

Chapter 2

Kinematics and Dynamics of the Feed Cable-Suspended Structure for Super Antenna

2.1 Preamble

So far the largest antenna is the Arecibo spherical radio telescope with the diameter of 305 m, which was built in the 1970s and located in Puerto Rico, USA. As for Arecibo-type telescope, there exist three problems: high cost, narrow frequency band and spherical error. Due to the above problems, one plan proposed by Chinese astronomers and engineers is to build a set of large spherical reflectors in the extensively existing karst landform in southwest China. A new-generation large radio telescope (LT) nowadays is referred to as the square kilometer array (SKA). Many different technological solutions have been brought forward and studied by institutes participating in the SKA [1, 2]; and the papers [3–6] propose a multi-disciplinary design project and made nonlinear analysis about it with many reasonable results. Figure 2.1 shows a number of outstanding innovations of the design project: (1) the unique karst landform in Guizhou province, southwest China; (2) the main active reflector enabling to overcome both shortcomings of narrow bandwidth due to the line feed, and spherical error due to spherical reflector; and (3) the light-weight feed cabin driven by cables plus stable Stewart platform, by which, the tracing accuracy of feed mounted on the movable plate of Stewart platform can be up to millimeters. As a multi-science platform, LT will provide treasures to astronomers, as well as bring prosperity to other research, for instance, space weather study, deep space exploration, and national security.

The millimeter positioning precision of the feed fixed at the moving platform of Stewart platform can be realized by varying the six legs' lengths. Although this project can make it easier to implement the LT engineering, it raises a much higher requirement for the control engineering. This is due to the use of the large span cables to drive the cabin structure weighing 20–30 t. For such a large lag and heavy inertia control system, it is difficult to guarantee the requirement for the tracking accuracy (less than 4 mm).

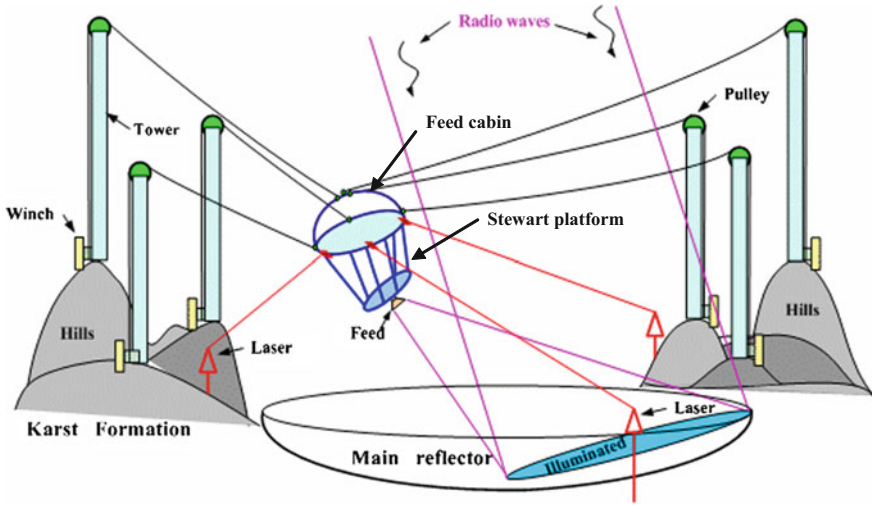


Fig. 2.1 Schematic diagram of a new design project of LT

The cooperative variation of lengths of six long cables pulls the feed cabin to track some radio source with six-DOFs. Similar to a parallel robot, the cable-cabin flexible structure of LT can be viewed as a cable-supporting parallel robot (CPR). Conventional robots with serial or parallel structures are impractical for some applications since the workspace requirements are higher than what the conventional robots can provide. CPR uses cables instead of links to manipulate objects. The cables are so light that actuators of CPR have only to drive their loads. Furthermore, CPR gives a wide range of motion, because drums of the mechanism can wind long cables. For the above reasons, cable-driven mechanisms have received attention and have been recently studied since 1980s [7–11].

As you know, dynamics is a huge field of study devoted to studying the forces required to cause motion. In order to accelerate the CPR from rest, glide at a constant end-effector velocity, and finally decelerate to a stop, a complex set of torque functions must be applied by the joint actuators [12]. One method of controlling the CPR to follow a desired path involves calculating these actuator torque functions by using the dynamic equations of motion of the CPR. In addition, as demonstrated in [13], servomechanism dynamics constitute an important component of the complete robotic dynamics. Therefore, the dynamics of the servomotors and its gears must be modeled for control design. However, the literature on the control of the CPR system including the actuator dynamics is sparse. In the following, we develop dynamic equations of the CPR system including actuator dynamics, which may be used to control and simulate the motion of the CPR.

This chapter presents design and experiment of the feed cable-suspended structure for the next generation large spherical radio telescope. To begin with, optimization design of the structure is performed to meet the requirement that the orientation of the feed cable-suspended structure need range from 40° to 60° . And aiming at the drawback of weak twisting stiffness of the feed cable-suspended system, a method to add stable cables is proposed to enhance the twisting stiffness of the structure. The experiments that developed combining with the inner FAST model of 5-m demonstrate the effectiveness and feasibility of the proposed method. And since the dynamic modeling of CPR (Cable-suspended parallel robot) is the precondition for its motion control. One objectives of this book is to find the precise kinematic and dynamic model of FAST and other similar CPRs (Cable-suspended parallel robot). Hence, according to the 5-m scaled model, the inverse dynamic formulation of CPR with non-negligible cable mass is established by using inverse kinematics analysis and Lagrangian dynamic formulation. Moreover, in this chapter, according to the 50-m scaled model of FAST, the inverse dynamic formulation of the mechanical system including servomechanism dynamics is established by using Lagrange's Equations. In Sect. 2.2, the optimization design of the telescope structure is performed and the connection between stable cables and twisting stiffness is verified. In Sect. 2.3, the inverse kinematic and dynamic model of CPR with non-negligible cable mass is carry out. Furthermore, the inverse dynamic formulation of the mechanical system including servomechanism dynamics is established in Sect. 2.4 by using Lagrange's Equations.

2.2 On Design and Experiment of the Feed Cable-Suspended

2.2.1 *Mechanics Equations of the Feed Cable-Suspended Structure*

Since the cables, in Fig. 2.1, are several hundred meters in length, and the movement of the cabin is pretty slow, the feed cabin can be viewed as static state on the points of trajectory. The cable-suspended system (CSS) suspends an object by six wires and restrains all motion degrees of freedom for the object using the wires and gravitational force when the cabin moves within the workspace. This type of mechanism is called “incompletely restrained wire-suspended mechanism,” [14] and is able to control three-dimensional position and orientation of object by changing only length of wires. Experimental studies on the inner LT 5-m and outer LT 50-m model demonstrate the conclusion as shown in Figs. 2.2 and 2.3. For example, the results of LT 50-m model show that the angular tracking error and the position error can be less than 2° and 25 mm when the cabin reaches a pose angle of $\pm 40^\circ$, respectively.

Fig. 2.2 The model of feed cable-suspended structure for LT 5-m



Fig. 2.3 The model of feed cable-suspended structure for LT 50-m



We have already developed mechanics equations of the feed cable-suspended structure [15], and briefly outline here.

$$\mathbf{A}(\mathbf{X})\mathbf{H} = \mathbf{B}(\mathbf{X}) \quad (2.1)$$

where $\mathbf{A} \in R^{6 \times 6}$ is Jacobin matrix. $\mathbf{H} = [H_1 H_2 H_3 H_4 H_5 H_6]^T$ is the vector describing the horizontal components of the active cable tensions. $\mathbf{B} \in R^{6 \times 1}$ is the load vector acting on the structure. For more details on the specification of the feed cable-suspended structure, refer to [15].

2.2.2 Optimization of the Feed Cable-Suspended Structure

In order to achieve a design scheme to meet the requirement of the orientation of 60° , it is necessary to analyze the original one, as shown in Fig. 2.1. In this scheme, six cable towers are designed as the same height, one group of the cables is

connected to the bottom circle of the feed cabin through the top of the three towers 120° apart; the other is connected to a small circle on the top of the feed cabin to eliminate the singularity [15]. In the following discussion, the cables connected to the bottom circle of the cabin are called lower cable, and the tower, through which the lower cable connected is called lower cable tower. As the same way upper cable and upper cable tower are also defined (see in Fig. 2.4). The cross-sectional shape and structure of cables is spiral strands, as shown in Fig. 2.5.

As the orientation of the feed cabin increased to 60° from 40° , the original scheme is not feasible from research results. So we have to find a new way from the original design. Two improved schemes can lead to this purpose from the results of computer simulation. One is to lower the lower cable towers and heighten the upper cable towers; the other is to change the connection way of the feed cable-suspended structure. So it is necessary to obtain the optimal design scheme to achieve the orientation of 60° by virtue of optimization strategy.

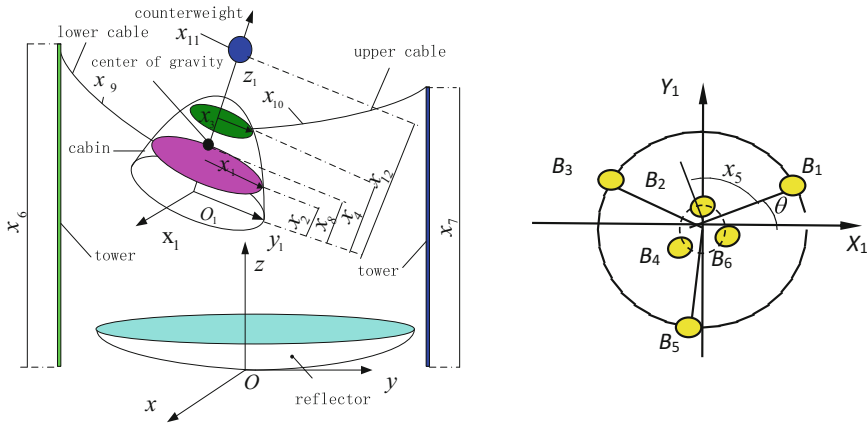
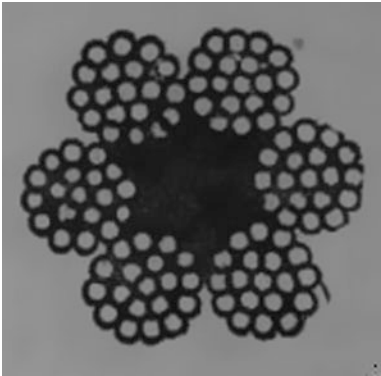


Fig. 2.4 Sketch map of design variables

Fig. 2.5 Sketch map of the structure of cable



A. Optimization model of the feed cable-suspended structure

In order that the optimized structure can meet the requirement for the orientation of 60° , little cable tension difference, and all kinds of the constraint conditions, the following work needed to be performed.

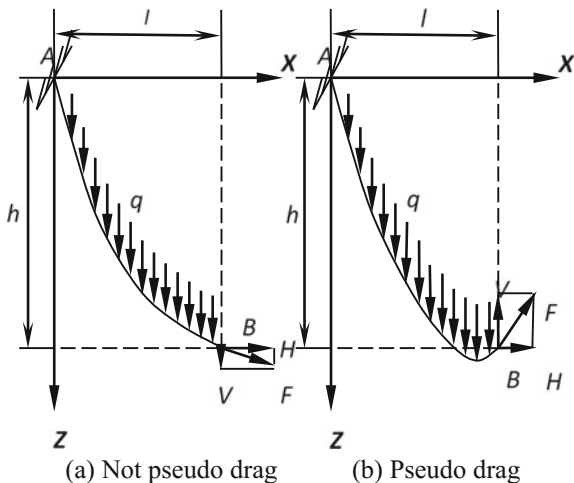
Firstly, we should determine definitely that parameters can be chosen as design variables. As shown in the Fig. 2.4, design variables include the radius (x_1) and the height (x_2) of bottom circle where lower cables are connected to the cabin, the radius (x_3) and the height (x_4) of top circle where upper cables are connected to the cabin, angle (x_5) between the adjacent lower cable and upper cable, the height of lower and upper cable towers (x_6 and x_7), the weight center of cabin (x_8), and the cross sectional area of the lower and upper cables (x_9 and x_{10}). The meaning of the counterweight (x_{11} and x_{12}) is to be given in detail afterward.

Secondly, the constraint condition considering Eq. (2.1) in the whole workspace is written as

$$\mathbf{A}(X)\mathbf{H} = \mathbf{B}(X) \quad (2.2)$$

The problem of pseudo drag of the cable has to be avoided. As shown in Fig. 2.6, it shows the condition that cables are not pseudo drag, which requires the vertical component V of cable tension F being not less than zero; and Fig. 2.6b describes the condition that cables are pseudo drag, where the vertical component V is negative.

Fig. 2.6 Sketch map of state of cable



However, cables are of the unique property—carrying loads in tension but not in compression [16]. Therefore, this needs to meet the requirement for the lower bound of F corresponding to H and V , denoted as $\underline{H}$ and $\underline{V}$. So the constraint that cables are not pseudo drag is written as follows

$$g_1(X) = -H + \underline{H} \leq 0 \quad (2.3)$$

$$g_2(X) = -V + \underline{V} \leq 0 \quad (2.4)$$

In addition, cables should meet the requirement for material strength, written as,

$$g_{3e}(X) = \sigma_e - [\sigma] \leq 0, \quad (e = 1, 2, \dots, 6) \quad (2.5)$$

Of course, all the variables have their own lower and upper bounds, i.e.,

$$\underline{x}_i \leq x_i \leq \overline{x}_i, \quad (i = 1, 2, \dots, n) \quad (2.6)$$

where n represents the total number of design variables.

Finally, the optimal objective is to make lower and upper cable tension difference as small as possible, which is beneficial to the control and economizing energy. Therefore, objective function can be formulated as

$$f(X) = \frac{2}{3}(F_{\max}^{(l)} - F_{\min}^{(l)}) + \frac{1}{3}(F_{\max}^{(u)} - F_{\min}^{(u)}) \quad (2.7)$$

where superscript l represents lower cable, u —upper cable, and F —cable tension that equals to the vector sum of horizontal tension H and vertical tension V . The reason why neither one coefficient nor the other is equal to $1/2$ is that the lower cables play an important role in the orientation of the cabin. As a result, optimization problem can be mathematically stated as

$$\begin{aligned} & \text{(P1)} \\ & \text{Find} \\ & \quad X = (x_1, x_2, \dots, x_n)^T \\ & \text{Min} \\ & \quad f(X) = \frac{2}{3}(F_{\max}^{(l)} - F_{\min}^{(l)}) + \frac{1}{3}(F_{\max}^{(u)} - F_{\min}^{(u)}) \\ & \text{s.t.} \\ & \quad A(X)H = B(X) \\ & \quad g_1(X) = -H + \underline{H} \leq 0 \\ & \quad g_2(X) = -V + \underline{V} \leq 0 \\ & \quad g_{3e}(X) = \sigma_e - [\sigma] \leq 0, \quad (e = 1, 2, \dots, 6) \\ & \quad \underline{x} \leq x_i \leq \overline{x}, \quad (i = 1, 2, \dots, n) \end{aligned} \quad (2.8)$$

It is noted that problem **P1** is concerned over whole space and it is difficult for cabin to reach the 60° orientation. So the process in detail is that the horizontal circle, whose orientation is 60° , is divided into 360 points equally. Each point is a

corresponding structure. If an optimum design can be found for each point, there will be an optimum design, $X^* = (x_1^*, x_2^*, \dots, x_n^*)^T$, for the whole working space.

B. Solution of the optimal problem

Problem **P1** is a nonlinear programming problem, and it is difficult to calculate its derivative. In recent years, textbooks are published and papers have appeared on genetic algorithm (GA) and its applications [17, 18]. Therefore, the optimal problem is solved with GA here.

The initial value, lower and upper bounds of variables and optimal results are shown in Table 2.1. The population size is 100; the number of generations is 600; the crossover rate is 0.6, and the mutation rate is 0.07. For the cable may be special made, the 10 design variables are continuous, and the size of the combinatorial space is mainly considered for the engineering. The time needed for one single evaluation is about 400 ms, and the first population is generated randomly. The initial values are the original design values before optimization.

The weight of the cabin is 300 kN; the material of cables is steel wire, whose allowable stress is 1850 N/mm^2 , and cables should avoid pseudo drag. Hence, $\underline{H}$ is equal to 200 N, and $\underline{V}$ is equal to 10 kN.

Figure 2.7 illustrates the variation of cable tension F and cable length L . The maximum, minimum cable tension and cable tension difference of lower cables (A_1, A_2, A_3) are 391, 20 and 371 kN, respectively, corresponding values for those of upper cables (A_4, A_5, A_6) are 358, 165 and 193 kN.

It can also be known that the variation of cable tension is smooth in the whole space, without spine and mutation. Therefore, it is surely driven by servo system.

Figure 2.8 shows the combination of cabin and six cables in the end. Compared with what it was, it extends truss from the top and bottom of the cabin. Obviously, it becomes easily to realize the orientation requirement of the cabin.

Table 2.1 Initial values and optimal results of the variables

Variables	Initial value	Optimal result	Lower bound	Upper bound	Unit
x_1	5.0	5.32	5.0	10.0	m
x_2	0.0	-1.5	-1.5	0.0	m
x_3	0.5	0.71	0.5	3.0	m
x_4	4.5	6.89	3.0	10.0	m
x_5	6.0	26.8	0.0	60.0	deg
x_6	280	298.5	235	300	m
x_7	280	260.8	235	288	m
x_8	1.0	2.65	0.0	2.7	m
x_9	5.0	5.3	2.0	10.0	cm^2
x_{10}	5.0	3.2	1.0	10.0	cm^2

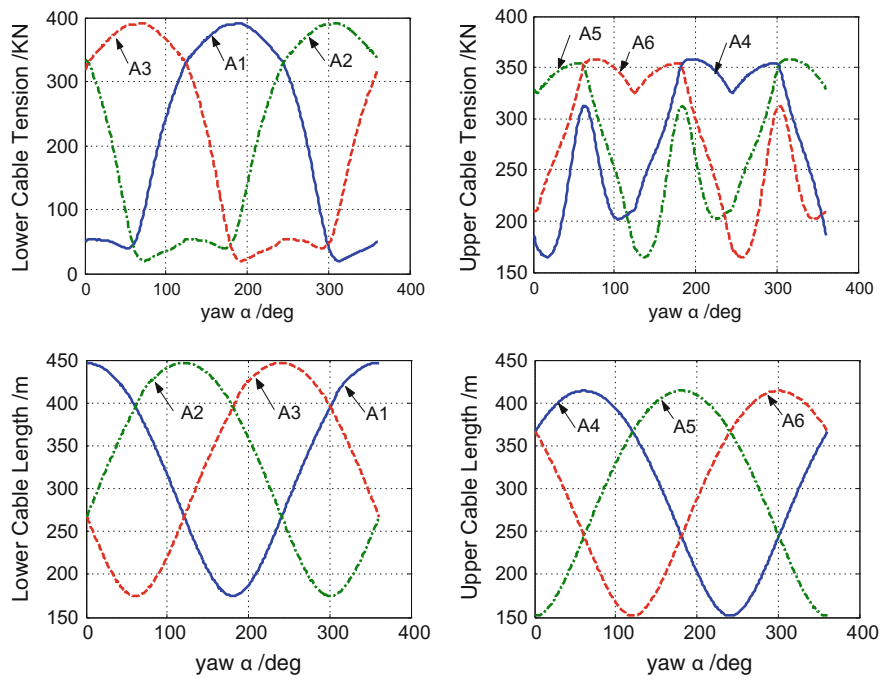
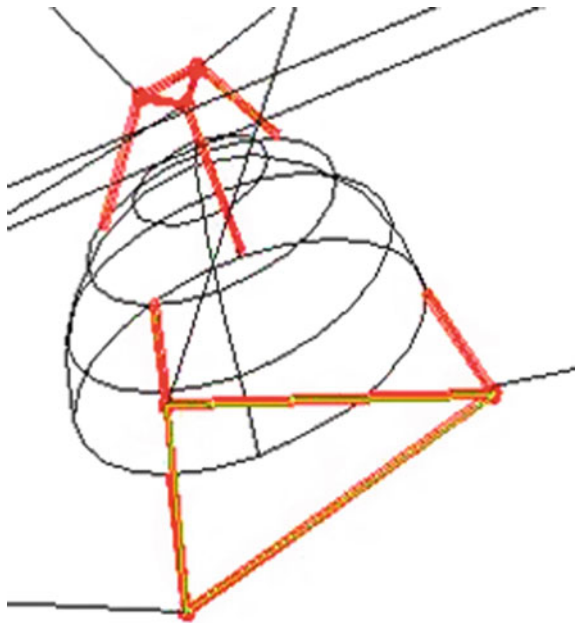


Fig. 2.7 Variation of cable tension and length with γ

Fig. 2.8 Connection between cables and cabin after optimization



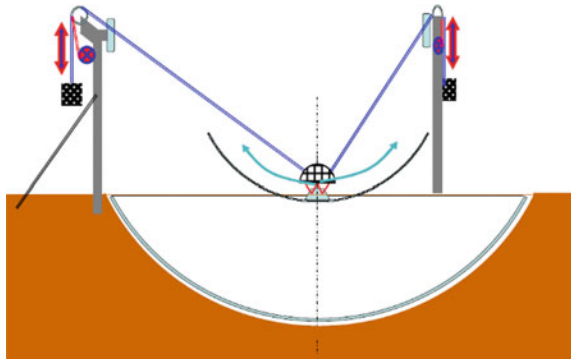
C. Optimization of counterweight and further reduction of cable tension difference

Based on the preceding analysis, the orientation angle of cabin can be increased up to 60° from 40° corresponding to the change of the structure of cabin (Fig. 2.4). Note that the tension difference of the cable is still large. The upper cable tension difference is 193 kN, and the lower is 371 kN.

In engineering, the clogs with 165 and 20 kN can be added to the upper and lower cable towers separately (Fig. 2.9) to reduce energy expense. As for the current results obtained, the required maximum power of servo motor decreased from 358 and 391 kN to 193 and 371 kN, respectively, which is our pursuance.

To reduce the tension difference of the cable further and make cable tension as smooth as possible, a counterweight is added on the cabin, which is proposed by Professor R.D. Nan in National Astronomical Observatories, Chinese Academy of Sciences. As shown in Fig. 2.4, variable x_{11} is the mass of counterweight and x_{12} is the distance from the counterweight to the bottom of the cabin. The objective is to minimize the tension difference by optimizing x_{11} and x_{12} in the whole workspace. This will be beneficial to the increment of orientation angle of the cabin. The process of calculation in detail is as follows. When γ is equal to 60° , x_{11} and x_{12} are taken as design variables, while other parameters keep the optimal results unchanged, as given in Table 2.1. The variables of x_{11} and x_{12} are optimized. Then, γ is given as 50° , 40° , 30° , 20° , 10° and 0° ; x_{11} is constant; x_{12} is varied, and the optimal objective is to minimize the tension difference of the cable.

Fig. 2.9 Clogs are added on cable towers



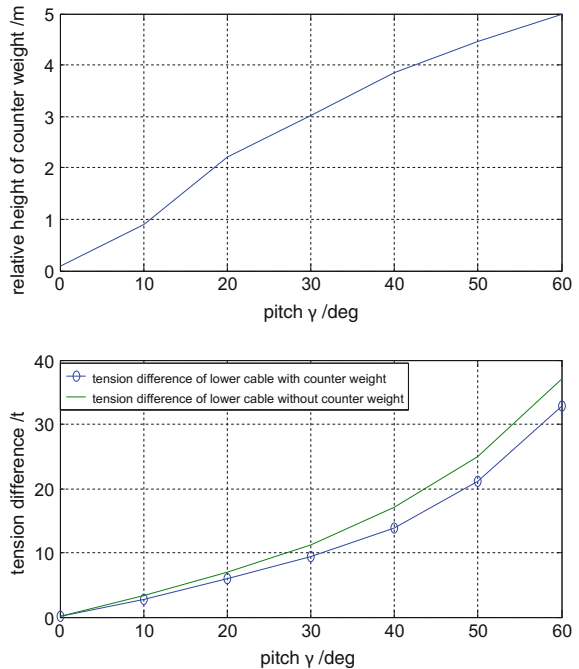
So problem **P1** is changed into **P2**.

$$\begin{aligned}
 & \text{(P2)} \\
 & \text{Find} \\
 & \quad x_{12} \\
 & \text{Min} \\
 & \quad f(\mathbf{X}) = \frac{2}{3}(F_{\max}^{(l)} - F_{\min}^{(l)}) + \frac{1}{3}(F_{\max}^{(u)} - F_{\min}^{(u)}) \\
 & \text{s.t.} \\
 & \quad A(\mathbf{X})H = B(\mathbf{X}) \\
 & \quad g_1(\mathbf{X}) = -H + \underline{H} \leq 0 \\
 & \quad g_2(\mathbf{X}) = -V + \underline{V} \leq 0 \\
 & \quad g_{3e}(\mathbf{X}) = \sigma_e - [\sigma] \leq 0, \quad (e = 1, 2, \dots, 6) \\
 & \quad \underline{x} \leq x_i \leq \bar{x}, \quad (i = 1, 2, \dots, n)
 \end{aligned} \tag{2.9}$$

Not that x_{12} is 0, $\bar{x}_{12}$ is equal to x_{12}^* , which is the optimal results when $\gamma = 60^\circ$. The results can be obtained by virtue of solving **P2** with GA, as shown in Fig. 2.10. Figure 2.10a shows the variation of distance from the counterweight to the top of cabin. We are told that when $x_{11} = 29.7$ kN and orientation varies from 0° to 60° , the distance varies from 0 to 5.03 m.

Figure 2.10b illustrates the variation of the maximum tension difference of the cable with angle γ . Moreover, we can also note that the tension difference of the lower cable decreases from 371 to 328 kN.

Fig. 2.10 Clogs are added on cable towers. **a** variation of bar length with γ , **b** variation of cable tension difference with γ



2.2.3 Experimental Results and Discussion of Enhancing the Stiffness of the Feed Cable-Suspended Structure

In order to enhance the system stiffness, especially, twisting stiffness, Paper [19] proposes a method by adding stable cables with useful results. Considering the length of paper, the following design process and results of experiment are given only.

To evaluate the performance of the proposed method, the LT 5-m model is selected for implementation. As shown in Fig. 2.11, when the cabin is located at the position (0, 0, 110) (cm), a rope is drawn along the tangent direction of the cabin. Then, the rope is cut off suddenly. At the same time three laser ranging equipments record the vibration of the cabin. For each cables there is a clog with 3 N connected at the free end. The weights of the cabin and tube with water are 60 and 40 N, respectively.

The vibration of the cabin along y-direction with 6-cable and 9-cable is shown in Fig. 2.11. It can be observed that the application of the stable cables introduces significant amount of passive damping. The time to be stable 6 and 9 cables are about 4 and 30 s, reduced almost 8 times. As for the amplitude, 5 times are obtained.

Actually, the above experiments have been done repeatedly in addition to adding the tube with liquid. Typical responses are given in Fig. 2.12, and compared with other three cases. It can be noted from Fig. 2.12 that the tube with liquid is not significant effectiveness. Hence, the nine-cable structure is positive.

The experiments shown in Figs. 2.11 and 2.12 are carried out when cabin is fixed at the given position. It is help to see the benefit from 3-added downward stable cables. However, it is not enough to evaluate the structural behavior comprehensive. Dynamic property of the structure is probably much more important. For the sake of this, the following dynamic experiment with LT5-m is done. During

Fig. 2.11 Comparison of vibration in y direction

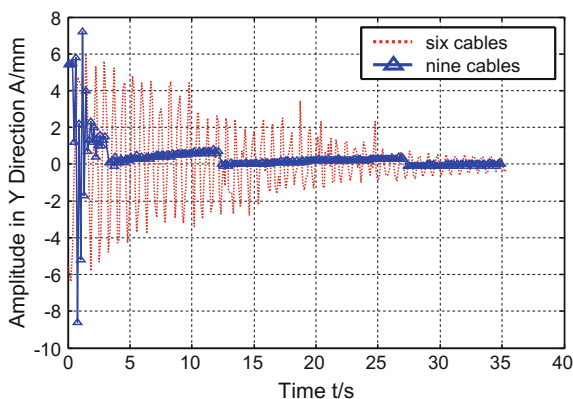
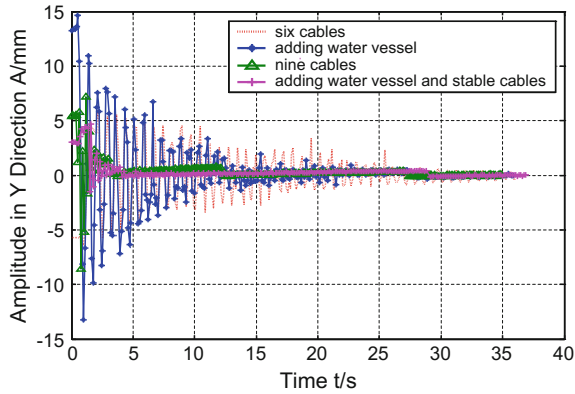


Fig. 2.12 Comparison of vibration in y direction



experiment, the cabin is asked to move along the given trajectory. The artificial wind forces, through a fan with 5 kW, on the structure with a mean velocity of 17 m/s is provided. A sample of the wind velocity persists 50 s, then, the fan is cut off and vibration recording is continued 50 s more. Vibration responses of the 6 cables and 9 cables are compared in Fig. 2.13. Obviously, the incensement in dynamic stiffness is significant by adding three downward stable cables.

For further observing the variety of the twisting stiffness, three moments in x, y and z directions (Fig. 2.14) are acted on the cabin. Figures 2.15 and 2.16 show the experimental results in the case of 6 and 9 cables. The clog is changed from 3, 5 and to 9 N. The incensement in the twisting stiffness is considerably. On the other hand, as far as the 6 cables are concerned, we do not worry about the stiffness in the x and y directions, while worrying about the stiffness about z direction. The experimental results in Fig. 2.15 demonstrate the conclusion. In addition, through optimizing the weight of clog, we examine the vibration response. Figure 2.16 shows the experimental results. It is observed from Fig. 2.16 that the stiffness about Z_1 with 9 cables is in the same level as the stiffness about X_1 and Y_1 of 6 cables.

Fig. 2.13 Comparison of vibration in x direction

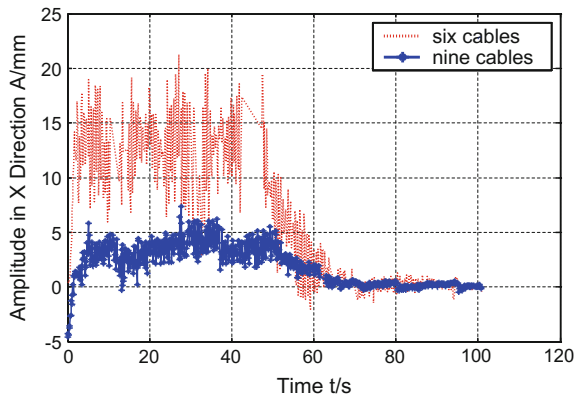


Fig. 2.14 Schematic representation of the cabin

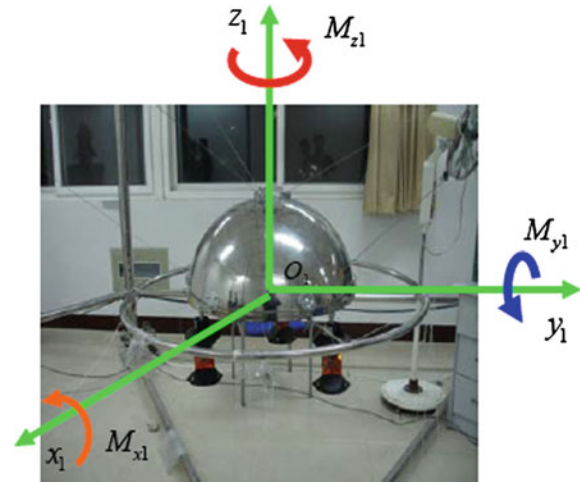


Fig. 2.15 Experimental results of twisting stiffness

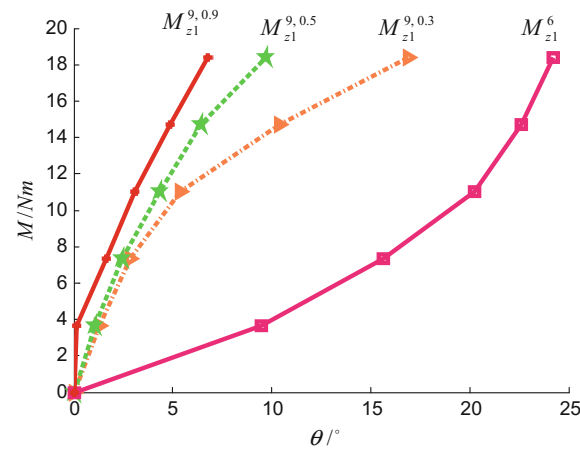
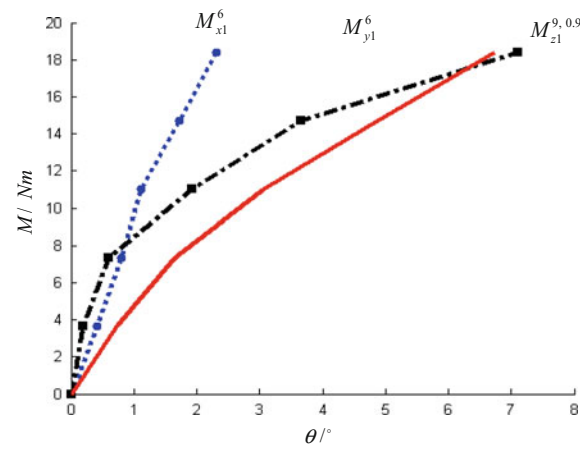


Fig. 2.16 Experimental results of system stiffness



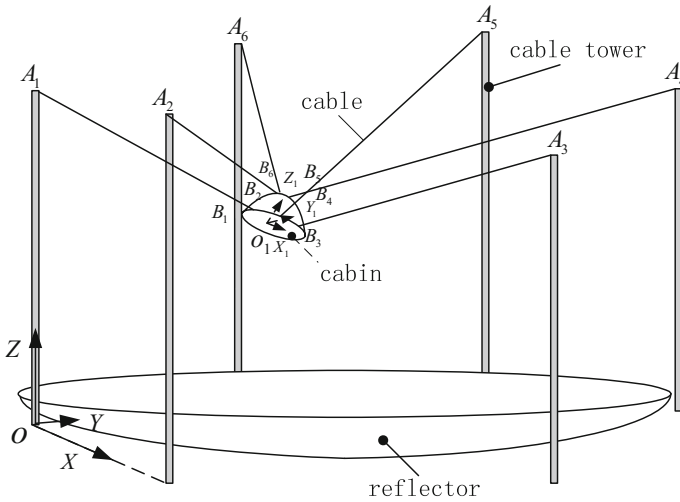


Fig. 2.17 Schematic diagram of CPR for LT

2.3 Dynamic Modeling of a Cable-Suspended Parallel Robot with Non-negligible Cable Mass

2.3.1 System Dynamic Model

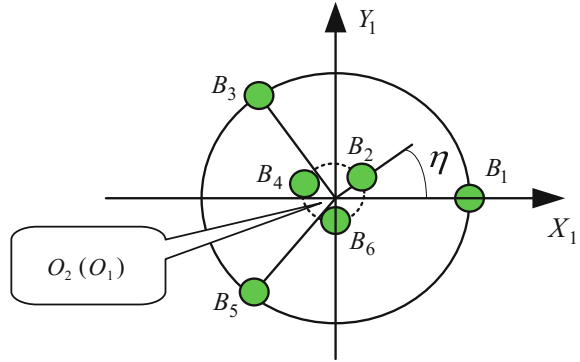
A. Model definition and kinematics

A simple schematic of the CPR for FAST 5-m representing the coordinate systems is shown in Fig. 2.2. The CPR is considered that is kinematically and statically determined. In the design, there is a cable tower/winch pair at each vertex of a regular hexagon of radius a , which actuate six cables that are linked to a cabin. Therefore, the cabin can translate and rotate in the inertial frame. Let the origin of the inertial frame $OXYZ$ be the bottom of tower A_1 . The origin of $O_1X_1Y_1Z_1$ (cabin coordinate system) is in the center of the bottom of the cabin as shown in Fig. 2.18.

In Fig. 2.18, towers A_1 – A_6 are distributed evenly on a circle with the diameter of D . The cables A_1B_1 , A_3B_3 , and A_5B_5 are connected from the bottom of the cabin to the towers A_1 , A_3 , and A_5 , respectively. The other three cables A_2B_2 , A_4B_4 , and A_6B_6 are connected from the towers A_2 , A_4 , and A_6 to the top points B_2 , B_4 , and B_6 of the cabin as shown in Fig. 2.18. Angle η between O_1B_2 and O_1B_1 indicates the relative position relationship of spherical joint B_1 and spherical joint B_2 . It can be noted from Fig. 2.18 that the coordinates of B_2 , B_4 , and B_6 have relationships with η .

The coordinates of the joint A_i on the tower i with respect to the frame $OXYZ$ can be described as

Fig. 2.18 Connection between the cables and the cabin



$$\left. \begin{aligned} \mathbf{r}_{A1} &= (0 \ 0 \ h)^T, & \mathbf{r}_{A2} &= (a \ 0 \ h)^T \\ \mathbf{r}_{A3} &= \left(\frac{3}{2}a \ \frac{\sqrt{3}}{2}a \ h\right)^T, & \mathbf{r}_{A4} &= (a \ \sqrt{3}a \ h)^T \\ \mathbf{r}_{A5} &= (a \ \sqrt{3}a \ h)^T, & \mathbf{r}_{A6} &= \left(-\frac{1}{2}a \ \frac{\sqrt{3}}{2}a \ h\right)^T \end{aligned} \right\} \quad (2.10)$$

where h is the height of tower and a is the side length between the tower A_1 and tower A_2 in regular hexagon. The length of the cable can be changed by rotating the winch to reel the cable in or let it out. The cabin is assumed to be a rigid body.

We proceed to consider the vector diagram for an i th cable. The position of frame $OXYZ$ is represented by vector $\mathbf{r}_{Bi} = [x_{Bi} \ y_{Bi} \ z_{Bi}]^T$, which contains the Cartesian coordinates x, y, z of the origin of frame $O_1X_1Y_1Z_1$ with respect to frame $OXYZ$. The length vector $\mathbf{l}_i$ expressed with respect to frame $OXYZ$ can be computed by

$$\mathbf{l}_i = \mathbf{r}_{Bi} - \mathbf{r}_{Ai} \quad (i = 1, 2, \dots, 6) \quad (2.11)$$

where, $\mathbf{r}_{Bi} = \mathbf{r}_{O1} + R \cdot \mathbf{r}_{Bi1}$ ($i = 1, 2, \dots, 6$) $\mathbf{r}_{O1}$ is the position vector of the origin of $O_1X_1Y_1Z_1$ with respect to $OXYZ$; $\mathbf{r}_{Bi1}$ is the position vector of the point B_i related to $O_1X_1Y_1Z_1$. $\mathbf{r}_{Bi1}$ can be expressed as

$$\left. \begin{aligned} \mathbf{r}_{B11} &= [r, 0, 0]^T \\ \mathbf{r}_{B31} &= [r \cos(\theta), r \sin(\theta), 0]^T \\ \mathbf{r}_{B51} &= [r \cos(2\theta), r \sin(2\theta), 0]^T \end{aligned} \right\} \quad (2.12)$$

$$\left. \begin{aligned} \mathbf{r}_{B21} &= \left[r_1 \cos(\eta), r_1 \sin(\eta), \sqrt{r^2 - (r_1)^2} \right]^T \\ \mathbf{r}_{B41} &= \left[r_1 \cos(\eta + \theta), r_1 \sin(\eta + \theta), \sqrt{r^2 - (r_1)^2} \right]^T \\ \mathbf{r}_{B61} &= \left[r_1 \cos(\eta + 2\theta), r_1 \sin(\eta + 2\theta), \sqrt{r^2 - (r_1)^2} \right]^T \end{aligned} \right\} \quad (2.13)$$

In which, r and r_1 represent the radii of the points B_1, B_3, B_5 and the points B_2, B_4, B_6 of the cabin, respectively. $\theta = 120^\circ$ as shown in Fig. 2.18.

R is the orientation matrix of the cabin with respect to $OXYZ$. By introducing the roll, pitch, and yaw angles into R , the matrix becomes,

$$\mathbf{R} = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix} \quad (2.14)$$

where roll is the rotation about the fixed axis Z by α , pitch is the rotation about the fixed axis Y by β , and yaw is the rotation about the fixed axis X by γ ; c represents cosine function; and s represents sine function.

The Jacobian matrix of CPR is defined as the relation between the velocity of the cabin and the velocity of the driven cable. We specify the velocity of the cable-driven as $\dot{\mathbf{l}} = [\dot{l}_1 \dot{l}_2 \dot{l}_3 \dot{l}_4 \dot{l}_5 \dot{l}_6]^T$ (m/s), define the velocity of the center of the cabin as $\mathbf{v} = (\dot{x}, \dot{y}, \dot{z})^T$ (m/s), and express the angular velocity of the cabin with respect to three axes Z, Y and X as $\mathbf{w} = (\dot{\alpha}, \dot{\beta}, \dot{\gamma})^T$ (rad/s), so the velocity of the cabin can be described as $\dot{\mathbf{r}}_{O1} = [\mathbf{v}\mathbf{w}]^T = [\dot{x} \dot{y} \dot{z} \dot{\alpha} \dot{\beta} \dot{\gamma}]^T$. The relation between $\dot{\mathbf{l}}_i$ and $\dot{\mathbf{r}}_{O1}$ can be defined as [20]

$$\dot{\mathbf{l}} = \mathbf{J} \cdot \dot{\mathbf{r}}_{O1} \quad (2.15)$$

where the Jacobian matrix $\mathbf{J}$ is

$$\mathbf{J} = \left[\frac{\partial l_i}{\partial x} \frac{\partial l_i}{\partial y} \frac{\partial l_i}{\partial z} \frac{\partial l_i}{\partial \alpha} \frac{\partial l_i}{\partial \beta} \frac{\partial l_i}{\partial \gamma} \right] \quad (i = 1, 2, \dots, 6) \quad (2.16)$$

For simplicity, we use the symbolism to develop the Jacobian matrix. The formula can be obtained.

Thus, we can compute the cable-driven lengths, i.e., norms of $\mathbf{l}_i$, from the given positions and orientations of the cabin. This problem is called an inverse kinematics problem of a CPR. The forward kinematics problem is the opposite of the inverse kinematics, i.e., to obtain the positions and orientations from the given cable-driven lengths.

B. Dynamic equations of the cabin

The general dynamic equations of motion can be obtained from the Lagrangian formulation. The generalized coordinates is $\mathbf{q} = [x, y, z, \alpha, \beta, \gamma]^T$. Lagrange's equation can be written in the form of potential energy V , kinetic energy T , and generalized forces or torques in the following form,

$$\frac{d}{dt} \left(\frac{\partial T(\mathbf{q}, \dot{\mathbf{q}})}{\partial \dot{\mathbf{q}}} \right) - \frac{\partial T(\mathbf{q}, \dot{\mathbf{q}})}{\partial \mathbf{q}} + \frac{\partial V(\mathbf{q})}{\partial \mathbf{q}} = \boldsymbol{\tau} \quad (2.17)$$

This results in a kinetic energy of the cabin, which can be written in Cartesian coordinates as,

$$T = \frac{1}{2}m\mathbf{v}^T\mathbf{v} + \frac{1}{2}\mathbf{w}^T\mathbf{I}_{O1}\mathbf{R}^T\mathbf{w} \quad (2.18)$$

or,

$$\begin{aligned} T = & \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2}[I_{xx}(\dot{\alpha}c\alpha c\beta + \dot{\beta}s\alpha c\beta - \dot{\gamma}s\beta)^2 \\ & + I_{yy}((c\alpha s\beta s\gamma - s\alpha c\gamma)\dot{\alpha} + (c\alpha c\gamma + s\alpha s\beta s\gamma)\dot{\beta} + \dot{\gamma}c\beta s\gamma)^2 \\ & + I_{zz}((c\alpha s\beta c\gamma + s\alpha s\gamma)\dot{\alpha} + (s\alpha s\beta c\gamma - c\alpha s\gamma)\dot{\beta} + \dot{\gamma}c\beta c\gamma)^2] \end{aligned} \quad (2.19)$$

where $\mathbf{v} = [\dot{x} \ \dot{y} \ \dot{z}]^T$; $\mathbf{w} = [\dot{\alpha} \ \dot{\beta} \ \dot{\gamma}]^T$; m and $\mathbf{I}_{O1}$ is the mass and the moment of inertia of the cabin, respectively. $\mathbf{I}_{O1}$ is expressed as

$$\mathbf{I}_{O1} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \quad (2.20)$$

In this study, each cable is assumed to be a force element. Therefore, the potential energy of the system is due only to gravitational forces. The potential energy takes the form:

$$V = mgz \quad (2.21)$$

where g is the acceleration due to gravity.

It is noted that there is a relation between externally applied wrench on the cabin and the cable tensions required to keep the system in equilibrium, this relationship [10] is

$$[F_x, F_y, F_z, M_\partial, M_\beta, M_\gamma]^T = -\mathbf{J}^T\mathbf{u} \quad (2.22)$$

where the external force on the cabin at the reference point is F_x, F_y, F_z and the external moment on the cabin body is given by M_x, M_β, M_γ ; $\mathbf{u}$ is the vector of cable-driven tensions.

Through a series of transformations, substitutions and simplifying the resulting expression, here, we write the equations of motion in terms of $\mathbf{q}$ generalized coordinates as the following general form

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) + \boldsymbol{\tau}_d = -\mathbf{J}^T\mathbf{u} \quad (2.23)$$

where $\mathbf{M}(\mathbf{q})$ is the inertia matrix of the system, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is the vector of Coriolis and centripetal terms, $\mathbf{G}(\mathbf{q})$ is the vector of gravity terms, $\boldsymbol{\tau}_d$ is the vector of external disturbance terms (e.g., random wind, etc.). Here, $\mathbf{u}$ is the vector of cable-driven

tensions T_{ji} for $i = 1, 2, \dots, 6$. In the following discussion, we will briefly describe how T_{ji} is derived.

2.3.2 Mechanics Analysis of the Cable

Six flexible cables are used to drive the cabin to implement high-precision trajectory tracking. Due to the eigenfrequency of the cable-cabin system is very low, the response of the cabin is very slow in the presence of external disturbances. The quasi-dynamics analysis is made by substituting static tension of cable at a static position for the dynamic force of the cable at the same position. When the cabin operates at the practically observed speed (0.5–2 cm/s), the difference between static tension and the dynamic force is very small [21]. However, this difference has little influence on the calculation of the cable length, so the dynamic characteristic of cables can be neglected. Owing to six identical cables, we only perform mechanics analysis of a cable.

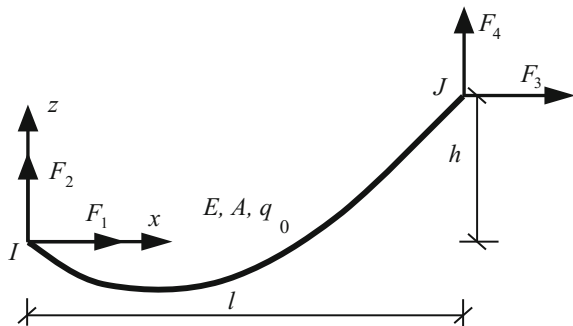
The first challenge for analyzing a cable-suspended system is to mathematically describe the static displacement (or the shape) of the cables under the influence of gravity [22]. To compute the static displacement of a cable, we will assume for the moment that the unstressed length of the cable, L_u , are known. The procedure in this section determines the complete geometry of the cable, end forces from a given strained or unstrained cable length and the positions of cable ends by using Newton-Raphson method.

Considering an elastic cable element stretched in vertical plane, with unstressed length L_u , corresponding stressed length L , elastic modulus E , cross sectional area A , and self-weight per unit length q_0 , as shown in Fig. 2.19, the rigorous relationship between cable projection and end force of the cable is subjected to [23]

$$L^2 = h^2 + \frac{l^2}{\eta^2} \sin^2 \eta \quad (2.24)$$

where

Fig. 2.19 Elastic catenary cable element



$$\eta = \frac{q_0 l}{2F_1} \quad (2.25)$$

$$L_u = L - \frac{1}{2EAq_0} \left(F_4 T_J + F_2 T_I + F_1^2 \ln \frac{F_4 + T_J}{T_I - F_2} \right) \quad (2.26)$$

and

$$F_2 = \frac{q_0}{2} \left[L - h \frac{\cosh \eta}{\sinh \eta} \right] \quad (2.27)$$

$$l = -F_1 \left[\frac{L_u}{EA} + \frac{1}{q_0} \ln \frac{F_4 + T_J}{T_I - F_2} \right] \quad (2.28)$$

$$h = \frac{1}{2EAq_0} (T_J^2 - T_I^2) + \frac{T_J - T_I}{q_0} \quad (2.29)$$

The variables $F_1, F_2, F_3, F_4, T_{fi}$ and T_{ji} (for $i = 1, 2, \dots, 6$) are subjected to the following equations

$$\left. \begin{aligned} F_3 &= -F_1, & F_2 + F_4 &= q_0 L_u \\ T_{fi} &= (F_1^2 + F_2^2)^{1/2}, & T_{ji} &= (F_3^2 + F_4^2)^{1/2} \end{aligned} \right\} \quad (2.30)$$

There exists an unique solution for the cable-driven tension using (2.13), for a given motion of the cabin, as long as $\mathbf{J}$ is not singular. In this case, the solution of (2.13) can be written as

$$\mathbf{u} = -(\mathbf{J}^T)^{-1} [\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) + \boldsymbol{\tau}_d] \quad (2.31)$$

2.4 Conclusions

In this chapter, design and optimization of the feed cable-suspended structures is presented. And the counterweight is added to the top of cabin, and its position varies with orientation in time, which is beneficial to the further reduction of cable tension difference, economizing energy and big orientation. In addition, to enhance the twisting stiffness of the cable-suspended structure, a method by adding stable cables is proposed. The experimental results have demonstrated the effectiveness and feasibility of the proposed method. Then, when the cabin locates at the pose angle of 0° and 20° , respectively, analytical calculations are conducted in three cases, and the antenna pattern is simulated with GRASP 8.0 (see Fig. 2.18). The numerical results show the stable cables have little influence on the electronic performance. Then, the inverse dynamics problem of the CPR with non-negligible

cable mass is formulated by the Lagrange formulation. modeling and analysis of the cable-supporting system including servomechanism dynamics for super antenna is presented. Subsequently, to improve the precision and performance of our model, the dynamic analysis of the cabin including servomechanism dynamics without regard to cable mass is established in detail for the controller design in next chapter.

References

1. Duan BY, Qiu YY, Zhang FS et al (2009) On design and experiment of the feed cable-suspended structure for super antenna. *Mechatronics* 19(4):503–509
2. Nan R (2006) Five hundred meter aperture spherical radio telescope (FAST). *Sci China Ser G* 49(2):129–148
3. Duan BY (1999) A new design project of the line feed structure for large spherical radio telescope and its nonlinear dynamic analysis. *Mechatronics* 9(1):53–64
4. Qiu YH (1998) A novel design for a giant Arecibo-type spherical radio telescope with an active main reflector. *Mon Not R Astron Soc* 301(3):827–830
5. Su YX, Duan BY (2000) The application of the Stewart platform in large spherical radio telescopes. *J Robotic Syst* 17(7):375–383
6. Luo YF, Deng CG, Li GQ et al (2001) Structural analysis of FAST reflector supporting system. *Astrophys Space Sci* 278(1–2):231–236
7. Albus J, Bostelman R, Dagalak N (1992) The NIST robocrane. *J Robot Syst* 10(5)
8. Shiang WJ, Cannon D, Gorman J (1999) Dynamic analysis of the cable array robotic crane. 1999 IEEE international conference on robotics and automation, 1999. Proceedings. vol 4. pp 2495–2500
9. Yamamoto M, Yanai N, Mohri A (2004) Trajectory control of incompletely restrained parallel-wire-suspended mechanism based on inverse dynamics. *IEEE Trans Rob* 20(5):840–850
10. Oh S, Agrawal SK (2005) A reference governor-based controller for a cable robot under input constraints. *IEEE Trans Control Syst Technol* 13(4):639–645
11. Kamman JW, Huston RL (2001) Multibody dynamics modeling of variable length cable systems. *Multibody SysDyn* 5(3):211–221
12. Craig JJ (2005) Introduction to robotics: mechanics and control. Pearson Prentice Hall, Upper Saddle River
13. Anupaju CM, Su CY, Oya M (2005) Adaptive motion tracking control of uncertain nonholonomic mechanical systems including actuator dynamics. *IEE Proc Control Theor Appl* 152(5):575–580
14. Ming A, Higuchi T (1994) Study on multiple degree-of-freedom positioning mechanism using wires. II: Development of a planar completely restrained positioning mechanism. *Int J Jpn Soc Precis Eng* 28(3):235–242
15. Zi B, Duan BY, Du JL et al (2008) Dynamic modeling and active control of a cable-suspended parallel robot. *Mechatronics* 18(1):1–12
16. Oh SR, Agrawal SK (2005) Cable suspended planar robots with redundant cables: controllers with positive tensions. *IEEE Trans Rob* 21(3):457–465
17. Hajela P (1990) Genetic search-an approach to the nonconvex optimization problem. *AIAA J* 28(7):1205–1210
18. Kaveh A, Rahami H (2006) Nonlinear analysis and optimal design of structures via force method and genetic algorithm. *Comput Struct* 84(12):770–778
19. Qiu YY, Duan BY, Wei Q et al (2002) Optimal distribution of the cable tensions and structural vibration control of the cable-cabin flexible structure. *Struct Eng Mech* 14(1):39–56

20. Guo Q, Liu Y, Liu Y (2002) Local structurization kinematic decoupling of six-leg virtual-axis NC machine tool. *IEEE/ASME Trans Mechatron* 7(4):515–518
21. Sun X (2001) A new project of the large spherical radio telescope suspend cable feed supporting system for nonlinear statics, kinematics and dynamics. Ph. D. Dissertation, Xidian University
22. Kozak K, Zhou Q, Wang J (2006) Static analysis of cable-driven manipulators with non-negligible cable mass. *IEEE Trans Rob* 22(3):425–433
23. Karoumi R (1999) Some modeling aspects in the nonlinear finite element analysis of cable supported bridges. *Comput Struct* 71(4):397–412

Design, Analysis and Control of Cable-Suspended
Parallel Robots and Its Applications

Zi, B.; Qian, S.

2017, XII, 299 p. 299 illus., 258 illus. in color.,

Hardcover

ISBN: 978-981-10-1752-0