Chapter 2
Conditions of Disturbances Rejection for Discrete First, Second Order and Repetitive Sliding Mode Controllers

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Abstract Harmonic disturbance rejection is an important field of control theory and applications. In this paper a discrete first and second order sliding mode control for multivariable systems are investigated. The necessary conditions of harmonic disturbances rejection using first and second order sliding mode control laws are elaborated. In order to improve the performances of sliding mode control in periodic disturbances rejection, a discrete repetitive sliding mode control is presented. A necessary condition for the choice of the discontinuous terms in discrete repetitive sliding mode control is then developed. The different proposed control strategies have been tested on numerical simulation example. The obtained results are very satisfactory in terms of compensation of periodic disturbances using discrete repetitive sliding mode control.

Keywords Conditions of disturbances rejection · Discrete sliding mode control · Second order sliding mode control · Repetitive sliding mode control · Multivariable control systems · Rejection of periodic disturbances

2.1 Introduction

Due to the development and the progress of technology, many industrial processes are became more complex (multivariable, non linear, with parameter uncertainties and external disturbances, ...). Then, the representation of such systems by an exact mathematical model and the development of an adequate control law are extremely difficult.

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To overcome this problem, various control strategies have been proposed in the last two decades. Sliding Mode Control (SMC) as a Variable Structure System (VSS) has been widely used in different engineering fields and carried out excellent performances in many industrial applications (Lopez and Nouri 2006; Yu and Kaynak 2009). The SMC is known by its robustness against uncertainties and external disturbances. The main idea behind SMC is to use a discontinuous control input to drive state trajectories towards desired sliding surface in finite time and maintain them on it. Nevertheless, the main drawback of the sliding mode control is the chattering phenomenon caused by the discontinuous part of the control law. The phenomenon of chattering is a harmful phenomenon and it often leads to undesirable results.

Due to the development of computer and the use of digital control, discrete sliding mode control (DSMC) has become more important research in many theoretical and practical control systems (Gao et al. 1995; Bartoszewicz 1998; Stoica 2008). However, the properties of continuous SMC is not available in the case of DSMC because of the finite sampling rate (Young et al. 1999). Then, the problem of chattering phenomenon is more difficult in discrete sliding mode control. Several methods have been proposed to reduce this problem. One solution is to replace the signum function by a smoother function in the boundary layer such as the saturation function. Another solution is to use a high order sliding mode control (HOSMC) (Mihoub et al. 2009; Romdhane et al. 2015; Cavallo and Natale 2004).

Another problem of the DSMC is its sensitivity to external disturbances. In a variety of industrial processes, these disturbances are, often, periodic signals (robotic, rotating machine tools, active noise control ...). In electrical networks, non-linear devices are the main source of harmonic disturbances. All power electronic converters can generate harmonic disturbances by injecting harmonic currents in the power system (Zhou and Wang 2003). Under the effect And turning fields, the couple can cause mechanical vibration in rotating machinery. For example, in the hard disk drives, the periodic disturbances are caused by the eccentricity on the track at the frequency of rotation of the disk (Chang et al. 2006). In the robot-assisted laparoscopic surgery of the digestive system, physiological movements caused by breathing and heartbeat can be considered as a periodic disturbances (Ginhoux 2003), etc.

In order to resolve this problem, several proposals have been made to modify the discrete classical sliding mode control. Bartoszewicz et al. (Ginhoux 2003) proposed a new non-switching reaching law for periodic review inventory systems. In (Mihoub et al. 2011), a multimodel discrete second order sliding mode control for non stationary system was formulated. Young et al. (Young et al. 1999) proposed a discrete sliding mode control with delayed disturbance compensator. In (Yan et al. 2013), a discrete sliding mode control with decoupled perturbation estimator based on computation time delay was developed. A discrete sliding mode control with a nonlinear observer is designed to estimate non measurable states and perturbation and applied to induction motors in (Castillo-Toledoa et al. 2008). Bandyopadhyay et al. (Bandyopadhyay and Fulwani 2009) proposed a nonlinear sliding surface and disturbance observer to design sliding mode control for discrete multiple-input multiple-output linear systems with matched perturbations.

In parallel a discrete adaptive sliding mode controller have been developed (Monsees 2002; Chan 1997).

Nevertheless, the use of disturbance observer can make control system more complex. Another approach is to combine the sliding mode control with repetitive control (Dehri et al. 2011, 2012a, b).

Repetitive control (RC) is one of the most common approach in dealing with the periodic disturbance rejection (Arimoto et al. 1984; Xuan et al. 2013). It has been investigated in the continuous and discrete time domain and in different engineering areas (robots (Xuan et al. 2007; Fateha et al. 2013), hard disk drives (Chang et al. 2006), pulse width modulation inverters (Zhou and Wang 2003), etc.). The main idea behind repetitive control is to remove errors that occur at the fundamental and harmonics frequency of the periodic signal. This control is based on the Internal Model Principle (IMP) (Francis and Wonham 1976) which consists in simply incorporating the model of the disturbance into the controller configuration. A periodic signal can be generated by a delay block with a positive feedback loop. However, the repetitive control is confronted with several problems: such as the problem of stability and the inability to take into account certain characteristics of processes (Doh and Ryoo 2006).

In order to resolve these problems, we have proposed to combine the repetitive control with the sliding mode control for discrete multivariable systems (Dehri et al. 2011, 2012a, b).

This work focuses on the synthesis of a necessary condition to reject periodic disturbances for multivariable systems using discrete first, second order and repetitive sliding mode controllers.

This paper is organized as follows. The synthesis of a necessary condition for rejection external disturbances using classical discrete multivariable sliding mode control is presented in Sect. 2.2. In Sect. 2.3, a necessary condition for rejection external disturbances in discrete second order sliding mode control for multivariable systems is developed. A necessary condition for rejection periodic disturbances of discrete repetitive sliding mode control is then proposed in Sect. 2.4. Section 2.5 gives the simulation results of the discrete first, second order and repetitive sliding mode controllers for a chemical reactor model subjected to periodic disturbances.

2.2 Conditions of Disturbances Rejection in Discrete Multivariable Sliding Mode Control

Consider a linear multivariable discrete-time system:

$$x(k+1) = Ax(k) + Bu(k) \tag{2.1}$$

where $x(k)$ and $u(k)$ are respectively the state and input vectors:

$$x(k) = \left[x_1(k) \ \ldots \ x_n(k) \right]^\top$$

$$u(k) = \left[u_1(k) \ \ldots \ u_m(k) \right]^\top$$

A and B are respectively $n \times n$ and $n \times m$ known constant matrices. We suppose that the matrix B is of full rank.

The sliding function is defined as:

$$S(k) = Cx(k) = [\, s_1(k) \ \cdots \ s_m(k)]^\top \tag{2.2}$$

where C is an (m, n) matrix chosen using the pole assignment method (Chang and Chen 2000).

The reaching law can be written as follows (Bartoszewicz 1998):

$$S(k+1) = \Phi S(k) - \begin{bmatrix} m_1 \text{sign}(s_1(k)) \\ m_2 \text{sign}(s_2(k)) \\ \vdots \\ m_m \text{sign}(s_m(k)) \end{bmatrix} \tag{2.3}$$

where Φ ($\Phi \in \Re^{m \times m}$) is a diagonal matrix such
$0 \leq \Phi_{i,\, i} < 1, \quad \forall \ i = 1 \ldots m$ and m_i ($m_i \in \Re$) is a positive gain.
And sign is the signum function defined as:

$$\text{sign}(s_i(k)) = \begin{cases} -1 & \text{si} \quad s_i(k) < 0 \\ \ \ 1 & \text{si} \quad s_i(k) > 0 \end{cases}; \qquad i \in \left[1 \ldots m \right]$$

Using the considered reaching law, the sliding mode control law can be expressed as:

$$u(k) = (CB)^{-1} \left[-CAx(k) + \Phi S(k) \right] - (CB)^{-1} \begin{bmatrix} m_1 \text{sign}(s_1(k)) \\ \vdots \\ m_m \text{sign}(s_m(k)) \end{bmatrix} \tag{2.4}$$

To study the robustness of the control law, we suppose that the system is subject to external disturbances as follows:

$$x(k+1) = Ax(k) + Bu(k) + f(k) \tag{2.5}$$

where $f(k)$ is the external disturbances vector.

Suppose the following matching condition is satisfied:

$$f(k) = Bd(k); \quad d(k) = \begin{bmatrix} d_1(k) \\ d_2(k) \\ \vdots \\ d_m(k) \end{bmatrix} \tag{2.6}$$

Then, the system (2.5) is equivalent to:

$$x(k+1) = Ax(k) + B[u(k) + d(k)] \tag{2.7}$$

Conditions of Disturbances Rejection

By applying the sliding control law (2.4) to the system (2.5), the sliding functions vector is given by:

$$\begin{aligned} S(k+1) &= Cx(k+1) = CAx(k) + CBu(k) + CBd(k) \\ &= \Phi S(k) - \begin{bmatrix} m_1 \operatorname{sign}(s_1(k)) \\ m_2 \operatorname{sign}(s_2(k)) \\ \vdots \\ m_m \operatorname{sign}(s_m(k)) \end{bmatrix} + CBd(k) \end{aligned}$$

A necessary and sufficient condition for the existence of a quasi sliding mode is defined as (Sarpturk et al. 1987):

$$|s_i(k+1)| < |s_i(k)|, \quad \forall\ i = 1 \ldots m \tag{2.8}$$

The previous condition is equivalent to:

$$\begin{cases} (s_i(k+1) - s_i(k)) \operatorname{sign}(s_i(k)) < 0, & \forall\ i = 1 \ldots m \\ (s_i(k+1) + s_i(k)) \operatorname{sign}(s_i(k)) > 0, & \forall\ i = 1 \ldots m \end{cases}$$

Suppose that:

$$\bar{d}(k) = CBd(k) = \left[\bar{d}_1(k)\ \bar{d}_2(k)\ \ldots\ \bar{d}_m(k) \right]^\top$$

We have:

$$s_i(k+1) = \Phi_{i,\,i} s_i(k) - m_i \operatorname{sign}(s_i(k)) + \bar{d}_i(k), \quad \forall\ i = 1 \ldots m$$

Case $s_i(k) > 0$:
In this case, the conditions of existence of a quasi-sliding mode are:

$$\begin{cases} s_i(k+1) - s_i(k) < 0 \\ s_i(k+1) + s_i(k) > 0 \end{cases}$$

or

$$s_i(k+1) - s_i(k) = -(1 - \Phi_{i,i})s_i(k) - m_i + \bar{d}_i(k)$$

then:

$$s_i(k+1) - s_i(k) = (\Phi_{i,i} - 1)s_i(k) - m_i + \bar{d}_i(k) < 0 \Rightarrow m_i > (\Phi_{i,i} - 1)s_i(k) + \bar{d}_i(k)$$

We have:

$$s_i(k+1) + s_i(k) = (1 + \Phi_{i,i})s_i(k) - m_i + \bar{d}_i(k)$$

then:

$$s_i(k+1) + s_i(k) = (\Phi_{i,i} + 1)s_i(k) - m_i + \bar{d}_i(k) > 0 \Rightarrow m_i < (\Phi_{i,i} + 1)s_i(k) + \bar{d}_i(k)$$

Case $s_i(k) < 0$:
The conditions of existence of a quasi-sliding mode are:

$$\begin{cases} s_i(k+1) - s_i(k) > 0 \\ s_i(k+1) + s_i(k) < 0 \end{cases}$$

or

$$s_i(k+1) - s_i(k) = -(1 - \Phi_{i,i})s_i(k) + m_i + \bar{d}_i(k)$$

then

$$s_i(k+1) - s_i(k) = (\Phi_{i,i} - 1)s_i(k) + m_i + \bar{d}_i(k) > 0 \Rightarrow m_i > -(\Phi_{i,i} - 1)s_i(k) - \bar{d}_i(k)$$

We have:

$$s_i(k+1) + s_i(k) = (\Phi_{i,i} + 1)s_i(k) + m_i + \bar{d}_i(k) < 0 \Rightarrow m_i < -(\Phi_{i,i} + 1)s_i(k) - \bar{d}_i(k)$$

Theorem 1 *The discrete sliding mode control law defined in (2.4) allows the rejection of the external disturbances if and only if the gains m_i satisfy:*

$$(\Phi_{i,i} - 1)\,|s_i(k)| + \bar{d}_i(k)\,\text{sign}(s_i(k)) < m_i < (\Phi_{i,i} + 1)\,|s_i(k)| + \bar{d}_i(k)\,\text{sign}(s_i(k)) \tag{2.9}$$

According to this condition, it is noted that the rejection of external disturbances is possible only if we know exactly these disturbances. If the disturbances are not known, an estimation phase is required.

Also, the minimum quasi-sliding mode band $2\mu_i$ depends on the maximum norm of the disturbances:

$$2\mu_i > \frac{2m_i}{1 - \Phi_{i,i}}$$

Consequently, if these disturbances are relatively important, a large amplitude oscillations is appeared which can excite the high frequencies and damage the controlled system.

To resolve this problem, the discrete second order sliding mode control is described in the following paragraph.

2.3 Conditions of Disturbances Rejection in Discrete Multivariable Second Order Sliding Mode Control

Discrete second order sliding mode control (2-DSMC) is a major approach being used of higher order sliding mode control. It consists in force the the state to move on the sliding surface and to keep it first derivative null.

There are some works, in literature, which used 2-DSMC for multiple-input multiple-output (MIMO) systems because it is well known that the control problems of MIMO systems are very difficult (Chang 2002; Romdhane et al. 2015; Monsees 2002; Dehri et al. 2011, 2012a, b; Chang and Chen 2000).

In the case of the multivariable second order sliding mode control, the sliding functions vector is selected as follows (Mihoub et al. 2011):

$$\sigma(k) = S(k) + \beta S(k-1) = \left[\sigma_1(k) \ \ldots \ \sigma_m(k)\right]^\top \tag{2.10}$$

where β is an (m, m) diagonal matrix.

The equivalent control law is obtained if the following relation is verified:

$$\sigma(k+1) = \sigma(k) = 0$$

Then, the expression of the equivalent control law can be calculated as:

$$u_{eq}(k) = (CB)^{-1}\left[-\beta S(k) - CAx(k)\right]$$

The robustness is ensured by the addition of a discontinuous term $u_{dis}(k)$ such as:

$$\begin{aligned} u_{dis}(k) &= u_{dis}(k-1) - (CB)^{-1}T_e \begin{bmatrix} m_1^{'}\operatorname{sign}(\sigma_1(k)) \\ \vdots \\ m_i^{'}\operatorname{sign}(\sigma_i(k)) \\ \vdots \\ m_m^{'}\operatorname{sign}(\sigma_m(k)) \end{bmatrix} \\ &= u_{dis}(k-1) - (CB)^{-1}\left[m_1^{''}\operatorname{sign}(\sigma_1(k)) \ \ldots \ m_m^{''}\operatorname{sign}(\sigma_m(k))\right]^\top \end{aligned}$$

with T_e is the sampling time.

The discrete second order sliding mode control is then expressed as:

$$u(k) = u_{eq}(k) + u_{dis}(k) \tag{2.11}$$

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Using (2.5) and (2.11), the sliding functions vector $S(k+1)$ can be written as:

$$S(k+1) = Cx(k+1) = -\beta S(k) + CBd(k) + CBu_{dis}(k)$$

Based on the last relation and (2.10), the sliding functions vector $\sigma(k+1)$ can be written as:

$$\begin{aligned}\sigma(k+1) &= \sigma(k) + CB(d(k) - d(k-1)) \\ &- \left[m_1^{''}\text{sign}(\sigma_1(k)) \ \ldots \ m_m^{''}\text{sign}(\sigma_m(k)) \right]^\top\end{aligned}$$

A necessary and sufficient condition for the existence of a quasi sliding mode in the case of discrete second order sliding mode control is defined as:

$$|\sigma_i(k+1)| < |\sigma_i(k)|\,, \quad \forall \ i = 1 \ldots m \tag{2.12}$$

The previous inequality is equivalent to the following two inequalities:

$$\begin{cases} (\sigma_i(k+1) - \sigma_i(k))\text{sign}(\sigma_i(k)) < 0 \\ (\sigma_i(k+1) + \sigma_i(k))\text{sign}(\sigma_i(k)) > 0 \end{cases}$$

Suppose that:

$$\tilde{d}(k) = CBd(k) - CBd(k-1) = \left[\tilde{d}_1(k) \ \tilde{d}_2(k) \ \ldots \ \tilde{d}_m(k) \right]^\top$$

We have:

$$\sigma_i(k+1) = \sigma_i(k) - m_i^{''}\text{sign}(\sigma_i(k)) + \tilde{d}_i(k), \quad \forall \ i = 1..m$$

Case $\sigma_i(k) > 0$:
The conditions of existence of a quasi-sliding mode becomes:

$$\begin{cases} \sigma_i(k+1) - \sigma_i(k) < 0 \\ \sigma_i(k+1) + \sigma_i(k) > 0 \end{cases}$$

We have

$$\sigma_i(k+1) - \sigma_i(k) = -m_i^{''} + \tilde{d}_i(k)$$

So:

$$\sigma_i(k+1) - \sigma_i(k) = -m_i^{''} + \tilde{d}_i(k) < 0 \Rightarrow m_i^{''} > \tilde{d}_i(k)$$

Using the following equality:

$$\sigma_i(k+1) + \sigma_i(k) = 2\sigma_i(k) - m_i^{''} + \tilde{d}_i(k)$$

We obtain:

$$\sigma_i(k+1) + \sigma_i(k) = 2\sigma_i(k) - m_i^{''} + \tilde{d}_i(k) > 0 \Rightarrow m_i^{''} < 2\sigma_i(k) + \tilde{d}_i(k)$$

Case $\sigma_i(k) < 0$:
The conditions of existence of a quasi-sliding mode are:

$$\begin{cases} \sigma_i(k+1) - \sigma_i(k) > 0 \\ \sigma_i(k+1) + \sigma_i(k) < 0 \end{cases}$$

which yields

$$\sigma_i(k+1) - \sigma_i(k) = m_i^{''} + \tilde{d}_i(k) > 0 \Rightarrow m_i^{''} > -\tilde{d}_i(k)$$

$$\sigma_i(k+1) + \sigma_i(k) = 2\sigma_i(k) + m_i^{''} + \tilde{d}_i(k) < 0 \Rightarrow m_i^{''} < -2\sigma_i(k) - \tilde{d}_i(k)$$

Theorem 2 *The discrete second order sliding mode control law defined in (2.11) allows the rejection of the external disturbances if and only if the gains $m_i^{''}$ satisfy:*

$$\tilde{d}_i(k)\,\mathrm{sign}(\sigma_i(k)) < m_i^{''} < 2\,|\sigma_i(k)| + \tilde{d}_i(k)\,\mathrm{sign}(\sigma_i(k)) \tag{2.13}$$

We can conclude that the discrete second order sliding mode control is able to reduce the chattering phenomenon and to reject constant disturbances without necessity of estimating them by the integration of the discontinuous term.

Also, the minimum quasi-sliding mode band depends on the maximum norm of the derivative of disturbances.

Conditions of Rejection of Periodic Disturbances

The disturbance vector is given as follows:

$$d(k) = \begin{bmatrix} \sum_{j=0}^{N_d} a_{1j}\cos(jwk) + b_{1j}\sin(jwk) \\ \vdots \\ \sum_{j=0}^{N_d} a_{ij}\cos(jwk) + b_{ij}\sin(jwk) \\ \vdots \\ \sum_{j=0}^{N_d} a_{mj}\cos(jwk) + b_{mj}\sin(jwk) \end{bmatrix} \tag{2.14}$$

The difference between $d(k)$ and $d(k-1)$ can be majored as:

$$|d(k) - d(k-1)| \leq wT_e \begin{bmatrix} \sum_{j=0}^{N_d} |a_{1j}| + |b_{1j}| \\ \vdots \\ \sum_{j=0}^{N_d} |a_{ij}| + |b_{ij}| \\ \vdots \\ \sum_{j=0}^{N_d} |a_{mj}| + |b_{mj}| \end{bmatrix} \leq wT_e d_{\max}$$

with $d_{\max} = \sup(d(k)) = \begin{bmatrix} d_{1\max} & \dots & d_{m\max} \end{bmatrix}^\top$

The condition of periodic disturbances rejection in discrete second order sliding mode control is formulated as:

$$-wT_e d_{i\max} < m_i^{''} < 2|\sigma_i(k)| + wT_e d_{i\max} \tag{2.15}$$

The discrete sliding mode control can reject periodic disturbances if the period of these disturbances is very high.

A solution for this problem was proposed in (Dehri et al. 2011, 2012a, b) by combining the discrete sliding mode control and the repetitive control.

2.4 Conditions of Disturbances Rejection in Discrete Multivariable Repetitive Sliding Mode Control

In the presence of periodic disturbances, the discrete multivariable first and second order sliding mode control performances are decreased considerably. In order to overcome these problems, we propose to use the discrete multivariable repetitive sliding mode control.

We suppose that the disturbances vector $d(k)$ is periodic with the period N:

$$d(k) = \begin{bmatrix} d_1(k) \\ \vdots \\ d_i(k) \\ \vdots \\ d_m(k) \end{bmatrix} = \begin{bmatrix} d_1(k-N) \\ \vdots \\ d_i(k-N) \\ \vdots \\ d_m(k-N) \end{bmatrix} = d(k-N) \tag{2.16}$$

The difference between $s(k+1)$ and $s(k+1-N)$ can be calculated as:

$$s(k+1) - s(k+1-N) = \begin{bmatrix} CAx(k) + CBu(k) + CBd(k) \\ -CAx(k-N) - CBu(k-N) - CBd(k-N) \end{bmatrix}$$

Using the last relation and (2.3), the discrete repetitive sliding mode control law for multivariable systems subjected to periodic disturbances can be expressed as:

$$\begin{aligned} u(k) = u(k-N) - (CB)^{-1}CA(x(k) - x(k-N)) \\ - (CB)^{-1}\begin{bmatrix} m_1\mathrm{sign}(s_1(k)) \\ m_2\mathrm{sign}(s_2(k)) \\ \vdots \\ m_i\mathrm{sign}(s_i(k)) \\ \vdots \\ m_m\mathrm{sign}(s_m(k)) \end{bmatrix} \\ + (CB)^{-1}(\Phi s(k) - s(k+1-N)) \end{aligned} \tag{2.17}$$

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Replacing the control law by its expression (2.17), the sliding functions vector is then given as follows:

$$\begin{aligned} & S(k+1) = Cx(k+1) = CAx(k) + CBu(k) + CBd(k) \\ & = CAx(k) + CBu(k-N) - CA(x(k) - x(k-N)) + \Phi S(k) - S(k+1-N) \\ & - \begin{bmatrix} m_1\mathrm{sign}(s_1(k)) \\ m_2\mathrm{sign}(s_2(k)) \\ \vdots \\ m_m\mathrm{sign}(s_m(k)) \end{bmatrix} + CBd(k) \\ & = \Phi S(k) - \begin{bmatrix} m_1\mathrm{sign}(s_1(k)) \\ m_2\mathrm{sign}(s_2(k)) \\ \vdots \\ m_m\mathrm{sign}(s_m(k)) \end{bmatrix} + CB(d(k) - d(k-N)) \end{aligned}$$

A necessary and sufficient condition for the existence of a quasi sliding mode is defined by (2.8).

Suppose that:

$$\underline{d}(k) = CB(d(k) - d(k-N)) = \begin{bmatrix} \underline{d}_1(k) \; \underline{d}_2(k) \; \ldots \; \underline{d}_m(k) \end{bmatrix}^{\top}$$

We have:

$$s_i(k+1) = \Phi_{i,\,i}s_i(k) - m_i\mathrm{sign}(s_i(k)) + \underline{d}_i(k), \quad \forall\; i = 1 \ldots m$$

Case $s_i(k) > 0$:
In this case, the conditions of existence of a quasi-sliding mode are:

$$\begin{cases} s_i(k+1) - s_i(k) < 0 \\ s_i(k+1) + s_i(k) > 0 \end{cases}$$

then:

$$s_i(k+1) - s_i(k) = (\Phi_{i,i} - 1)s_i(k) - m_i + \underline{d}_i(k) < 0 \Rightarrow m_i > (\Phi_{i,i} - 1)s_i(k) + \underline{d}_i(k)$$

$$s_i(k+1) + s_i(k) = (\Phi_{i,i} + 1)s_i(k) - m_i + \underline{d}_i(k) > 0 \Rightarrow m_i < (\Phi_{i,i} + 1)s_i(k) + \underline{d}_i(k)$$

Case $s_i(k) < 0$:
The conditions of existence of a quasi-sliding mode becomes

$$\begin{cases} s_i(k+1) - s_i(k) > 0 \\ s_i(k+1) + s_i(k) < 0 \end{cases}$$

Which gives:

$$s_i(k+1) - s_i(k) = (\Phi_{i,i} - 1)s_i(k) + m_i + \underline{d}_i(k) > 0 \Rightarrow m_i > -(\Phi_{i,i} - 1)s_i(k) - \underline{d}_i(k)$$

$$s_i(k+1) + s_i(k) = (\Phi_{i,i} + 1)s_i(k) + m_i + \underline{d}_i(k) < 0 \Rightarrow m_i < -(\Phi_{i,i} + 1)s_i(k) - \underline{d}_i(k)$$

Theorem 3 *The discrete multivariable repetitive sliding mode control law defined in (2.17) allows the rejection of the external disturbances if and only if the gains m_i satisfy:*

$$(\Phi_{i,i} - 1)\,|s_i(k)| + \underline{d}_i(k)\ \mathrm{sign}(s_i(k)) < m_i < (\Phi_{i,i} + 1)\,|s_i(k)| + \underline{d}_i(k)\ \mathrm{sign}(s_i(k)) \tag{2.18}$$

We remark that the discrete multivariable repetitive sliding mode control is able to reject periodic disturbances without necessity of estimating them.

Also, the minimum quasi-sliding mode band depends on the maximum norm of the difference of disturbances between instances k and $k - N$.

2.5 Simulation Results

Consider the mathematical model of a chemical reactor (Stoica 2008):

$$x(k+1) = Ax(k) + Bu(k) + Bd(k)$$

where:

$$A = \begin{bmatrix} 0.9580 & 0 & 0 & 0 \\ 0 & 0.9418 & 0 & 0 \\ 0 & 0 & 0.9048 & 0 \\ 0 & 0 & 0 & 0.9277 \end{bmatrix}; \quad B = \begin{bmatrix} 0.25 & 0 \\ 0.25 & 0 \\ 0 & 0.5 \\ 0 & 0.5 \end{bmatrix}$$

with

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix}; \quad u(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix}; \quad d(k) = \begin{bmatrix} d_1(k) \\ d_2(k) \end{bmatrix}$$

The periodic disturbances vector is chosen as:

$$\begin{cases} d_1(k) = 0.1 + 0.5\sin\left(\frac{2\pi k}{N}\right) + 0.2\cos\left(\frac{2\pi k}{N}\right) \\ d_2(k) = 0.2 + 0.4\sin\left(\frac{2\pi k}{N}\right) + 0.3\cos\left(\frac{2\pi k}{N}\right) \end{cases}; \quad N = 30$$

The retained synthesis parameters are:

$$\Phi_{1,\,1} = \Phi_{2,\,2} = 0.1; \qquad C = \begin{bmatrix} 286.7629 & -282.7629 & 16.9876 & -16.9876 \\ -12.6797 & 12.6797 & -96.1930 & 98.1930 \end{bmatrix}$$

The evolution of disturbances $d_1(k)$ and $d_2(k)$ is given in Fig. 2.1.

In this section, three kinds of controllers are considered: the first order discrete multivariable sliding mode control (DSMC), the second order discrete multivariable sliding mode control (2-DSMC) and the discrete multivariable repetitive sliding mode control (DRSMC).

Firstly, a first order discrete multivariable sliding mode control is used.

The simulation results are shown in Figs. 2.2, 2.3 and 2.4. Figure 2.2 gives the evolutions of the states. Figure 2.3 presents the evolutions of the control signals $u_1(k)$ and $u_2(k)$. The evolution of the sliding surfaces $s_1(k)$ and $s_2(k)$ is illustrated by Fig. 2.4.

It can be noted that all the states and sliding functions track the origin with periodic error due to the presence of external periodic disturbances. As a conclusion, the first order DSMC is not able to reject effectively the considered periodic disturbances.

This is proved by Theorem 1. Indeed, the selected gains are not satisfied the condition (2.9) without knowing perfectly the considered disturbances.

For classical discrete sliding mode control, it was noted:

- the presence of chattering phenomenon.
- the rejection of constant or harmonic disturbances is only possible when these disturbances are known or estimated.

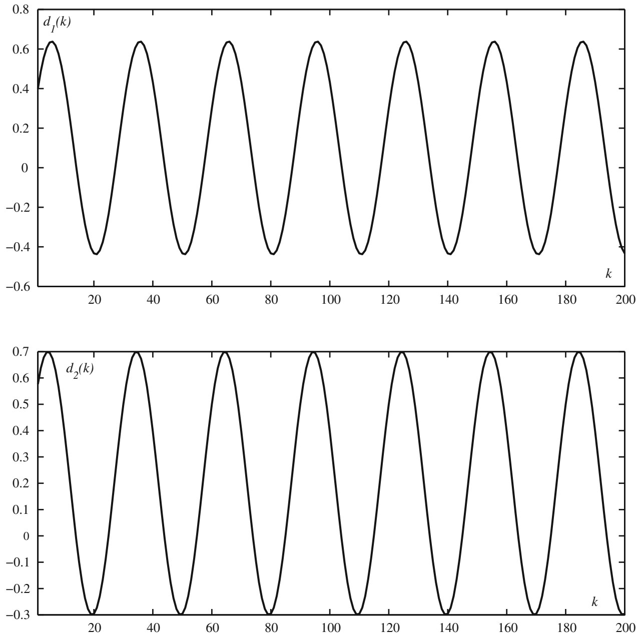

Fig. 2.1 Evolutions of periodic disturbances $d_1(k)$ and $d_2(k)$

Secondly, in order to ameliorate the performance of the first order discrete multivariable sliding mode control, we use the discrete multivariable second order sliding mode control.

The simulation results are shown in Figs. 2.5, 2.6 and 2.7. The evolution of the states is given in Fig. 2.5. Figure 2.6 shows the evolutions of the inputs $u_1(k)$ and $u_2(k)$. The evolution of sliding surfaces is presented in Fig. 2.7.

We observe that the presence of periodic disturbances can cause periodic errors at sliding surfaces. Also, it can be seen that a minor periodic errors are obtained using 2-DSMC as shown by comparing Figs. 2.2 and 2.5. This is due to the ability of discrete second order sliding mode control to reject constant terms of external disturbances.

Thirdly, the discrete multivariable repetitive sliding mode control (2.17), which is a combination between DSMC and repetitive approach, is used in order to reject periodic disturbances.

The simulation results of the system with the considered control law are shown in Figs. 2.8, 2.9 and 2.10. Figure 2.8 gives the evolutions of the states $x_1(k)$, $x_2(k)$, $x_3(k)$ and $x_4(k)$,. Figure 2.9 presents the evolutions of the inputs $u_1(k)$ and $u_2(k)$. The evolution of the sliding functions $s_1(k)$ and $s_2(k)$ is presented in Fig. 2.10. These figures proves that a relatively satisfactory performances are recorded in terms of rejecting periodic disturbances.

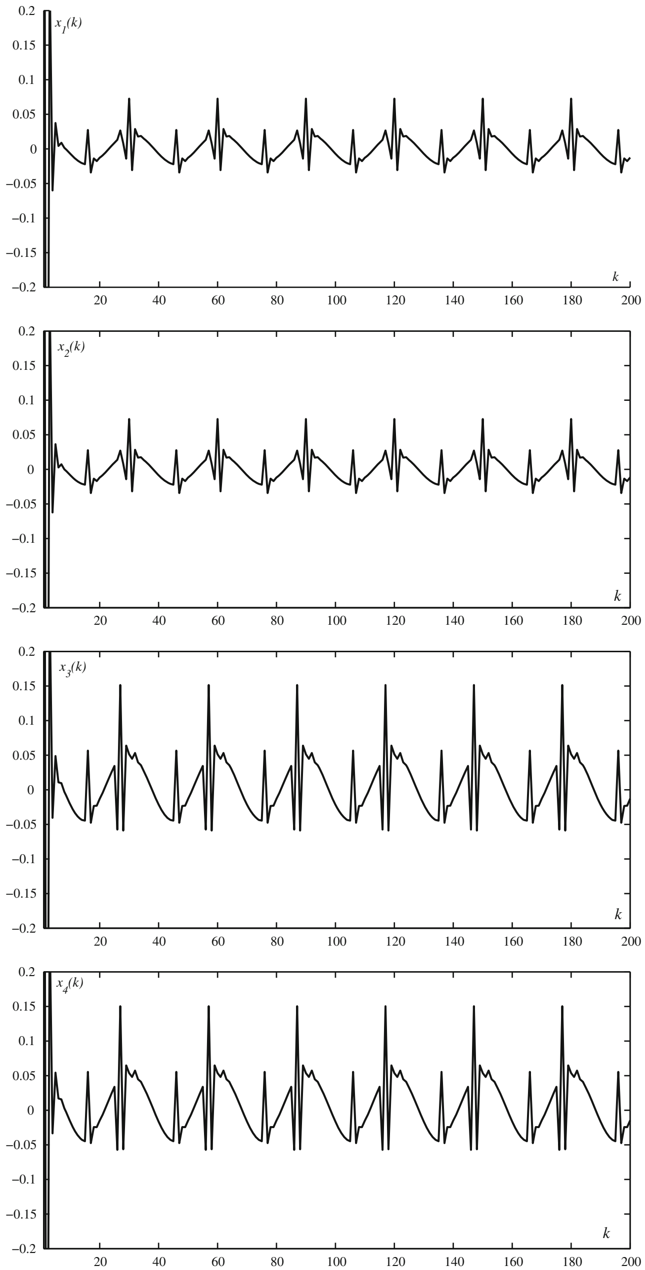

Fig. 2.2 Evolutions of the states $x_1(k)$, $x_2(k)$, $x_3(k)$ and $x_4(k)$ (DSMC)

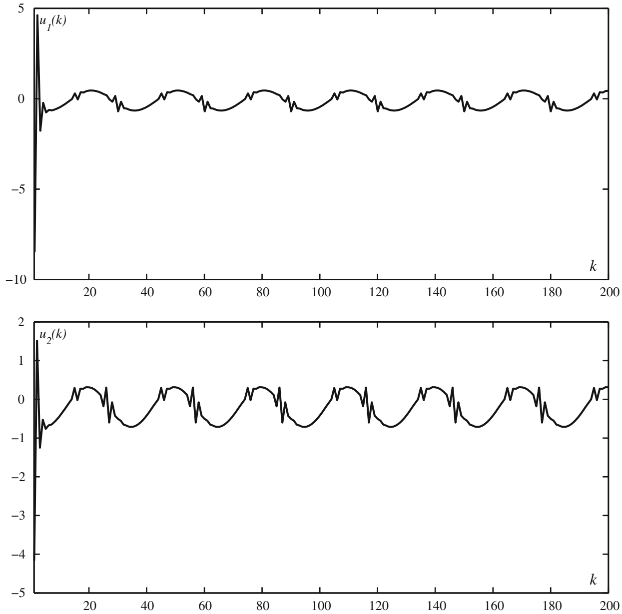

Fig. 2.3 Evolutions of the control signals $u_1(k)$ and $u_2(k)$ (DSMC)

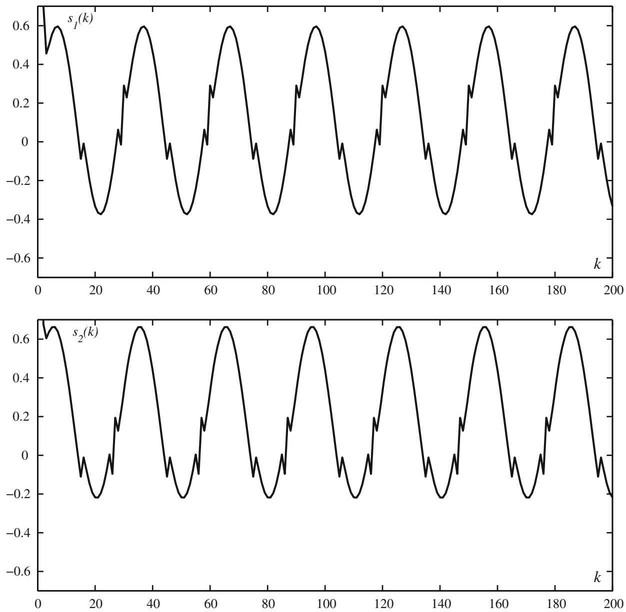

Fig. 2.4 Evolutions of the sliding functions $s_1(k)$ and $s_2(k)$ (DSMC)

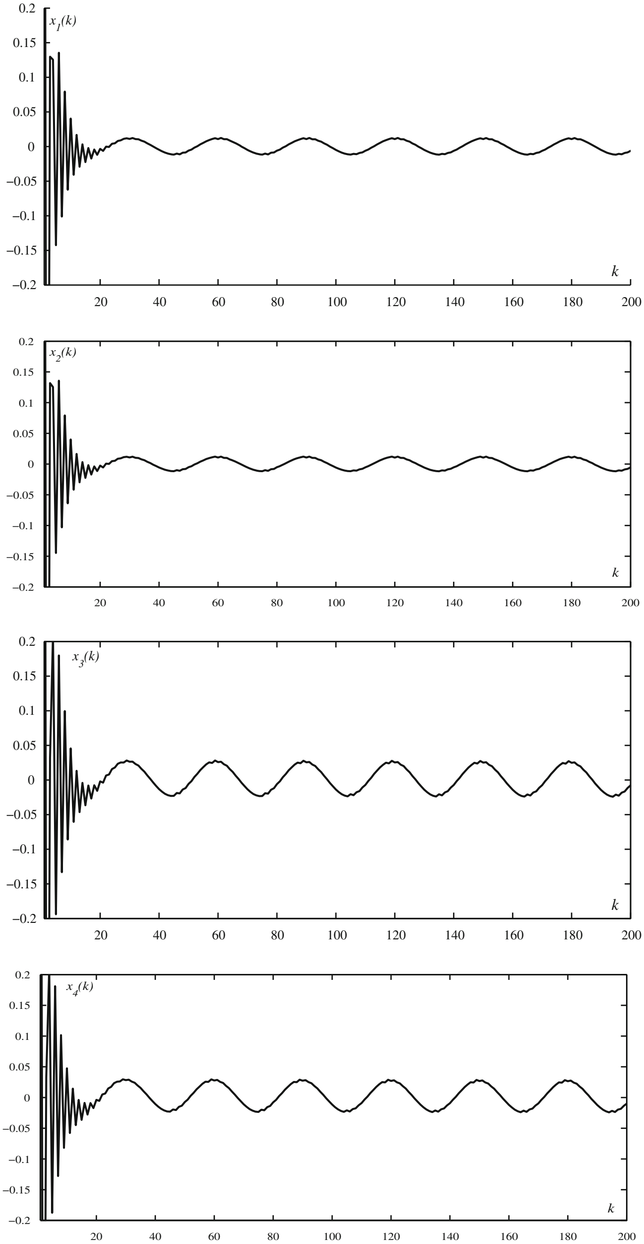

Fig. 2.5 Evolutions of the states $x_1(k)$, $x_2(k)$, $x_3(k)$ and $x_4(k)$(2-DSMC)

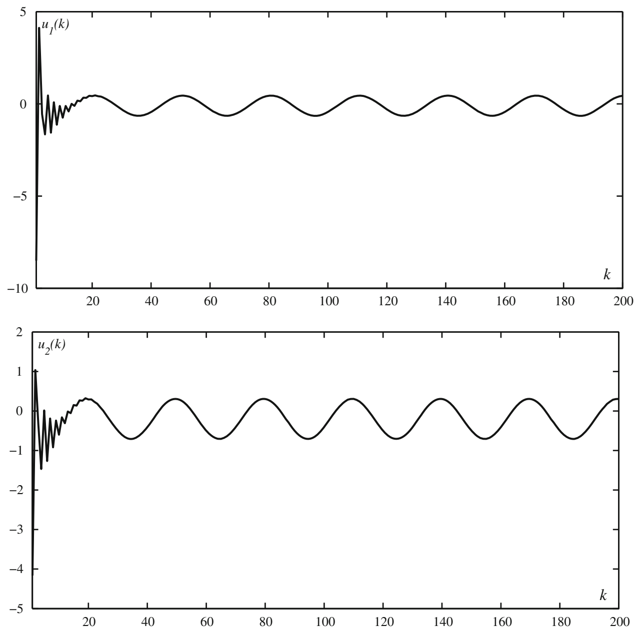

Fig. 2.6 Evolutions of the control signals $u_1(k)$ and $u_2(k)$ (2-DSMC)

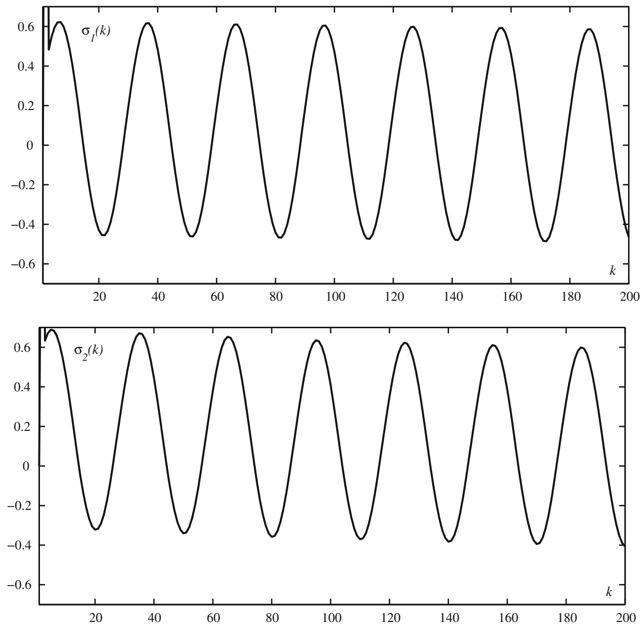

Fig. 2.7 Evolutions of the sliding functions $\sigma_1(k)$ and $\sigma_2(k)$ (2-DSMC)

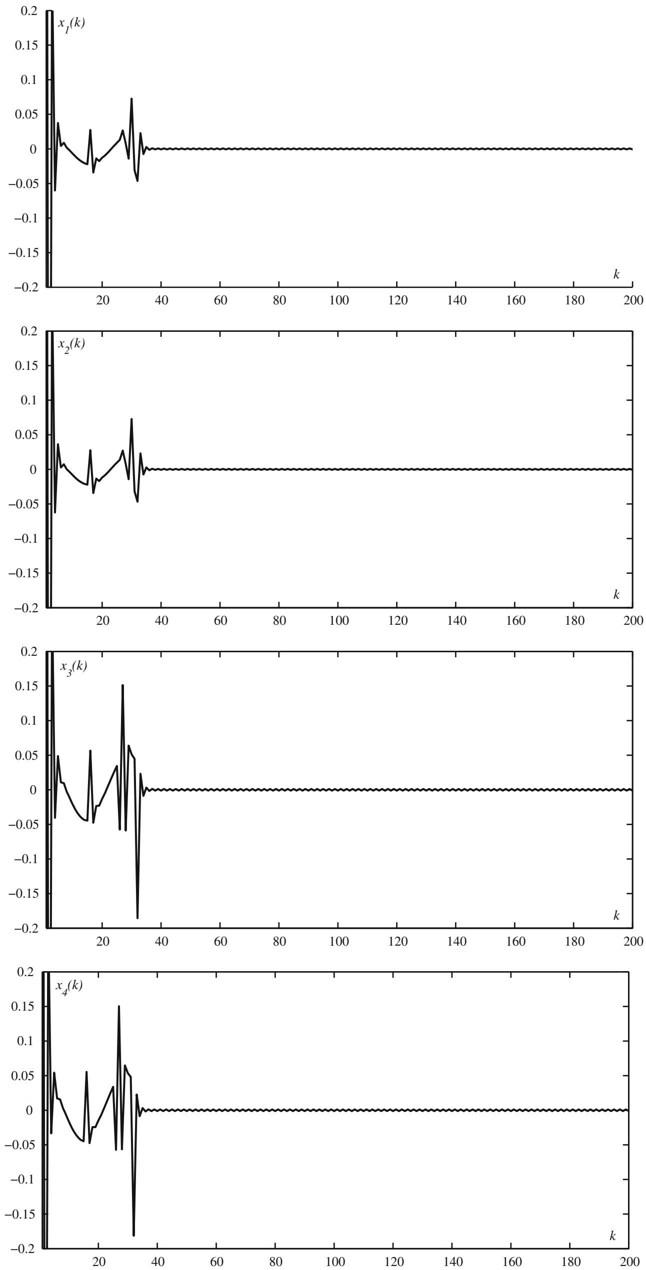

Fig. 2.8 Evolutions of the states $x_1(k)$, $x_2(k)$, $x_3(k)$ and $x_4(k)$ (DRSMC)

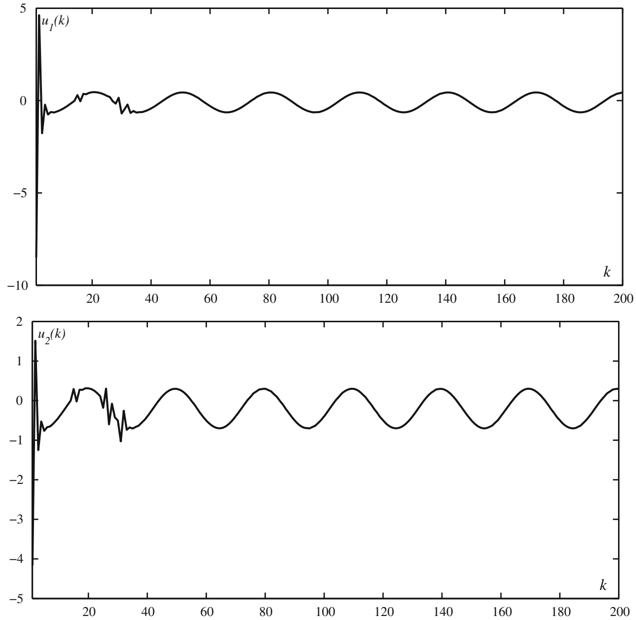

Fig. 2.9 Evolutions of the control signals $u_1(k)$ and $u_2(k)$ (DRSMC)

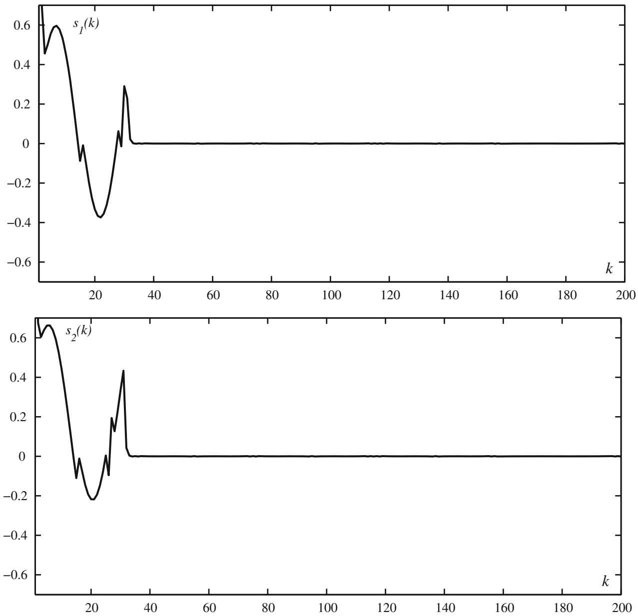

Fig. 2.10 Evolutions of the sliding functions $s_1(k)$ and $s_2(k)$ (DRSMC)

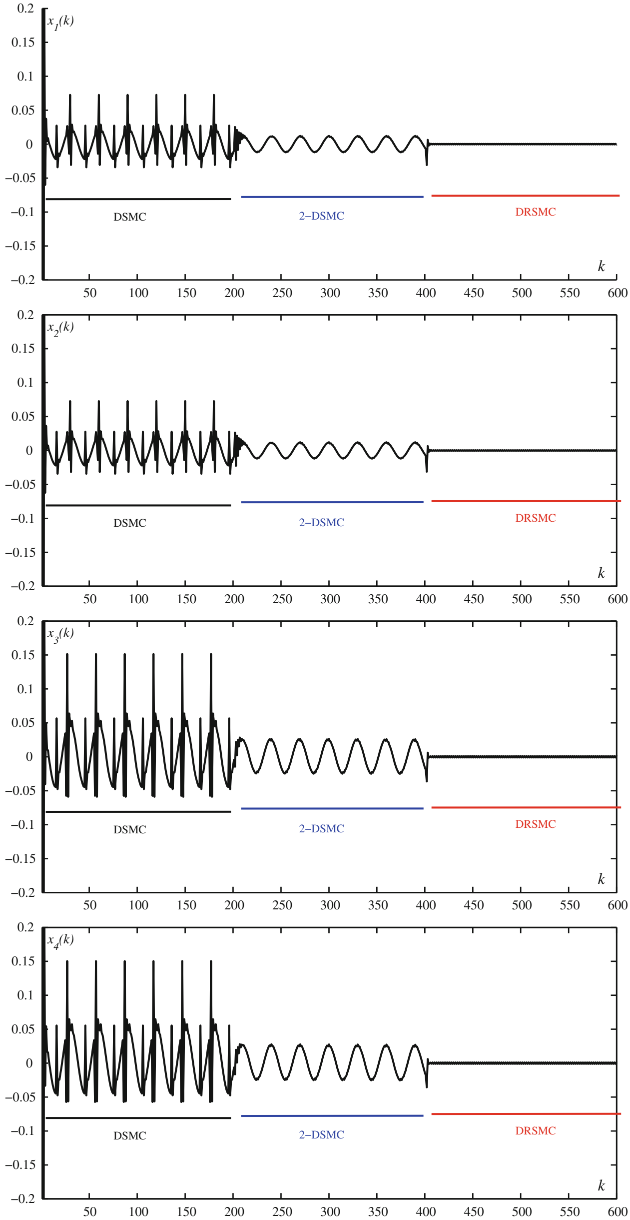

Fig. 2.11 Evolutions of the states $x_1(k)$, $x_2(k)$, $x_3(k)$ and $x_4(k)$

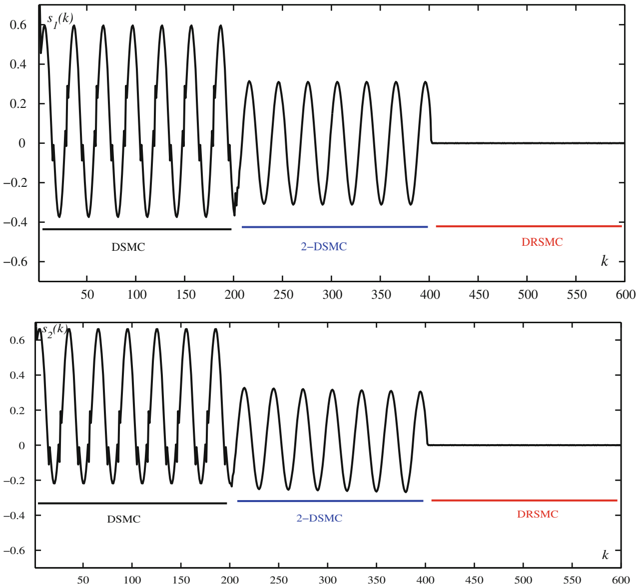

Fig. 2.12 Evolutions of the sliding functions $s_1(k)$ and $s_2(k)$

A comparison between the three kinds of sliding mode controllers: DSMC, 2-DSMC and DRSMC, as shown by Figs. 2.11 and 2.12, reveals that the use of DRSMC strategy reduce effectively the periodic disturbances.

2.6 Conclusion

In this paper, the problem of rejecting external disturbances in the discrete sliding mode control is studied. A solution to reject constant disturbances can be given by the discrete multivariable second order sliding mode control. To reject periodic disturbances, the discrete multivariable repetitive sliding mode control is proposed. Also, a necessary conditions of disturbances rejection in first, second order and repetitive sliding mode controllers has been developed for a discrete multivariable systems.

The effectiveness of the multivariable repetitive sliding mode control was validated through numerical simulation for a chemical reactor model. A comparison between the results obtained by the three kinds of sliding mode controllers showed a good rejection of periodic disturbances of the discrete multivariable repetitive sliding mode control.

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References

Arimoto, S., Kawamura, S., & Miyazaki, F. (1984). Bettering operation of dynamic systems by learning: a new control theory for servomechanism or mechatronics systems. *Proceedings of the 23rd Conference on Decision and Control*, pp. 1064–1069.

Bandyopadhyay, B., & Fulwani, D. (2009). High-performance tracking controller for discrete plant using nonlinear sliding surface. *IEEE Transactions on Indusrial Electronics*, *56*(9), 3628–3637.

Bartoszewicz, A. (1998). Discrete-time quasi-sliding-mode control strategies. *IEEE Transactions on Industrial Electronics*, *45*(4), 633–637.

Bartoszewicz, A., & Lesniewski, P. (2014). Reaching law approach to the sliding mode control of periodic review inventory systems. *IEEE Transactions on Automation Science and Engineering*, *11*(3), 810–817.

Castillo-Toledoa, B., Gennaro, S. D., Loukianov, A. G., & Rivera, J. (2008). Discrete time sliding mode control with application to induction motors. *Automatica*, *44*, 3036–3045.

Cavallo, A., & Natale, C. (2004). High-order sliding control of mechanical systems: theory and experiments. *Automatica*, *12*, 1139–1149.

Chan, C. Y. (1997). Discrete-time adaptive sliding mode control of a linear system in statespace form. *International Journal of Control*, *67*(6), 859–868.

Chang, J. L. (2002). Discrete sliding-mode control of mimo linear systems. *Asian Journal of Control*, *4*(2), 217–222.

Chang, J. L., & Chen, Y. P. (2000). Sliding vector design based on the pole assignment method. *Asian Journal of Control*, *2*, 10–15.

Chang, K., Shim, I., & Park, G. (2006). Adaptive repetitive control for an eccentricity compensation of optical disk drivers. *IEEE Transactions on Consumer Electronics*, *52*(2), 445–450.

Dehri, K., Ltaief, M., & Nouri, A.S. (2011). New discrete multivariable sliding mode control for multi-periodic disturbances rejection. *Proceedings of the International Multi-Conference on Systems, Signals and Devices*, Tunisia.

Dehri, K., Ltaief, M., Nouri, A. S. (2012a). Repetitive sliding mode control for nondecouplable systems: periodic disturbances rejection. *Proceedings of the 20th Mediterranean Conference on Control and Automation, MED'12*, July 3–6, Barcelona, Spain.

Dehri, K., Ltaief, M., & Nouri, A. S. (2012b). New discrete sliding mode control for nonlinear multivariable systems: multi-periodic disturbances rejection and stability analysis. *International Journal of Control Science and Engineering*, *2*(2), 7–15.

Doh, T.Y., & Ryoo, J.R. (2006). Robust stability condition of repetitive control systems and analysis on steady-state tracking errors. *Proceedings of the International Joint Conference on SICE-ICASE*, pp. 5169–5174.

Fateha, M. M., Tehranib, H. A., & Karbassic, S. M. (2013). Repetitive control of electrically driven robot manipulators. *International Journal of Systems Science*, *44*(4), 775–785.

Francis, B., & Wonham, W. (1976). The internal model principle of control theory. *Automatica*, *12*(5), 457–465.

Gao, W., Wang, Y., & Homaifa, A. (1995). Discrete-time variable structure control systems. *IEEE Transactions on Industrial Electronics*, *42*(2), 117–122.

Ginhoux, R. (2003). *Compensation des mouvements physiologiques en chirurgie robotisée par commande prédictive*. PhD thesis, Université Louis Pasteur Strasbourg.

Lopez, P., & Nouri, A.S. (2006). Théorie élémentaire et pratique de la commande par les régimes glissants. Berlin: Springer.

Mihoub, M., Nouri, A.S., Abdennour, R.B. (2009). Real-time application of discrete second order sliding mode control to a chemical reactor. *Control Engineering Practice*, *17*(9), 1089–1095.

Mihoub, M., Nouri, A.S., & Abdennour, R.B. (2011). *Multimodel discrete second order sliding mode control: stability analysis and real time application on a chemical reactor* pp. 473–490. In Tech.

Monsees, G. (2002). Discrete-time sliding mode control. *PHD Thesis, University of Gabes, Tunisia*.

Romdhane, H., Dehri, K., & Nouri, A. S. (2015). Second order sliding mode control for discrete decouplable multivariable systems via input-output models. *International Journal of Automation and Computing*, *12*(6), 630–638.

Sarpturk, S. Z., Istefanopulos, Y., & Kaynak, O. (1987). On the stability of discrete-time sliding mode control systems. *IEEE Transactions on Automatic Control*, *32*, 930–932.

Stoica, C. N. (2008). Robustification de lois de commande prédictives multivariables. *Thesis*.

Xuan, S. M., Xiong, H. X., & Yu, C. B. (2007). Repetitive Learning Control for Time-varying Robotic Systems: A Hybrid Learning Scheme. *Automatica*, *33*(11), 1189–1195.

Xuan, S. M., Jiang, Y. L., & Gang, H. H. (2013). Discrete Adaptive Repetitive Control: Convergence Analysis and Implementation. *Acta Automatica Sinica*, *39*(4), 400–7406.

Yan, T. H., He, B., Chen, X. D., & Xu, X. S. (2013). The discrete-time sliding mode control with computation time delay for repeatable run-out compensation of hard disk drives. *Mathematical Problems in Engineering*, *2013*(2), 1–13.

Young, K. D., Utkin, V. I., & Ozguner, U. (1999). A control engineer's guide to sliding mode control. *IEEE Transactions on Control Systems Technology*, *7*(3), 328–342.

Yu, X., & Kaynak, O. (2009). Sliding mode control with soft computing: a survey. *IEEE Transactions on Industrial Electronics*, *56*(9).

Zhou, K., & Wang, D. (2003). Digital repetitive controlled three-phase PWM rectifier. *IEEE Transactions on Power Electronics*, *18*(1), 309–316.

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