

Chapter 2

Pipe-Z Optimization

Abstract This chapter outlines the optimization methodology for a single-branch tubular structure created with Pipe-Z system. In this process the geometrical properties of the Pipe-Z module (PZM), and the shape of the entire structure comprised of PZM replicas are simultaneously optimized. The total number of modules to follow a given guide path is minimized under constraints. The algorithm is illustrated with planar simplification of the three-dimensional problem. The results produced by random search are presented and discussed. Additionally, the search domain has been sampled and visualized. Finally, as an example for three-dimensional shape optimization, 6_3 knot is constructed with the minimal number of congruent modules.

Keywords Pipe-Z · Multicriterial optimization · Discrete optimization · Trapezoid-tiling · Domain visualization

2.1 Introduction

Any modern design must meet a number of independent and often conflicting criteria. In the engineering design, the most common requirements for construction systems are:

1. structural soundness,
2. feasibility,
3. economy,
4. and often—free-form.

Regarding the first issue, here PZMs and entire PZs are assumed to be rigid, in other words, structurally sound. Feasibility is represented here by modularity of the system. Moreover, additional constraints on size and possible shapes of the modules are imposed. Economy is addressed here by the minimization of the number of modules used for a given task. Finally, the free-form issue is represented relatively complex forms of mathematical prime knots.

The problem described above can be formulated as optimization:

- Construct a modular structure which follows given free-form path, so called guide path (*GP*) Sect. 1.2.

- *GP* must be contained within this modular structure.
- Modules to be congruent.
- Construction must not self-intersect.
- The number of modules to be *minimal*.

In the following section, the methodology is outlined using a planar *GP* formed by the simplest prime knot, *Unknot*.

2.2 Trapezoid-Tiling of a Planar *Unknot*

So called basis splines (B-splines) have been originally presented in [1, 2] and further introduced to geometric design community by [3]. B-splines are a generalization of Bézier curves, and *NURBS* (Non-uniform rational basis spline) is a further generalization of B-Splines. They have a number of practical properties which are particularly useful in various types of design. The list of important characteristics of the B-spline curves can be found in [4].

In order to make the case general, let us consider an *Unknot* formed by a planar closed B-spline. The start point (S) and initial direction ($\mathbf{v}$) are given, as shown in Fig. 2.1(1).

The trapezoidal unit (TU) shown in Fig. 2.1(2) is a two-dimensional analog of PZM introduced in Sect. 1.1.1. In fact, TU can be considered as a lateral section through PZM. Thus in order to avoid confusion, the denotation of control parameters is exactly the same as in PZM: r , ζ , and s .

However, since every subsequent TU can be added in two possible ways, the angular parameter ζ has been given additional function. Here it is used not only to control the shape of the TU, but also to indicate whether a subsequent unit “turns right” or “left” as shown in Fig. 2.2.

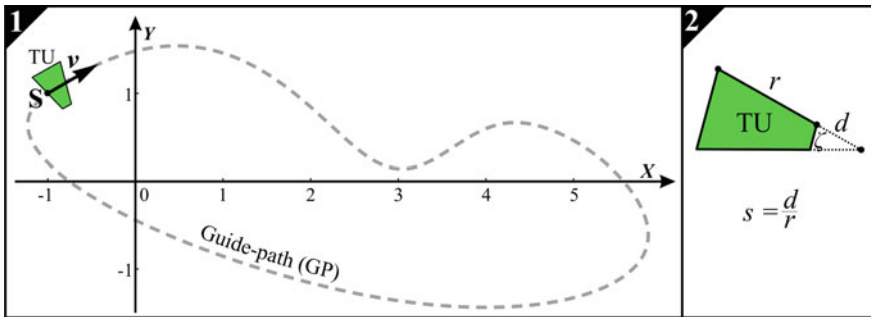


Fig. 2.1 1 B-spline *GP* to be tiled with congruent trapezoidal units (TU). 2 The parameters of the trapezoidal unit (TU)

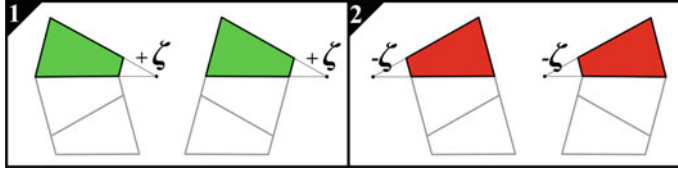
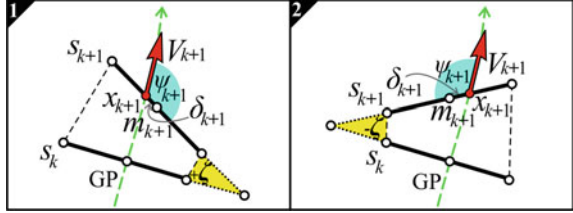


Fig. 2.2 Negative and positive ζ s correspond to the attachment of the “left” and “right” TUs

Fig. 2.3 Sub-figures 1 and 2 correspond to the attachment of: “right” and “left” TUs, respectively. Previously and currently added TU segments are denoted by s_k and s_{k+1} , respectively



2.2.1 Alignment of Trapezoidal Units to the Guide Path

The alignment of TUs to *GP* is explained and illustrated in Fig. 2.3. For an interactive demonstration see [5].

At each step one of two TUs (“left” or “right”) which gives a smaller value of the cost function is chosen:

$$\text{Minimize} \left(\delta_i + w \left| \psi_i - \frac{\pi - \zeta}{2} \right| \right) \quad (2.1)$$

where for an i th segment:

δ_i is the distance between the midpoint of the segment (m_i) and the intersection between this segment and *GP* (x_i);

ψ_i is the angle between direction V_i of *GP* in point x_i and the i th segment;

w : $w = [0, 100]$, is a weight parameter. It balances the influence between the distance δ_i and the angle ψ_i , as illustrated in Fig. 2.4.

As Fig. 2.4 indicates, the influence of parameter w is substantial. Most importantly, for optimal results, its value should be set individually for every case. The cost function L_j returns the *length* of a candidate solution j that is an allowable trapezoidal tiling. L is a function of four independent variables, and the optimization problem is expressed as follows:

$$\text{Minimize} (L_j[r, s, \zeta, w]) \quad (2.2)$$

for: $0.5 \leq r \leq 3$;

$0 \leq s \leq 1.5$;

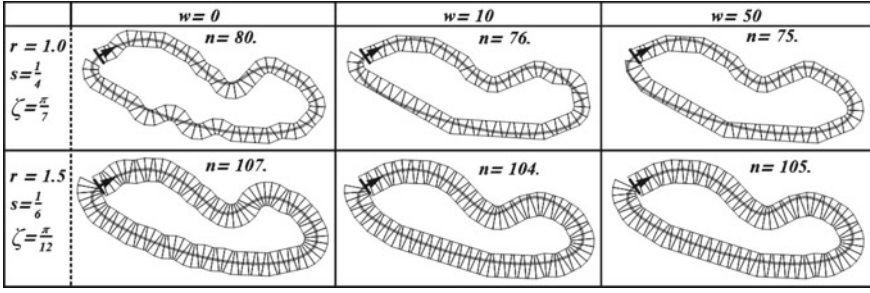


Fig. 2.4 Alignment of two different TUs along GP at three values of weight w . The number of units used in the tiling (n) is shown for each example

$$12 \leq \zeta \leq \frac{\pi}{4};$$

$$0 \leq w \leq 100.$$

The above ranges have been determined by trial-and-error. The self-intersection prohibition has been implemented by so called “death penalty” [6]. In other words, any self-intersecting individual is automatically considered as infeasible candidate solution. A number of preliminary experiments lead to the following observations regarding the cost function L :

- It returns only feasible solutions, however, it is very sensitive to the values of the variables.
- Although the values of variables are continuous, L is not continuous.
- The feasible solutions are scarce and their distribution seems unclear.

2.3 Random Search

Random search (RS) is a simple and quick way of preliminary exploration of such irregular solution domains. Two trials of 200,000 random solutions have been generated and evaluated. Each trial took approximately two hours. The best solutions have been recorded with the time of their occurrence, as shown in Fig. 2.5.

Only the solutions with lower L value than previously generated have been recorded. Trial 1 produced better results, which are shown in Fig. 2.6.

2.3.1 Domain Visualization

As mentioned above, the “feasible solutions are scarce and most importantly, their *distribution seems unclear*”. The following experiment explores the solution space on a more systematical basis. In principle, the cost function L_j of a j th solution is calculated for a four-dimensional vector (r, s, ζ, w) . In order to intuitively visualize

the results three-dimensional space, this vector has been reduced to three variables: (r, s, ζ) . The weight w has been assigned an arbitrary value: $w = 50$. The ranges of the remaining parameters for this experiment have been determined by trial-and-error.

The numbers of samples have been assumed arbitrarily as follows:

$$0.5 \leq r \leq 3; \text{ number of samples (NOS): } 20$$

$$0 \leq s \leq 1.5; \text{ NOS: } 15$$

$$\frac{\pi}{12} \leq \zeta \leq \frac{\pi}{4}; \text{ NOS: } 20$$

In other words, the three-dimensional “slice” (through $w = 50$) of the four-variable domain has been sampled by 6000 ($= 20 \times 15 \times 20$) points. Figure 2.7 shows the values of the cost function L for the feasible solutions.

As Fig. 2.7 indicates, the previous observation regarding the distribution of good solutions in the solution space might be false. The regularity of the “sliced” domain is striking. In fact, there is strong coupling among the three variables (r, s, ζ) . As a result, the solutions form layers of equipotent cost function L . This indicates that a single ideal solution in this “3D slice” might not exist, or if it does, it would be difficult to find. On the other hand, it seems relatively simple to find (generate) practically any number of extremely good solutions for this w . In order to draw more general assumptions, this experiment should be repeated for various values of weight w .

2.4 Optimization of a PZ Knot 6₃

Since the results produced by RS for the planar case described above are quite satisfactory, the same approach has been implemented for the regular three-dimensional PZ knot. 6₃ prime knot has been selected to illustrate the optimization of a spatial tubular PZ structure. The evaluation of three-dimensional candidate solutions is substantially more computationally expensive. Thus a trial of 500 random solutions

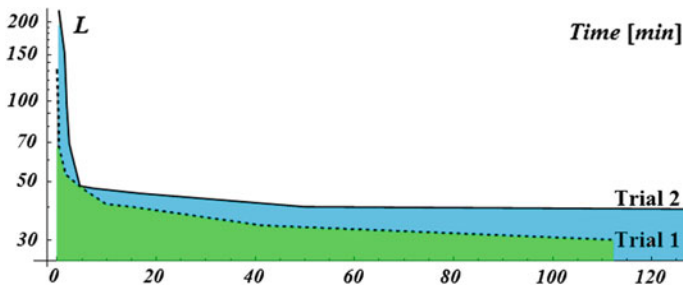


Fig. 2.5 Two trials of RS. Trial 2 took approximately 15 min longer and produced worse results

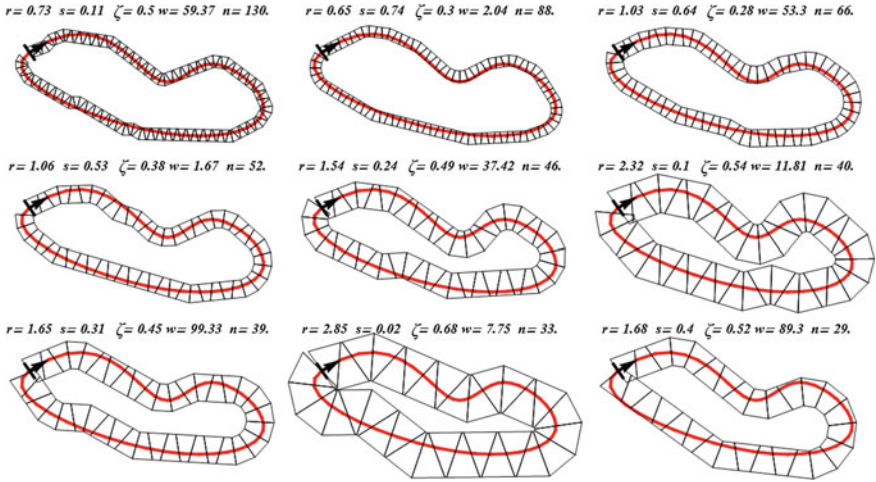


Fig. 2.6 The results of Trial 1 sorted from the worst/earliest (*top left*) to the best/latest (*bottom right*). The values of all four TU parameters + the number of units (n) are shown above each solution

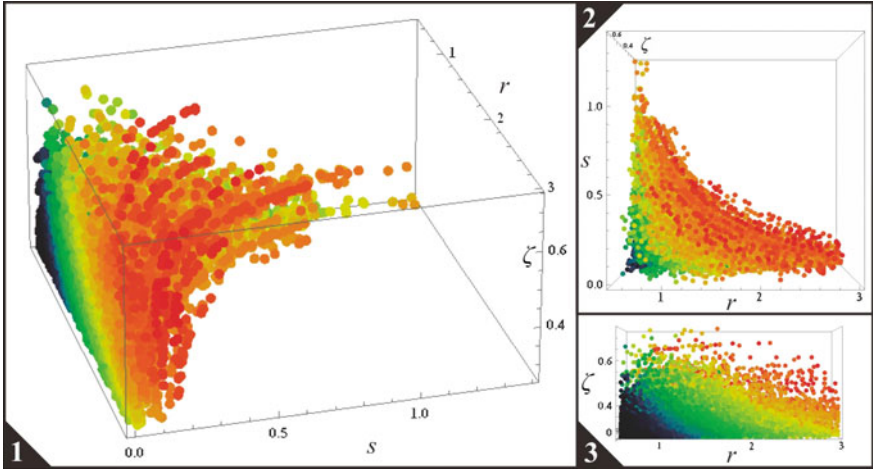


Fig. 2.7 The visualization of the "slice" through $w = 50$ of the sampled solution space. The best/lowest and worst/highest values of L are shown in: red and black, respectively. Sub-figures 1, 2, and 3 show: the axonometric view, *top* and *front* views, respectively

has been performed as described above. The best parameters for PZM and the best configuration for PZ 6₃ knot composed of 108 modules are shown in: Fig. 2.8(3) and 2.8(2), respectively.

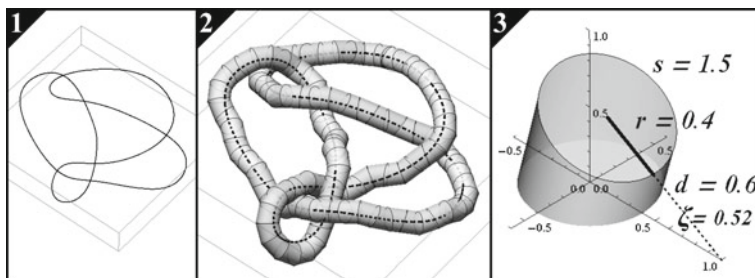


Fig. 2.8 The result of the RS optimization of a PZ 6_3 knot. **1** GP: the spatial curve of the 6_3 knot. **2** The best approximation of GP by PZ found by RS. The number of PZMs is 108. **3** The best parameters for PZM

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