We would find that another marble in a bowl standing beside it would show *exactly the same energy levels*. The only difference with the first bowl would be that the thunderclaps coming from each bowl would come at different times. As precisely defined as the energy levels are, as unpredictable are the times: sometimes there would be a long time between the separate bursts of energy, so much so that we could easily put the kettle on and drink some tea. And at other times, the thunderclaps would follow each other rapidly, without any predictable intervals.

We would certainly call this “magical behaviour” in the case of a real marble. Yet, this is how marbles at atomic scale (or rather: electrons) behave.

The story of strange behaviour is not finished yet. One would expect that the process of shedding energy, although it is unconventional, would nevertheless finally result in the situation where the electron would come to rest in the centre of the potential well. Once again: not so! There is a minimum energy level, the so-called “zero-point energy” at which we will never see any more energy-bursts, no matter how long we wait.

To speak metaphorically in macroscopic terms: the marble never comes to rest completely in the bottom of the bowl. It forever keeps on “wiggling” a little bit in the bottom part of the bowl, but never gives up all of its remaining energy.

Let us now return to the situation where the electron (or using the metaphor again: our marble) still has plenty of energy, before it has emitted all these light flashes. In the macroscopic world we are used to see a smooth motion, the marble gradually accelerating and decelerating between the high points where it reverses its motion and the lowest point where it has maximum velocity.

What we see in the quantum world is again something completely different. It appears that there are certain, fixed points in the potential well where we hardly ever “see” the electron, whilst in between these points the electron is frequently spotted. It is as if the electron is “frightened” away from certain “forbidden” points.

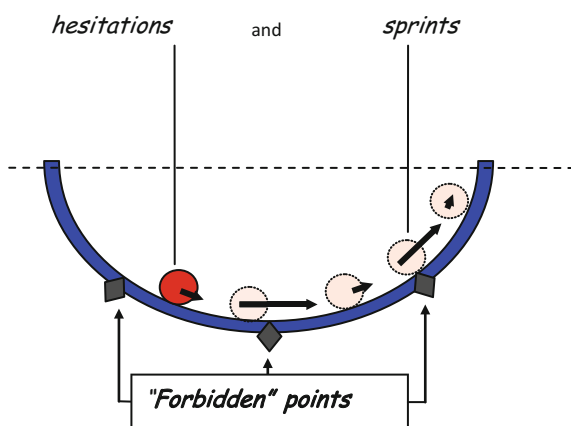
When a macroscopic marble would show such behaviour we would interpret it as a very irregular velocity. We know that the marble is rolling from one side of the bowl to the other side, so that it has to pass all the points on the contour of the bowl, including these forbidden points. If we nevertheless see that the marble spends a much longer time in the regions between the forbidden points and hardly any time in the areas directly around these points, we will interpret it as a wild variation of the velocity of the marble. Before it reaches such a forbidden point it decelerates—it “hesitates” so to speak—and then takes a run to race past the “danger point” at very high speed. This is followed by a new hesitation before it takes very quickly the next hurdle (Fig. 2.5).

We humans do the same thing when for instance at night the lights on our bicycle have failed again and we see further on the road policemen on watch. We race past the “danger point” hoping that we will not be noticed. This is indeed a rational action because thanks to our high speed we are dwelling only a short time in sight of the patrolling police officers and can therefore hope that they are exactly at the most critical moment looking somewhere else. By racing past we have increased our chance not to be spotted.

The interpretation of “forbidden points” in terms of a wildly variable velocity must not be taken literally in the case of an atomic particle. Caution is needed if such a macroscopic picture is applied to the atomic world, and much more will be said about it in Chap. 3 and later again, in Chap. 10. For the time being, the picture of alternating “hesitations” and “sprints” is useful to remember the *phenomenology* of the atomic marble’s behaviour.

We can make a different picture of this phenomenon. Let us take a camera and shoot at random times snapshots of the irregularly moving marble. Afterwards collecting all the photographs and printing them on the same frame, we will see that

**Fig. 2.5** The “forbidden points” in an atomic sized potential well, and the phenomenologic (macroscopic) interpretation in terms of jumps of the particle (not to be taken literally!)





the marbles on the photo's are not evenly distributed. Take for instance the vicinity around a “forbidden point”. When the marble is in this vicinity it is racing to pass as quickly as possible the forbidden region. We get hardly the chance to catch it on randomly taken photographs, so short is the time it spends at these points. When making random snapshots of the marble that is racing past a danger point on the bowl surface, the chance to get a picture of it is much smaller than at the places where the marble is hesitating and has a small speed. Consequently, when overlaying all the pictures taken, we will find many pictures of the “hesitating” marble and just a few or none at all of the marble speeding past the forbidden points.

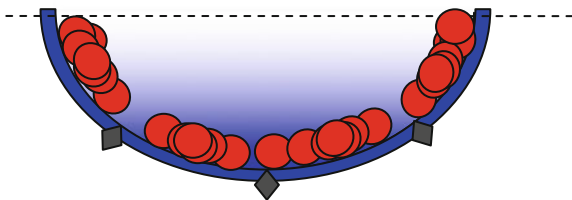
As will be seen later, in quantum mechanics it is customary to present results in the latter form, the picture one gets by taking snapshots at random. In fact, this is nothing else than determining the *chance* to find the marble at different points along its trajectory. Of course, in more dignified publications such information is usually presented in the form of graphs. They have the posh name “position probability density graphs” but really give the same type of information as contained in the picture of overlayed snapshots. In the next chapter we will have the opportunity to show several of such graphs, as calculated by using Schrödinger's equation (Fig. 2.6).

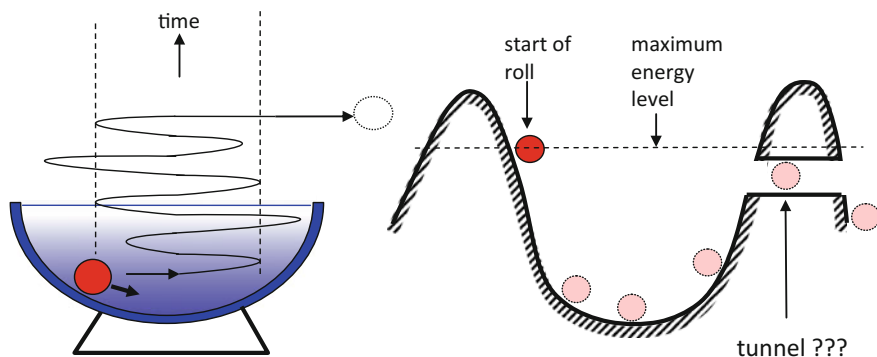
And now the most extreme behaviour of the marble at atomic scale. Alas the plot of this part of the story has already been given away in the introduction so that the surprise effect is lost a bit. Nevertheless it remains a remarkable thing: the marble sometimes jumps over the rim, out of the bowl! In the macroscopic world we would never expect this to happen, just because the marble has initially a given amount of energy which can only decrease, and which is insufficient ever to reach the rim of the bowl. After all, when we laid the marble in the bowl, it was somewhere *inside* the bowl, and we would not expect it to jump out of the bowl on its own accord, it simply does not have enough energy to ever come higher than where it started. At least, that is true in the macroscopic world (Fig. 2.7).

In the world of atoms an elementary particle which is trapped *inside* a potential well (so that it becomes an oscillator) is sometimes suddenly found *outside* the well! It would seem to have violated all the laws we know about conservation of energy.

And here we come back to our earlier introduced cyclist freewheeling down a hill and rolling up the next hill (still freewheeling). If he would not have sufficient energy to pass over the top of that next hill, and if we would nevertheless find him in a valley further on where he never could be whilst freewheeling, we ask

**Fig. 2.6** Overlayed snapshots of the marble within a bowl with forbidden points





**Fig. 2.7** The origin of the expression “tunnelling effect” for the energy-wise impossible escape of particles from their potential well

ourselves probably: “was he cheating and added some human energy by pedalling, or was there a tunnel through the hill, so that he has avoided a further—impossible—climb and sneaked *through* the hill instead of *over* it?”

By analogy the strange, impossible behaviour of an electron which is sometimes found in places where it energy-wise cannot be, is called “the tunnelling effect”. Although the phenomenon is impossible to understand (at first sight, but read on), it is very real. A special type of microscope can function only thanks to the tunnelling effect of electrons, and is therefore called “tunnelling microscope”.

In conclusion: when consulting our two different books on mechanics (classical mechanics and quantum mechanics), the books tell us entirely different things about exactly the same situation, i.e. the same types of oscillator. And not only the stories are different, nature itself behaves entirely different according to whether we have in our oscillator just a few electrons or a great many of them. No wonder that we need completely different theories when we want to describe what we see happening, and want to make further predictions. The same model can only have different outcomes if different “laws of nature” are assumed to be applicable.

The aim of this book is however, to show that a *deformable* droplet of charge inside a potential well will show most of this weird behaviour, even if we analyse it using the “normal” laws of nature.

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