

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Introduction	1
1.1.1	Elastic Blade	1
1.1.2	Horizontal Force Equilibrium	4
1.1.3	Boundary Conditions	5
1.1.4	Initial Conditions	6
1.1.5	Cantilever Beam Vibrations (Non-rotating)	6
1.1.6	Beam Functions	10
1.1.7	Rotating Beam Vibration	11
1.2	Galerkin Method	12
1.3	Rayleigh–Ritz Method	18
1.4	Finite Element Method	39
1.4.1	Element Properties	40
1.4.2	Energy Expressions	42
1.4.3	Assembly of Elements	45
1.4.4	Cantilever	48
<b>2</b>	<b>Stiff String Basis Functions</b>	<b>63</b>
2.1	Stiff String Equation	63
2.2	Stiff String Basis Functions	65
2.3	Uniform Rotating Beam	68
2.4	Tapered Rotating Beam	69
2.5	Hybrid Basis Functions	72
2.6	Finite Element	77
2.6.1	Uniform Rotating Beam	79
2.7	Tapered Rotating Beam	82
2.8	Summary	87
	References	88

<b>3</b>	<b>Rational Interpolation Functions</b>	89
3.1	Governing Differential Equation	89
3.2	Hermite Shape Functions	91
3.3	New Shape Functions	93
3.4	Static Finite Element Analysis	95
3.5	Dynamic Finite Element Analysis	98
3.5.1	Uniform Beam	98
3.5.2	Tapered Rotating Beam	100
3.6	Summary	104
	References	105
<b>4</b>	<b>Fourier-<math>p</math> Superelement</b>	107
4.1	Governing Equation of Rotating Beams	108
4.2	Shape Functions	109
4.3	Superelement Matrices	111
4.4	Numerical Results	111
4.4.1	Uniform Rotating Beam	112
4.4.2	Tapered Rotating Beam	112
4.5	Summary	118
	References	118
<b>5</b>	<b>Physics Based Basis Functions</b>	121
5.1	Basis Function	122
5.2	Finite Element Analysis	129
5.3	Numerical Results	130
5.3.1	Uniform Beam	130
5.3.2	Tapered Beam	134
5.3.3	Beams with Hub Offset	137
5.4	Summary	138
	References	141
<b>6</b>	<b>Collocation Approach</b>	143
6.1	Governing Differential Equation	143
6.2	Point Collocation Approach	146
6.2.1	Collocation Point at a Variable Location Within Beam Element	146
6.2.2	Collocation Point Near the Left Node of Beam Element	149
6.2.3	Collocation Point at the Midpoint of Beam Element	150
6.2.4	Collocation Point Near the Right Node of Beam Element	152
6.2.5	Two Point Collocation	153
6.2.6	Analysis of Shape Functions	158
6.3	Finite Element Formulation	160

6.4	Numerical Results. . . . .	162
6.4.1	Uniform Rotating Beam . . . . .	162
6.4.2	Tapered Rotating Beam. . . . .	166
6.5	Summary . . . . .	169
	References. . . . .	169
<b>7</b>	<b>Rotor Blade Finite Element . . . . .</b>	<b>171</b>
7.1	Energy Expressions . . . . .	173
7.2	Governing Differential Equations . . . . .	175
7.3	Derivation of the Shape Functions . . . . .	177
7.3.1	Shape Functions for Flapwise Bending. . . . .	179
7.3.2	Shape Functions for Lead-Lag Bending . . . . .	181
7.3.3	Shape Functions for Axial Deflection . . . . .	183
7.3.4	Shape Functions for Torsion. . . . .	184
7.4	Finite Element Method. . . . .	186
7.5	Numerical Results. . . . .	186
7.5.1	Analysis of Shape Functions. . . . .	186
7.5.2	Validation Study . . . . .	194
7.6	Convergence Study of New FEM Element and Polynomials . . . . .	194
7.7	Summary . . . . .	198
	Appendix 1 . . . . .	200
	References. . . . .	202
<b>8</b>	<b>Spectral Finite Element Method. . . . .</b>	<b>205</b>
8.1	Governing Differential Equation. . . . .	206
8.2	Spectral Finite Element Formulation. . . . .	207
8.2.1	Interpolating Function for SFER. . . . .	208
8.2.2	Interpolating Function for SFEN. . . . .	208
8.2.3	Dynamic Stiffness Matrix in Frequency Domain. . . . .	209
8.3	Free Vibration Results . . . . .	211
8.3.1	Uniform Beam . . . . .	211
8.3.2	Tapered Beam 1-Linear Mass and Cubic Flexural Stiffness Variation. . . . .	214
8.3.3	Tapered Beam 2-Linear Mass and Flexural Stiffness Variation. . . . .	216
8.4	Wave Propagation Study . . . . .	219
8.4.1	Convergence Study . . . . .	219
8.4.2	Numerical Results. . . . .	221
8.5	Summary . . . . .	225
	References. . . . .	226
<b>9</b>	<b>Violin String Shape Functions . . . . .</b>	<b>229</b>
9.1	Timoshenko Rotating Beam and Violin String. . . . .	230
9.2	Violin String Shape Functions . . . . .	234
9.3	Results and Discussion. . . . .	240
9.3.1	Uniform Beam . . . . .	240

9.3.2 Tapered Beam. . . . .	242
9.4 Summary . . . . .	248
References. . . . .	249
<b>Appendix A: Stiffness Matrix. . . . .</b>	<b>251</b>
<b>Appendix B: MATLAB Code. . . . .</b>	<b>273</b>
<b>Appendix C: Governing Equation for Rotating Timoshenko Beam . . . .</b>	<b>279</b>

Finite Element Analysis of Rotating Beams

Physics Based Interpolation

Ganguli, R.

2017, XII, 283 p. 108 illus., 19 illus. in color., Hardcover

ISBN: 978-981-10-1901-2