

Chapter 2

Dynamic Models of Satellite Relative Motion Around an Oblate Earth

Accurate dynamic model of relative motion is basic and critical to the study of satellite formation flying. Hence, accurate nonlinear and linear dynamic models of satellite relative motion considering J_2 perturbation are derived in this chapter. Firstly, an exact J_2 nonlinear model of satellite relative motion is developed based on the Lagrangian mechanics. Subsequently, with the aid of Gegenbauer polynomials, the nonlinear model is linearized to a complete J_2 linear relative model. Finally, by means of eliminating the second-order J_2 effect, the linear model is further approximated to a first-order J_2 linear relative model. Simulation results show that the exact J_2 nonlinear model produces exact results, and the first-order J_2 linear model also performs well under conditions that the inter-satellite distance is small and time duration is short.

Many relative dynamic models have been derived in the literature under different assumptions and using different methodologies. A comparison study is necessary to select an appropriate model for a specific mission and determine what kind of perturbation should be considered for specific applications. Thus, in this chapter, a simulation method with a modeling error index is also introduced for comparing and evaluating various existing models for relative motion of satellites flying in formation. The comparison results show that, when the Earth aspherical gravity and the air drag are present, the accuracy of some models is affected adversely by eccentricity, semimajor axis, inclination, and formation size. The numerical results provide valuable information for formation design.

2.1 Introduction

The autonomous formation flying of multiple small satellites to replace a single large satellite will be an enabling technology for many future space missions. Potential applications include surveillance missions, field measurement missions, and atmospheric survey missions. With the desire to maneuver or keep a long-term flying

satellite formation comes the need to predict accurate relative position and velocity between satellites. Many studies on relative motion of satellites flying in motion have been reported in the literature. Researchers initially use a set of linearized differential equations, i.e., Hill's equations (Hill 1878), which is also known as Clohessy-Wiltshire (CW) equations (Clohessy and Wiltshire 1960), to describe the relative motion of two satellites in near-circular orbits. Though CW equations had been successfully used in rendezvous scenario, its model error accumulates over time such that its solution becomes erroneous and unacceptable for long duration formation flying.

There has been active research interest to derive a simple and accurate dynamic model for formation design, guidance and control. Many dynamic models have been developed to cater various applications. Generally speaking, these dynamic models can be classified into three categories. The first category is the direct ordinary differential equation (ODE) models, which are mostly extensions or modifications of CW equations (Tschauner and Hempel 1965; Kechichian 1998; Schweighart and Sedwick 2002, 2005; Tillerson and How 2002; Vadali et al. 2001; Ross 2003; Roberts and Roberts 2004; Pluym and Damaren 2006; Inalhan et al. 2002; Gurfil 2005; Xu and Wang 2008; Morgan et al. 2012). As these models are in the form of differential equations, they have significant applications in controller design. The second category is the indirect models, which are usually expressed in differences of orbit elements (Schaub and Alfriend 2001; Schaub 2002, 2004). It is easier to use the second category of models to design the satellite formation because they describe the formation geometry directly. The third category is the solution-based models, which are usually in the form of state transition matrix (STM) (Gim and Alfriend 2005; Sengupta et al. 2007; Palmer and Imre 2007; Lee et al. 2007). Using STM, though it is very complicated, one can directly generate the satellite relative motion.

For the direct ODE models, CW equations were first extended to unperturbed relative motion that takes into account of eccentricity and/or nonlinearity. Tschauner and Hempel (1965) solved the satellite relative motion in elliptical orbits. Analytical solution to their model can be derived both in true anomaly and time domain (Tschauner and Hempel 1965; Tillerson and How 2002). The second order J_2 effect is the dominant perturbation for satellite formation missions. As such, a J_2 dynamic model of satellite relative motion in terms of differential equations in the local vertical local horizontal (LVLH) coordinate is significantly useful for the study of satellite formation flying in low Earth orbits. In fact, different J_2 dynamic equations have been developed. A noteworthy work was reported by Kechichian (1998), in which an exact nonlinear relative model that includes both the J_2 perturbation and the air drag was developed. Kechichian applied Newtonian mechanics and applied vector calculus to derive the relative dynamics. His result is very complex and represented equivalently by 12 first-order differential equations. In particular, some components of J_2 acceleration are not explicitly expressed in the relative model, and instead, a tedious algorithm is provided to calculate these J_2 acceleration components.

Some recent attempts have been done on the development of J_2 linear dynamics. Schweighart and Sedwick (2002, 2005) developed a hybrid J_2 linear model in near-circular orbits using averaged J_2 acceleration. Equations of this model on the

in-plane motion are linear time invariant (LTI) and somewhat similar to CW model (Clohessy and Wiltshire 1960), while equations to describe the cross-track motion are linear time varying (LTV). To simplify the cross-track dynamics, Tillerson and How (2002) combined the cross-track dynamics proposed by Vadali et al. (2001) with the in-plane dynamics of Schweighart and Sedwick. Some other results are reported (Ross 2003; Roberts and Roberts 2004; Pluym and Damaren 2006). A common assumption of those works is that the reference satellite flies in an unperturbed Keplerian orbit. As a result, the accuracy and the applicability of these models are limited.

In this chapter, similar to Kechichian's work, the satellite relative dynamics is studied based on the perturbed reference orbit, which is accurately described by a set of differential equations (Xu and Wang 2008b). A modification to the Kechichian's method is that the reference orbit is expressed simpler in terms of Reference Satellite Variables (RSV). Different from Kechichian's techniques, Lagrangian mechanics is used to derive the satellite relative dynamics. Consequently, an exact J_2 nonlinear relative model is obtained, which is independent of the right ascension of ascending node such that the satellite relative motion is completely expressed by 11 simple first-order differential equations. Furthermore, the complete J_2 linear relative model and the first-order J_2 linear relative model are derived by removing the nonlinear effect and the second-order J_2 effect (Xu and Wang 2008b). By means of numerical study, it is found that the nonlinear effect and the second-order J_2 effects introduce errors in opposite directions, so that they counteract each other somewhat. As a result, the first-order J_2 linear relative model is suitable for short-duration formation maneuver and keeping.

2.2 Nonlinear Dynamic Model of Relative Motion

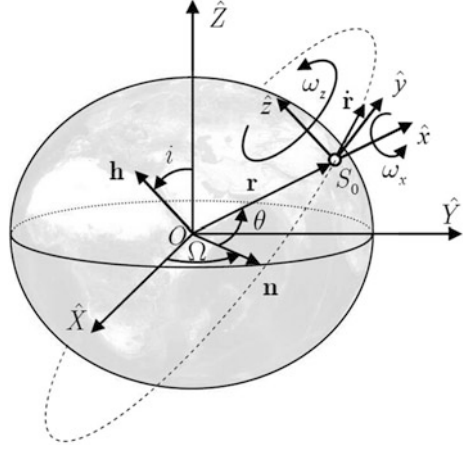
2.2.1 J_2 Reference Satellite Dynamics in LVLH Frame

In the study, one satellite, or a virtual satellite, is taken as reference satellite and others as member satellites. Without loss of generality, a 2-satellite system is considered, i.e., a free-flying reference satellite S_0 (without control force) and a controlled member satellite S_j (with control force). This subsection is devoted to explicitly establish the J_2 dynamics of the single reference satellite S_0 in the local rotating frame.

2.2.1.1 Properties of LVLH Frame

Two Cartesian coordinates are used in this study. As shown in Fig. 2.1, Earth-centered inertial (ECI) coordinate is spanned by unit vectors $(\hat{\mathbf{X}}, \hat{\mathbf{Y}}, \hat{\mathbf{Z}})$. LVLH coordinate $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ is attached on reference satellite S_0 . Vectors $\mathbf{r}$ and $\dot{\mathbf{r}}$ denote the position and the velocity of satellite S_0 , respectively. Hence, the vector

Fig. 2.1 ECI and LVLH coordinates



of angular momentum per unit mass is defined by $\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}}$. Furthermore, $r = \|\mathbf{r}\|$ and $h = \|\mathbf{h}\|$ denote the geocentric distance and the magnitude of angular momentum of satellite S_0 , where $\|\cdot\|$ denotes Euclidean norm. Then, the LVLH coordinate is spanned by unit vectors

$$\hat{\mathbf{x}} = \frac{\mathbf{r}}{r} \quad \hat{\mathbf{y}} = \hat{\mathbf{z}} \times \hat{\mathbf{x}} \quad \hat{\mathbf{z}} = \frac{\mathbf{h}}{h} \quad (2.1)$$

Vectors in LVLH coordinate and ECI coordinate can be transformed (Battin 1999) to each other by the rotation matrix R as

$$\begin{bmatrix} \hat{\mathbf{X}} & \hat{\mathbf{Y}} & \hat{\mathbf{Z}} \end{bmatrix}^T = R \begin{bmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \end{bmatrix}^T \quad (2.2)$$

with

$$R = \begin{bmatrix} c_\theta c_\Omega - s_\theta c_i s_\Omega & -s_\theta c_\Omega - c_\theta c_i s_\Omega & s_i s_\Omega \\ c_\theta s_\Omega + s_\theta c_i c_\Omega & -s_\theta s_\Omega + c_\theta c_i c_\Omega & -s_i c_\Omega \\ s_\theta s_i & c_\theta s_i & c_i \end{bmatrix} \quad (2.3)$$

where $s_\circ = \sin(\circ)$, $c_\circ = \cos(\circ)$, i is the inclination, θ is the true latitude, and Ω is the right ascension of ascending node.

The angular velocity of rotating LVLH frame is

$$\boldsymbol{\omega} = \omega_x \hat{\mathbf{x}} + \omega_z \hat{\mathbf{z}} \quad (2.4)$$

Note that the component around y -axis is zero, i.e., $\omega_y = 0$ (Kechichian 1998). The component ω_z is referred as the orbital rate, and the component ω_x denotes the steering rate of the orbital plane. Angular velocity $\boldsymbol{\omega}$ can be expressed by Eulerian angles (Ω, i, θ) , and its three components are (Kechichian 1998; Breakwell 1974):

$$\omega_x = \dot{i}c_\theta + \dot{\Omega}s_\theta s_i \quad (2.5)$$

$$\omega_y = -\dot{i}s_\theta + \dot{\Omega}c_\theta s_i = 0 \quad (2.6)$$

$$\omega_z = \dot{\theta} + \dot{\Omega}c_i \quad (2.7)$$

Using Eq. (2.4), velocities of unit vectors of LVLH frame are expressed as follows:

$$\dot{\hat{\mathbf{x}}} = \boldsymbol{\omega} \times \hat{\mathbf{x}} = \omega_z \hat{\mathbf{y}} \quad \dot{\hat{\mathbf{y}}} = \boldsymbol{\omega} \times \hat{\mathbf{y}} = \omega_x \hat{\mathbf{z}} - \omega_z \hat{\mathbf{x}} \quad \dot{\hat{\mathbf{z}}} = \boldsymbol{\omega} \times \hat{\mathbf{z}} = -\omega_x \hat{\mathbf{y}} \quad (2.8)$$

Then, the velocity of satellite S_0 is computed to

$$\dot{\mathbf{r}} = \frac{d}{dt}(r\hat{\mathbf{x}}) = \dot{r}\hat{\mathbf{x}} + r\omega_z\hat{\mathbf{y}} \quad (2.9)$$

Using Eq. (2.9), the angular momentum can be expressed as

$$\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}} = r^2\omega_z\hat{\mathbf{z}} \quad (2.10)$$

Comparing Eq. (2.10) with the last equation in (2.1), it is concluded that the magnitude of angular momentum is

$$h = \omega_z r^2 \quad (2.11)$$

Taking time derivative to Eq. (2.11), the rate of angular momentum is obtained as

$$\dot{h} = 2\omega_z r \dot{r} + \dot{\omega}_z r^2 \quad (2.12)$$

2.2.1.2 J_2 Dynamics of a Satellite in LVLH Frame

The dynamics of reference satellite S_0 in LVLH frame considering both the spherical gravitational potential and the oblate (J_2) gravitational potential is derived in this section. The governing equation is

$$\ddot{\mathbf{r}} = -\nabla U \quad (2.13)$$

with

$$U = -\frac{\mu}{r} - \frac{3J_2\mu R_e^2}{2r^3} \left(\frac{1}{3} - s_\phi^2 \right) \quad (2.14)$$

where U is the gravitational potential energy of satellite. ∇ denotes the vector differential operator. μ is the Earth gravitational constant. J_2 is the second zonal harmonic coefficient of the Earth. R_e is the Earth equatorial radius, and ϕ is the geocentric latitude of satellite S_0 .

Using Eqs. (2.8) and (2.9), the left side of Eq. (2.13) is calculated as

$$\ddot{\mathbf{r}} = (\ddot{r} - \omega_z^2 r)\hat{\mathbf{x}} + (\dot{\omega}_z r + 2\omega_z \dot{r})\hat{\mathbf{y}} + (\omega_x \omega_z r)\hat{\mathbf{z}} \quad (2.15)$$

Using Eqs. (2.11) and (2.12), ω_z and $\dot{\omega}_z$ in Eq. (2.15) can be replaced by h and $\dot{h}$, and then, Eq. (2.15) is converted to

$$\ddot{\mathbf{r}} = \left(\ddot{r} - \frac{h^2}{r^3} \right) \hat{\mathbf{x}} + \frac{\dot{h}}{r} \hat{\mathbf{y}} + \frac{\omega_x h}{r} \hat{\mathbf{z}} \quad (2.16)$$

Next, the gradient of U in Eq. (2.14) is computed in LVLH frame to be

$$\nabla U = \frac{\mu}{r^2} \hat{\mathbf{x}} + \frac{k_{J2}}{r^4} (1 - 3s_i^2 s_\theta^2) \hat{\mathbf{x}} + \frac{k_{J2} s_i^2 s_{2\theta}}{r^4} \hat{\mathbf{y}} + \frac{k_{J2} s_{2i} s_\theta}{r^4} \hat{\mathbf{z}} \quad (2.17)$$

where the constant k_{J2} is defined as

$$k_{J2} = \frac{3J_2 \mu R_e^2}{2} \quad (2.18)$$

Substituting Eqs. (2.16) and (2.17) into Eq. (2.13), the below three equations are obtained

$$\omega_x = -\frac{k_{J2} s_{2i} s_\theta}{hr^3} \quad (2.19)$$

$$\ddot{r} = -\frac{\mu}{r^2} + \frac{h^2}{r^3} - \frac{k_{J2}}{r^4} (1 - 3s_i^2 s_\theta^2) \quad (2.20)$$

$$\dot{h} = -\frac{k_{J2} s_i^2 s_{2\theta}}{r^3} \quad (2.21)$$

Replacing ω_x in Eq. (2.5) by Eq. (2.19), differential terms $(\dot{\Omega}, \dot{i})$ are solved using Eqs. (2.5) and (2.6)

$$\dot{\Omega} = -\frac{2k_{J2} c_i s_\theta^2}{hr^3} \quad (2.22)$$

$$\dot{i} = -\frac{k_{J2} s_{2i} s_{2\theta}}{2hr^3} \quad (2.23)$$

Substituting Eq. (2.22) into Eq. (2.7), and considering Eq. (2.11), $\dot{\theta}$ is also solved

$$\dot{\theta} = \frac{h}{r^2} + \frac{2k_{J_2}c_i^2s_\theta^2}{hr^3} \quad (2.24)$$

The results established are presented in the following theorem.

Theorem 2.1 *Considering spherical gravity and J_2 gravity of the Earth, the motion of the reference satellite S_0 can be described by a set of differential equations as in Eqs. (2.20), (2.21), (2.23) and (2.24) in terms of Reference Satellite Variables (RSV) $(\dot{r}, r, h, \Omega, i, \theta)$.*

Remark 2.1 It is noticed that five variables $(\dot{r}, r, h, i, \theta)$ are independent of Ω . This is expected because J_2 gravity is independent of the change of Ω .

The rotation rate of LVLH frame can be expressed conveniently in terms of RSV and also independent of Ω . The steering rate ω_x is obtained in Eq. (2.19). The orbital rate ω_z is the direct result of (2.11), i.e.,

$$\omega_z = \frac{h}{r^2} \quad (2.25)$$

Taking time derivative Eqs. (2.19) and (2.25), respectively, and considering Eqs. (2.21), (2.23), and (2.24), the steering acceleration α_x and the orbital acceleration α_z are derived as

$$\alpha_x = \dot{\omega}_x = -\frac{k_{J_2}s_{2i}c_\theta}{r^5} + \frac{3\dot{r}k_{J_2}s_{2i}s_\theta}{r^4h} - \frac{8k_{J_2}^2s_i^3c_is_\theta^2c_\theta}{r^6h^2} \quad (2.26)$$

$$\alpha_z = \dot{\omega}_z = -\frac{2h\dot{r}}{r^3} - \frac{k_{J_2}s_i^2s_{2\theta}}{r^5} \quad (2.27)$$

2.2.2 Derivation of Exact J_2 Nonlinear Relative Dynamics

2.2.2.1 Lagrangian Formulation of Relative Motion

The Lagrangian formulation is used to develop the relative dynamics of the member satellite S_j . The Lagrangian formulation for satellite relative motion is

$$\frac{d}{dt} \left(\frac{\partial L_j}{\partial \dot{\mathbf{q}}_j} \right) - \frac{\partial L_j}{\partial \mathbf{q}_j} = \mathbf{a}_j \quad (2.28)$$

where $\mathbf{q}_j = [x_j \ y_j \ z_j]^T$ and $\mathbf{a}_j = [a_{jx} \ a_{jy} \ a_{jz}]^T$ are respectively the configurations and the control accelerations of satellite S_j in LVLH coordinate, and L_j is its Lagrangian, which can be further expressed in the form of

$$L_j(\mathbf{q}_0, \dot{\mathbf{q}}_0, \mathbf{q}_j, \dot{\mathbf{q}}_j) = K_j(\mathbf{q}_0, \dot{\mathbf{q}}_0, \mathbf{q}_j, \dot{\mathbf{q}}_j) - U_j(\mathbf{q}_0, \mathbf{q}_j) \quad (2.29)$$

where $\mathbf{q}_0$ are configurations of reference satellite S_0 in ECI coordinate. K_j and U_j are respectively the kinetic and potential energies of the j -th member satellite. Since kinetic energy K_j arises from inertial motion, it depends on the relative motion $(\mathbf{q}_j, \dot{\mathbf{q}}_j)$ of satellite S_j in LVLH coordinate as well as the transport motion $(\mathbf{q}_0, \dot{\mathbf{q}}_0)$ of LVLH frame in ECI coordinate. On the other hand, potential energy U_j is solely due to gravity and thus is independent of velocities. Substituting Eq. (2.29) into Eq. (2.28) yields

$$\frac{d}{dt} \left(\frac{\partial K_j}{\partial \dot{\mathbf{q}}_j} \right) - \frac{\partial K_j}{\partial \mathbf{q}_j} + \frac{\partial U_j}{\partial \mathbf{q}_j} = \mathbf{a}_j \quad (2.30)$$

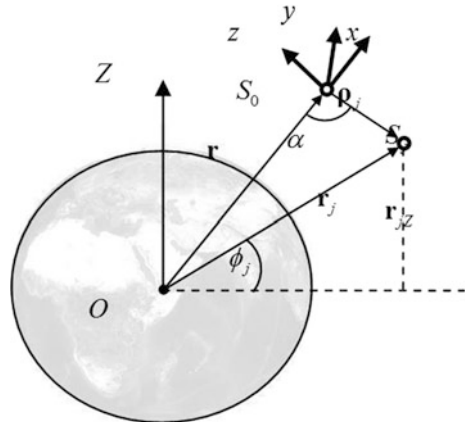
which is the Lagrangian formulation for the relative motion of member satellite S_j in LVLH frame. Next, Eq. (2.30) is used to develop the dynamics of satellite relative motion. The key steps are the precise calculations of kinetic energy K_j and potential energy U_j .

2.2.2.2 Kinetic Energy

As shown in Fig. 2.2, the position of satellite S_j in LVLH frame is

$$\boldsymbol{\rho}_j = x_j \hat{\mathbf{x}} + y_j \hat{\mathbf{y}} + z_j \hat{\mathbf{z}} \quad (2.31)$$

Fig. 2.2 Geometry of the member satellite



Then, in ECI frame, it is

$$\mathbf{r}_j = \mathbf{r} + \boldsymbol{\rho}_j = (x_j + r)\hat{\mathbf{x}} + y_j\hat{\mathbf{y}} + z_j\hat{\mathbf{z}} \quad (2.32)$$

Using the identities in Eq. (2.8), the velocity of satellite S_j in inertial space is calculated by taking time derivative of Eq. (2.32)

$$\dot{\mathbf{r}}_j = v_{jx}\hat{\mathbf{x}} + v_{jy}\hat{\mathbf{y}} + v_{jz}\hat{\mathbf{z}} \quad (2.33)$$

where

$$v_{jx} = \dot{x}_j + \dot{r} - y_j\omega_z \quad v_{jy} = \dot{y}_j + (r + x_j)\omega_z - z_j\omega_x \quad v_{jz} = \dot{z}_j + y_j\omega_x \quad (2.34)$$

Then, the kinetic energy per unit mass of satellite S_j is computed as

$$\begin{aligned} K_j &= \frac{1}{2} \left(v_{jx}^2 + v_{jy}^2 + v_{jz}^2 \right) \\ &= \frac{1}{2} (\dot{x}_j + \dot{r} - y_j\omega_z)^2 + \frac{1}{2} (\dot{y}_j + (r + x_j)\omega_z - z_j\omega_x)^2 + \frac{1}{2} (\dot{z}_j + y_j\omega_x)^2 \end{aligned} \quad (2.35)$$

where time-varying variables $(\dot{r}, r, \omega_x, \omega_z)$ are functions of RSV.

2.2.2.3 Potential Energy

Considering J_2 perturbation, the gravitational potential energy of member satellite S_j is

$$U_j = -\frac{\mu}{r_j} - \frac{k_{J2}}{r_j^3} \left(\frac{1}{3} - s_{\phi_j}^2 \right) \quad (2.36)$$

where ϕ_j and r_j are the geocentric latitude and the geocentric distance of satellite S_j , respectively. From Eq. (2.32), geocentric distance r_j is immediately obtained as

$$r_j = \sqrt{(r + x_j)^2 + y_j^2 + z_j^2} \quad (2.37)$$

From the geometry shown in Fig. 2.2, it is obtained

$$s_{\phi_j} = \frac{r_{jZ}}{r_j} \quad (2.38)$$

where r_{jZ} is the projection of $\mathbf{r}_j$ on Z axis of ECI frame. Since transformation R in Eq. (2.3) is an orthogonal matrix, $\mathbf{r}_j$ in Eq. (2.32) can be transformed to

$$\begin{bmatrix} r_{jX} & r_{jY} & r_{jZ} \end{bmatrix}^T = R \begin{bmatrix} x_j + r & y_j & z_j \end{bmatrix}^T \quad (2.39)$$

where r_{jX} , r_{jY} and r_{jZ} are the respective components of $\mathbf{r}_j$ expressed in ECI frame and

$$r_{jZ} = (r + x_j)s_i s_\theta + y_j s_i c_\theta + z_j c_i \quad (2.40)$$

Substituting Eq. (2.38) into Eq. (2.36), the potential energy of the j -th member satellite is given by

$$U_j = -\frac{\mu}{r_j} - \frac{k_{J2}}{3r_j^3} + \frac{k_{J2}r_{jZ}^2}{r_j^5} \quad (2.41)$$

where r_j and r_{jZ} are expressed in equations Eqs. (2.37) and (2.40).

2.2.2.4 Exact Nonlinear J_2 Relative Dynamics

Having kinetic energy in Eq. (2.35) and potential energy in Eq. (2.41), it is ready to use Lagrangian formulation of Eq. (2.30) to derive the satellite relative dynamics. Substituting Eq. (2.35) into the first two terms of Eq. (2.30), it is obtained

$$\frac{d}{dt} \left(\frac{\partial K_j}{\partial \dot{\mathbf{q}}_j} \right) - \frac{\partial K_j}{\partial \mathbf{q}_j} = \begin{bmatrix} \ddot{x}_j - 2\dot{y}_j\omega_z - x_j\omega_z^2 - y_j\alpha_z + z_j\omega_x\omega_z - r(\omega_z)^2 + (\ddot{r}) \\ \ddot{y}_j + 2\dot{x}_j\omega_z - 2\dot{z}_j\omega_x + x_j\alpha_z - y_j\omega_z^2 - y_j\omega_x^2 - z_j\alpha_x + 2\dot{r}(\omega_z) + r(\alpha_z) \\ \ddot{z}_j + 2\dot{y}_j\omega_x + x_j\omega_x\omega_z + y_j\alpha_x - z_j\omega_x^2 + r(\omega_x\omega_z) \end{bmatrix} \quad (2.42)$$

Next RSV dynamics are applied. The variables in round brackets in Eq. (2.42) are replaced with Eqs. (2.19), (2.20), (2.25), and (2.27). Furthermore, define

$$\zeta = \frac{2k_{J2}s_i s_\theta}{r^4} \quad n^2 = \frac{\mu}{r^3} + \frac{k_{J2}}{r^5} - \frac{5k_{J2}s_i^2 s_\theta^2}{r^5} \quad (2.43)$$

With some manipulations, Eq. (2.42) is converted to

$$\frac{d}{dt} \left(\frac{\partial K_j}{\partial \dot{\mathbf{q}}_j} \right) - \frac{\partial K_j}{\partial \mathbf{q}_j} = \begin{bmatrix} \ddot{x}_j - 2\dot{y}_j\omega_z - x_j\omega_z^2 - y_j\alpha_z + z_j\omega_x\omega_z - rn^2 - \zeta s_i s_\theta \\ \ddot{y}_j + 2\dot{x}_j\omega_z - 2\dot{z}_j\omega_x + x_j\alpha_z - y_j(\omega_z^2 + \omega_x^2) - z_j\alpha_x - \zeta s_i c_\theta \\ \ddot{z}_j + 2\dot{y}_j\omega_x + x_j\omega_x\omega_z + y_j\alpha_x - z_j\omega_x^2 - \zeta c_i \end{bmatrix} \quad (2.44)$$

On the other hand, inserting Eq. (2.41) into the third term in Eq. (2.30), and defining

$$\zeta_j = \frac{2k_{J2}r_{jz}}{r_j^5} \quad n_j^2 = \frac{\mu}{r_j^3} + \frac{k_{J2}}{r_j^5} - \frac{5k_{J2}r_{jz}^2}{r_j^7} \quad (2.45)$$

it is obtained

$$\frac{\partial U_j}{\partial \mathbf{q}_j} = \begin{bmatrix} n_j^2(r + x_j) + \zeta_j s_i s_\theta & n_j^2 y_j + \zeta_j s_i c_\theta & n_j^2 z_j + \zeta_j c_i \end{bmatrix}^T \quad (2.46)$$

Then, substituting Eqs. (2.44) and (2.46) into Eq. (2.30), the exact J_2 nonlinear model of the satellite relative motion is derived to be

$$\begin{aligned} \ddot{x}_j &= 2\dot{y}_j\omega_z - x_j(n_j^2 - \omega_z^2) + y_j\alpha_z - z_j\omega_x\omega_z - (\zeta_j - \zeta)s_i s_\theta - r(n_j^2 - n^2) + a_{jx} \\ \ddot{y}_j &= -2\dot{x}_j\omega_z + 2\dot{z}_j\omega_x - x_j\alpha_z - y_j(n_j^2 - \omega_z^2 - \omega_x^2) + z_j\alpha_x - (\zeta_j - \zeta)s_i c_\theta + a_{jy} \\ \ddot{z}_j &= -2\dot{y}_j\omega_x - x_j\omega_x\omega_z - y_j\alpha_x - z_j(n_j^2 - \omega_x^2) - (\zeta_j - \zeta)c_i + a_{jz} \end{aligned} \quad (2.47)$$

Now, the nonlinear dynamic equations of the satellite relative motion can be presented in the following theorem.

Theorem 2.2 Consider a 2-satellite system of the reference satellite S_0 and the member satellite S_j , as shown in Fig. 2.1. In the presence of spherical gravity and J_2 gravity of the Earth, the relative motion of the satellite S_j in the LVLH coordinate can be described by (2.47). In dynamics (2.47), all time varying parameters $(r, i, \theta, \omega_x, \omega_z, \alpha_x, \alpha_z, \zeta, n^2)$ are functions of RSV, which are given by Eqs. (2.19), (2.23)–(2.27) and (2.43). Variables (ζ_j, n_j^2) are nonlinear terms of configuration (x_j, y_j, z_j) , which are given by Eqs. (2.45), (2.37) and (2.40).

Remark 2.2 Dynamics (2.47) is independent of the motion of right ascension of ascending node Ω . This interesting observation is understandable because only the spherical and J_2 gravities of the Earth are included in the developed dynamics, and both of them are axial symmetric and independent of the motion of Ω . Therefore, the satellite relative motion under J_2 perturbation is actually described by 11 first-order differential equations of $(x_j, y_j, z_j, \dot{x}_j, \dot{y}_j, \dot{z}_j)$ and $(r, \dot{r}, h, i, \theta)$.

Remark 2.3 When relative model (2.47) is applied to study the problems of formation guidance and control, Eqs. (2.20)–(2.24) in RSV dynamics are good candidate to propagate time-varying parameters $(r, \dot{r}, h, i, \theta)$. However, they are not necessary in practice and other techniques can be used to evaluate these parameters, e.g., the technique that combines the advantages of orbit propagator and statistical orbit determination.

2.3 Linearized Dynamic Models of Relative Motion

In dynamics of Eq. (2.47), n_j^2 and ζ_j are nonlinear terms of configurations (x_j, y_j, z_j) , because they both include polynomials of the reciprocal of r_j . The technique of Gegenbauer polynomials is applied in this section to linearize the nonlinear model of Eq. (2.47).

Based on Fig. 2.2 and using the cosine theorem, r_j can be expressed as

$$r_j^2 = r^2 - 2r\rho_j \cos \alpha + \rho_j^2 \quad (2.48)$$

with

$$\cos \alpha = \frac{(-\mathbf{r}) \cdot \mathbf{\rho}_j}{r\rho_j} = -\frac{x_j}{\rho_j} \quad (2.49)$$

Equation (2.48) can be converted to

$$\frac{1}{r_j^{2\lambda}} = \frac{1}{r^{2\lambda}} \left(1 - 2 \cos \alpha \left(\frac{\rho_j}{r} \right) + \left(\frac{\rho_j}{r} \right)^2 \right)^{-\lambda} \quad (2.50)$$

Next, Gegenbauer polynomials (ultraspherical polynomials) is introduced, which are generalizations of the associated Legendre polynomials and expressed as

$$C_n^{(\lambda)}(u) = \frac{\Gamma(\lambda + 1/2)}{\Gamma(2\lambda)} \frac{\Gamma(n + 2\lambda)}{\Gamma(n + \lambda + 1/2)} \frac{(-1)^n}{2^n n!} (1 - u^2)^{-\lambda + 1/2} \frac{d^n}{du^n} (1 - u^2)^{n + \lambda - 1/2} \quad (2.51)$$

where $\Gamma(\circ)$ is the gamma function. The first two terms of Eq. (2.51) are

$$C_0^{(\lambda)}(u) = 1 \quad C_1^{(\lambda)}(u) = 2\lambda u \quad (2.52)$$

The generating function of Gegenbauer polynomials is, for $(\lambda + 1/2) > 0$, $|v| < 1$ and $|u| \leq 1$,

$$(1 - 2uv + v^2)^{-\lambda} = \sum_{n=0}^{\infty} C_n^{(\lambda)}(u) v^n \approx 1 + (2\lambda u)v \quad (2.53)$$

Let

$$u = \cos \alpha \quad v = \frac{\rho_j}{r} \quad (2.54)$$

Equation (2.50) can be expanded in the form of Eq. (2.53) as

$$\frac{1}{r_j^{2\lambda}} \approx \frac{1}{r^{2\lambda}} \left(1 + 2\lambda \left(-\frac{x_j}{r} \right) \left(\frac{\rho_j}{r} \right) \right) = \frac{1}{r^{2\lambda}} \left(1 - \frac{2\lambda x_j}{r} \right) \quad (2.55)$$

For $\lambda = (3/2), (5/2), (7/2)$, Eq. (2.55) gives

$$\frac{1}{r_j^3} \approx \frac{1}{r^3} - \frac{3x_j}{r^4} \quad \frac{1}{r_j^5} \approx \frac{1}{r^5} - \frac{5x_j}{r^6} \quad \frac{1}{r_j^7} \approx \frac{1}{r^7} - \frac{7x_j}{r^8} \quad (2.56)$$

Substituting Eqs. (2.56) and (2.40) into Eq. (2.45), and removing higher-order terms, ζ_j and n_j^2 are linearized to

$$\zeta_j \approx \zeta - \frac{8k_{J2}x_j s_i s_\theta}{r^5} + \frac{2k_{J2}y_j s_i c_\theta}{r^5} + \frac{2k_{J2}z_j c_i}{r^5} \quad (2.57)$$

$$n_j^2 \approx n^2 - \frac{3\mu x_j}{r^4} - \frac{5k_{J2}x_j(1 - 5s_i^2 s_\theta^2)}{r^6} - \frac{5k_{J2}y_j s_i^2 s_{2\theta}}{r^6} - \frac{5k_{J2}z_j s_{2i} s_\theta}{r^6} \quad (2.58)$$

Now, Eqs. (2.57) and (2.58) are substituted into Eq. (2.47). After some operations and removing higher-order terms, the complete J_2 linear model is obtained

$$\begin{aligned} \ddot{x}_j &= 2\dot{y}_j \omega_z + x_j \left(2n^2 + \omega_z^2 + \frac{2k_{J2}}{r^5} (1 - s_i^2 s_\theta^2) \right) \\ &\quad + y_j \left(\alpha_z + \frac{4k_{J2} s_i^2 s_{2\theta}}{r^5} \right) - 5z_j \omega_x \omega_z + a_{jx} \\ \ddot{y}_j &= -2\dot{x}_j \omega_z + 2\dot{z}_j \omega_x + x_j \left(\frac{4k_{J2} s_i^2 s_{2\theta}}{r^5} - \alpha_z \right) \\ &\quad - y_j \left(\frac{2k_{J2} s_i^2 c_\theta^2}{r^5} + n^2 - \omega_z^2 - \omega_x^2 \right) \\ &\quad + z_j \left(\alpha_x - \frac{k_{J2} s_{2i} c_\theta}{r^5} \right) + a_{jy} \\ \ddot{z}_j &= -2\dot{y}_j \omega_x - 5x_j \omega_x \omega_z - y_j \left(\frac{k_{J2} s_{2i} c_\theta}{r^5} + \alpha_x \right) \\ &\quad - z_j \left(n^2 - \omega_x^2 + \frac{2k_{J2} c_i^2}{r^5} \right) + a_{jz} \end{aligned} \quad (2.59)$$

The above equation can be rewritten into the following form

$$\frac{d}{dt} \begin{bmatrix} \dot{x}_j \\ \dot{y}_j \\ \dot{z}_j \end{bmatrix} = A_1(t) \begin{bmatrix} \dot{x}_j \\ \dot{y}_j \\ \dot{z}_j \end{bmatrix} + A_2(t) \begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix} + \begin{bmatrix} a_{jx} \\ a_{jy} \\ a_{jz} \end{bmatrix} \quad (2.60)$$

where

$$A_1(t) = \begin{bmatrix} 0 & 2\omega_z & 0 \\ -2\omega_z & 0 & 2\omega_x \\ 0 & -2\omega_x & 0 \end{bmatrix} \quad (2.61)$$

$$A_2(t) = \begin{bmatrix} 2n^2 + \omega_z^2 + \frac{2k_{J2}}{r^5}(1 - s_i^2 s_\theta^2) & \alpha_z + \frac{4k_{J2}s_i^2 s_{2\theta}}{r^5} & -5\omega_x \omega_z \\ \frac{4k_{J2}s_i^2 s_{2\theta}}{r^5} - \alpha_z & \frac{2k_{J2}s_i^2 c_\theta^2}{r^5} + n^2 - \omega_z^2 - \omega_x^2 & \alpha_x - \frac{k_{J2}s_{2i}c_\theta}{r^5} \\ -5x_j \omega_x \omega_z & \frac{k_{J2}s_{2i}c_\theta}{r^5} + \alpha_x & n^2 - \omega_x^2 + \frac{2k_{J2}c_\theta^2}{r^5} \end{bmatrix} \quad (2.62)$$

The above results can be summarized in the following corollary.

Corollary 2.1 *Consider a 2-satellite system of the reference satellite S_0 and the member satellite S_j , as shown in Fig. 2.1. In the presence of spherical gravity and J_2 gravity of the Earth, linear model of the relative motion of the satellite S_j in the LVLH coordinate can be derived as in (2.60)–(2.62) by removal of nonlinear effect.*

Since the nonlinear effect is removed in the linear model of Eq. (2.59), it is only expected to work for the satellite relative motion with small inter-satellite distance.

Further approximation can be made to the linear model of Eq. (2.59) by dropping the second-order J_2 terms. By removing ω_x^2 and replacing steering acceleration α_x with

$$\alpha_x \approx -\frac{k_{J2}s_{2i}c_\theta}{r^5} + \frac{3\dot{r}k_{J2}s_{2i}s_\theta}{r^4\dot{h}} \quad (2.63)$$

and substituting $(n^2, \omega_x, \omega_z, \alpha_z)$ in Eqs. (2.43), (2.19), (2.25), and (2.27) into Eq. (2.59), the first-order J_2 linear model is derived as

$$\begin{aligned} \ddot{x}_j &= \frac{2\dot{y}_j\dot{h}}{r^2} + x_j \left(\frac{2\mu}{r^3} + \frac{h^2}{r^4} + \frac{4k_{J2}(1 - 3s_i^2 s_\theta^2)}{r^5} \right) \\ &\quad - y_j \left(\frac{2\dot{r}\dot{h}}{r^3} - \frac{3k_{J2}s_i^2 s_{2\theta}}{r^5} \right) + \frac{5k_{J2}\dot{z}_j s_{2i} s_\theta}{r^5} + a_{jx} \\ \ddot{y}_j &= -\frac{2\dot{x}_j\dot{h}}{r^2} - \frac{2k_{J2}\dot{z}_j s_{2i} s_\theta}{r^3\dot{h}} + x_j \left(\frac{2\dot{r}\dot{h}}{r^3} + \frac{5k_{J2}s_i^2 s_{2\theta}}{r^5} \right) \\ &\quad - y_j \left(\frac{\mu}{r^3} - \frac{h^2}{r^4} + \frac{k_{J2}(1 + 2s_i^2 - 7s_i^2 s_\theta^2)}{r^5} \right) + z_j \left(\frac{3k_{J2}\dot{r} s_{2i} s_\theta}{r^4\dot{h}} - \frac{2k_{J2}s_{2i}c_\theta}{r^5} \right) + a_{jy} \\ \ddot{z}_j &= \frac{2k_{J2}\dot{y}_j s_{2i} s_\theta}{r^3\dot{h}} + \frac{5k_{J2}x_j s_{2i} s_\theta}{r^5} - \frac{3k_{J2}y_j \dot{r} s_{2i} s_\theta}{r^4\dot{h}} \\ &\quad - z_j \left(\frac{\mu}{r^3} + \frac{k_{J2}(3 - 2s_i^2 - 5s_i^2 s_\theta^2)}{r^5} \right) + a_{jz} \end{aligned} \quad (2.64)$$

The above equation can be rewritten into the following form

$$\frac{d}{dt} \begin{bmatrix} \dot{x}_j \\ \dot{y}_j \\ \dot{z}_j \end{bmatrix} = A_3(t) \begin{bmatrix} \dot{x}_j \\ \dot{y}_j \\ \dot{z}_j \end{bmatrix} + A_4(t) \begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix} + \begin{bmatrix} a_{jx} \\ a_{jy} \\ a_{jz} \end{bmatrix} \quad (2.65)$$

where

$$A_3(t) = \begin{bmatrix} 0 & \frac{2h}{r^2} & 0 \\ -\frac{2h}{r^2} & 0 & -\frac{2k_{J2}s_{2i}s_{2\theta}}{r^3h} \\ 0 & \frac{2k_{J2}s_{2i}s_{2\theta}}{r^3h} & 0 \end{bmatrix} \quad (2.66)$$

$$A_4(t) = \begin{bmatrix} \frac{2\mu}{r^3} + \frac{h^2}{r^4} + \frac{4k_{J2}(1-3s_i^2s_\theta^2)}{r^5} & -\frac{2\dot{r}h}{r^3} + \frac{3k_{J2}s_i^2s_{2\theta}}{r^5} & \frac{5k_{J2}s_{2i}s_{2\theta}}{r^5} \\ \frac{2\dot{r}h}{r^3} + \frac{5k_{J2}s_i^2s_{2\theta}}{r^5} & -\frac{\mu}{r^3} + \frac{h^2}{r^4} - \frac{k_{J2}(1+2s_i^2-7s_i^2s_\theta^2)}{r^5} & \frac{3k_{J2}\dot{r}s_{2i}s_{2\theta}}{r^4h} - \frac{2k_{J2}s_{2i}c_\theta}{r^5} \\ \frac{5k_{J2}s_{2i}s_{2\theta}}{r^5} & -\frac{3k_{J2}\dot{r}s_{2i}s_{2\theta}}{r^4h} & -\frac{\mu}{r^3} - \frac{k_{J2}(3-2s_i^2-5s_i^2s_\theta^2)}{r^5} \end{bmatrix} \quad (2.67)$$

The above result can be presented in the following corollary.

Corollary 2.2 Consider a 2-satellite system of the reference satellite S_0 and the member satellite S_j , in the presence of spherical gravity and J_2 gravity of the Earth, relative motion model in the LVLH coordinate can be further simplified into linear equations as in (2.65)–(2.67) by removal of nonlinear effect and second order J_2 terms.

Remark 2.4 The relative motion dynamics described by (2.60) and (2.65) are both linear time-varying system. The time-varying parameters in $A_1(t)$ and $A_2(t)$, $A_3(t)$ and $A_4(t)$ are the parameters of the reference satellite and are explicitly expressed in terms of RSV $(r, \dot{r}, h, i, \theta)$.

Remark 2.5 Both the nonlinear effect and the second-order J_2 effect have been removed in dynamics of Eq. (2.64). As a result, it is simpler than the exact nonlinear model in Eq. (2.47) and the complete linear model in Eq. (2.59). Moreover, in the dynamics of Eq. (2.64), all time-varying parameters of the reference orbit are explicitly expressed in terms of RSV $(r, \dot{r}, h, i, \theta)$.

2.4 Validation of Proposed Dynamic Models by Simulation

Simulations are carried out in MATLAB for three newly developed models, i.e., the exact J_2 nonlinear model in Eq. (2.47), the complete J_2 linear model in Eq. (2.59), and the first-order J_2 linear model in Eq. (2.64). In the simulations, each newly

developed model is compared with an exact J_2 propagator of satellite relative motion, which actually works by taking the difference of integrated solutions of two absolute J_2 dynamics in ECI frame. To do the comparison, the same initial conditions of the reference and member satellites are applied to the exact J_2 propagator and the newly developed models that are combined with RSV dynamics of Eqs. (2.20), (2.21), (2.23), and (2.24).

Different low Earth reference orbits are simulated by changing the initial values of osculating orbital elements on eccentricity and inclination, while the initial values of other osculating orbital elements remain the same as

$$a(0) = 7100 \text{ km}, \omega(0) = -20^\circ, f(0) = 20^\circ, \Omega(0) = 0 \quad (2.68)$$

It is known that the projected circular orbit (PCO) of satellite relative motion does not exist around an eccentric and perturbed reference orbit. However, when the eccentricity is small, the satellite relative orbit may nearly be a circle. Those relative orbits are referred as the Quasi-PCO. In the model simulations, Quasi-PCO is used as the motion of the member satellite, whose initial conditions are given by

$$x_j(0) = \rho/2, \quad y_j(0) = 0, \quad x_j(0) = \rho, \quad \dot{x}_j(0) = 0, \quad \dot{z}_j(0) = 0 \quad (2.69)$$

and

$$\dot{y}_j(0) \text{ is the solution of the energy matching condition.} \quad (2.70)$$

In Eq. (2.69), ρ is the initial radius of Quasi-PCO. Under conditions of Eqs. (2.68) and (2.69), as well as $e = 0.05, i = 45^\circ$ and $\rho = 2.5 \text{ km}$, the Quasi-PCO under J_2 perturbation is shown in Fig. 2.3.

Figures 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, and 2.10 show the simulation results in terms of errors between each newly developed model and the exact J_2 propagator in LVLH frame. Errors of the exact J_2 nonlinear model, the complete J_2 linear model, and the first-order J_2 linear model are presented by dash-dotted lines, dashed lines, and solid lines, respectively.

It can be seen from each figure in Figs. 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, and 2.10 that the model errors of the exact J_2 nonlinear model of Eq. (2.47) are always nearly zero in every scenario. This is because no approximation is taken in the derivation of the exact J_2 nonlinear model, so that it covers all effects of eccentricity, non-linearity, and J_2 perturbation. The simulation results demonstrate that the exact J_2 nonlinear model of Eq. (2.47) is correct and performs perfectly.

Remark 2.6 It is noticed that the primary errors of the two linear models are the drifts in the along-track direction (y). Compared with errors in other two directions (x and z directions, as shown in (a) and (c) of Figs. 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, and 2.10), the error magnitude in the along-track direction (as shown in Fig. 2.10b) is at least one order bigger.

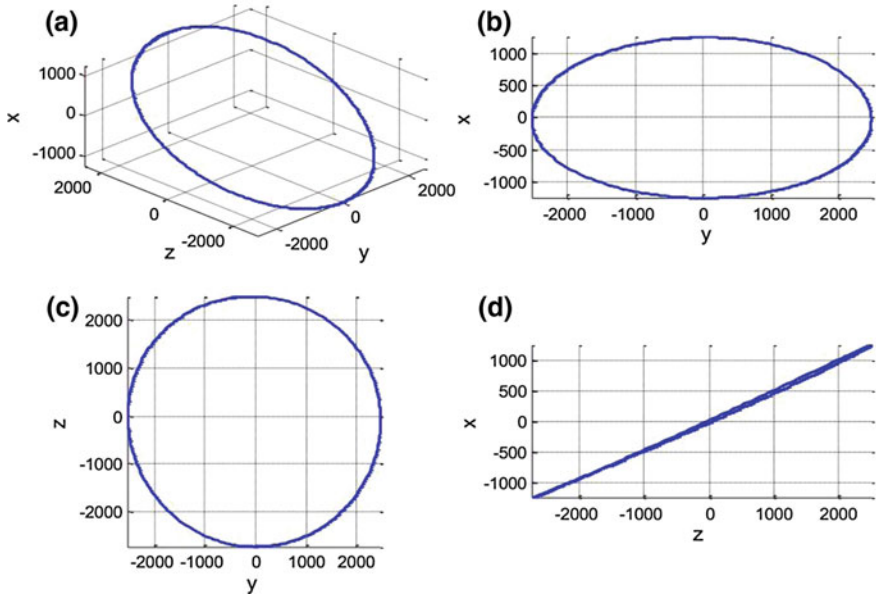
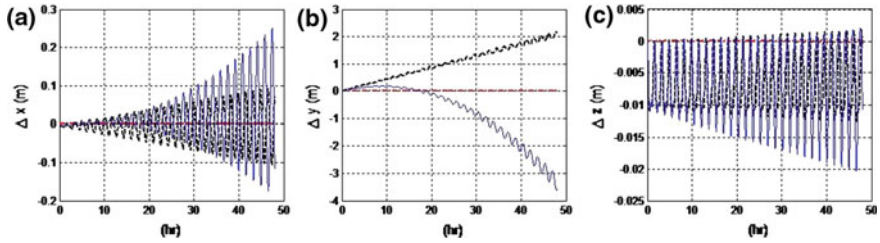
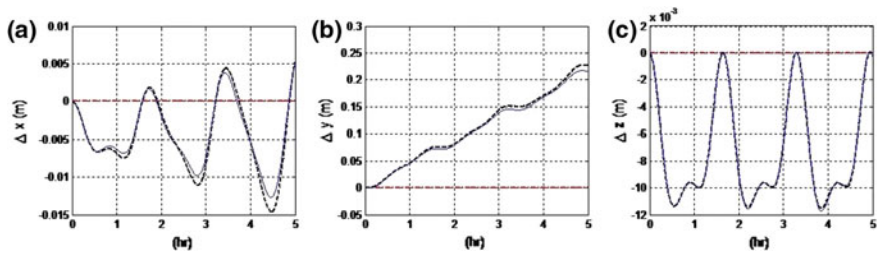


Fig. 2.3 Quasi-PCO of 5 km diameter

Fig. 2.4 Model errors at $\rho = 250$ m, $i = 45^\circ$, $e = 0.05$, $t = 48$ hFig. 2.5 Model errors at $\rho = 250$ m, $i = 15^\circ$, $e = 0.05$, $t = 5$ h

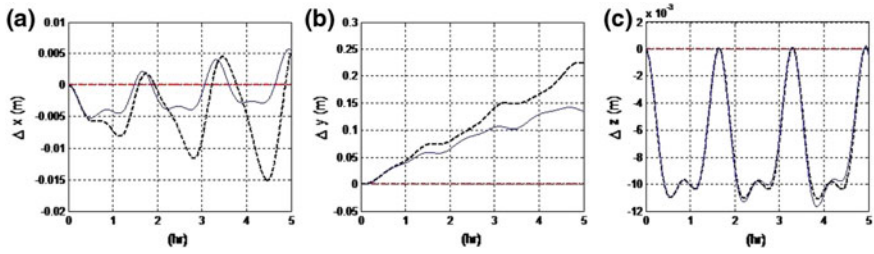


Fig. 2.6 Model errors at $\rho = 250$ m, $i = 45^\circ$, $e = 0.05$, $t = 5$ h

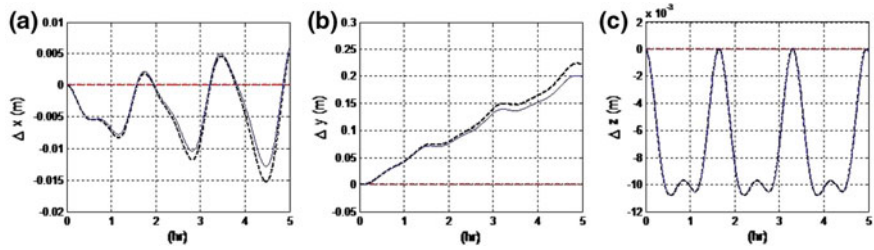


Fig. 2.7 Model errors at $\rho = 250$ m, $i = 85^\circ$, $e = 0.05$, $t = 5$ h

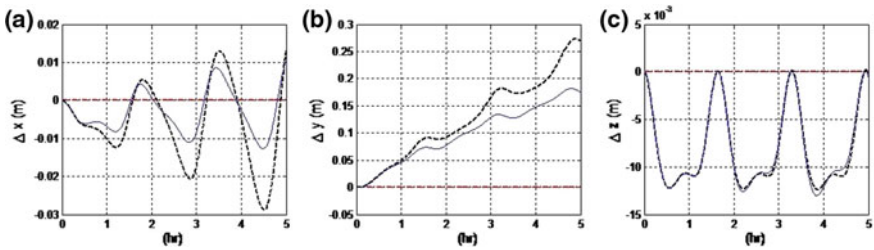


Fig. 2.8 Model errors at $\rho = 250$ m, $i = 45^\circ$, $e = 0.1$, $t = 5$ h

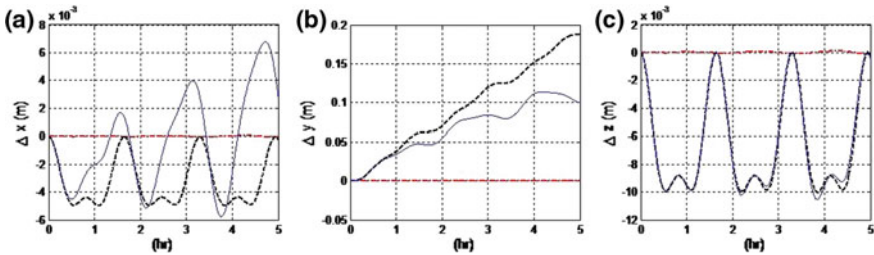


Fig. 2.9 Model errors at $\rho = 250$ m, $i = 45^\circ$, $e = 0$, $t = 5$ h

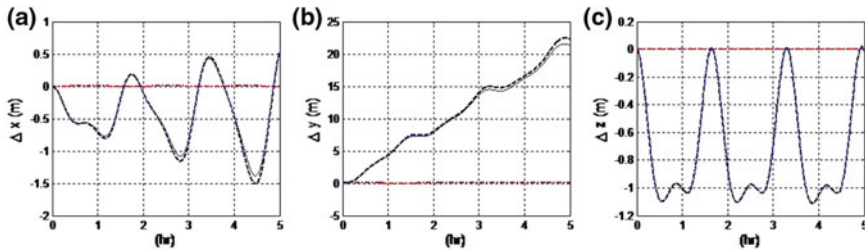


Fig. 2.10 Model errors at $\rho = 2500$ m, $i = 45^\circ$, $e = 0.05$, $t = 5$ h

Next, the drift errors in different satellite flying durations, different orbital inclinations, and eccentricities, as well as different formation sizes, are analyzed.

For a longer satellite flying such as 48 h as shown in Fig. 2.4b, the drift error of the complete J_2 linear model in Eq. (2.59) increases monotonously in positive direction over time, while the drift error of the first-order J_2 linear model in Eq. (2.64) increases firstly in positive direction and then turns down to negative direction. This phenomenon demonstrates that the removed nonlinear effect causes the drift error to increase positively, while the removed second-order J_2 effect is negative. Nevertheless, the secular nonlinear effect is approximately linear over time, while the secular second-order J_2 effect is higher order than the nonlinear effect. Consequently, the nonlinear effect is primary at the beginning so that the drift error increases positively. However, after some hours of flight, the second-order J_2 effect overwhelms the nonlinear effect and becomes dominant. Then, the drift error tends to negative direction.

Actually, the behaviors of the two linear models in the first several hours are more concerned, because the potential application of linear models is the formation maneuvering, which is usually carried out in a relative short duration such as one or few revolution(s). For this reason, analysis are subsequently focused on different scenarios in the first few hours of flight.

Figures 2.5, 2.6, and 2.7 compare the model performances in different inclinations. It is found that the drift errors of two linear models are similar in a smaller inclination (15° as shown Fig. 2.5b) or a bigger inclination (85° as shown Fig. 2.7b), while the drift error of the first-order J_2 linear model in a middle inclination (45° as shown Fig. 2.6b) is smaller. It shows that the second-order J_2 effect becomes dominant later in a smaller or larger inclination orbit and earlier in a middle inclination orbit.

Figures 2.6, 2.7, 2.8, and 2.9 compare the model performances in different eccentricities. It is found that the difference of drift errors between two linear models is smaller in a bigger eccentricity. It demonstrates that the second-order J_2 effect becomes significant earlier in a smaller inclination.

Figures 2.6, 2.7, 2.8, 2.9, and 2.10 compare the model performances in different formation sizes. They show that the two linear models tend to perform worse for a larger satellite formation size. This result agrees with the assumption used to derive the linear models. It is also observed that the second-order J_2 effect becomes evident later for a larger formation.

Remark 2.7 It is concluded that, when satellites fly in low Earth orbits such as a semimajor axis of 7100 km, the removed second-order J_2 effect compensates some removed nonlinear effects in the first several hours of satellite relative motion. So, the first-order J_2 linear model of Eq. (2.64) is more accurate and performs better than the complete J_2 linear model of Eq. (2.59) in a short duration. In particular, the second-order J_2 effect may manifest early and counteract the nonlinear effect more if the satellite flies in a middle inclination orbit which is near circular, and the formation size is small.

2.5 Comparison Study of Relative Dynamic Models

From the literature review in Sect. 2.1, it is known that the complexity of the relative dynamic models increases as the required accuracy becomes greater. The available models were derived under different assumptions and by different methodologies. A comparison study is necessary to select an appropriate model for a specific mission and determine what kind of perturbation should be considered for specific applications. Thus, in this section, a simulation method with a modeling error index is introduced for comparing and evaluating various theories of relative motion of satellite formation dynamics (Hang et al. 2008). The index captures error components of relative position and velocity between a selected satellite relative model and a precise propagator. It is applicable to evaluate the accuracy performance of any valid relative dynamic model including both linear and nonlinear models.

Commercial satellite software Satellite Tool Kit (STK) (www.STK.com) is used as a benchmarking tool to compare various dynamic models. The simulation results show that, when the Earth aspherical gravity and the air drag are present, the accuracy of some models is affected tremendously by eccentricity, semimajor axis, inclination, and formation size. The simulation results may serve for the future dynamic model development and the satellite formation mission design. In this section, the comparison focuses on the direct ODE models (ordinary differential equations in LVLH coordinate). However, the proposed evaluation method is generic and can be applied to other formulations.

2.5.1 Comparison Method with Model Error Index

In this study, all selected dynamic models are compared with a propagator, which is precise, standard, and acceptable in the practical application. In this research, STK is chosen as the key propagator. STK is popular commercial satellite software. It uses advanced algorithm and field data to generate satellite orbit, in which different perturbations can be customized and included.

Figure 2.11 shows the function chart of the model comparison method. In the first step, a group of initial conditions are assigned for both chief and deputy satellites,

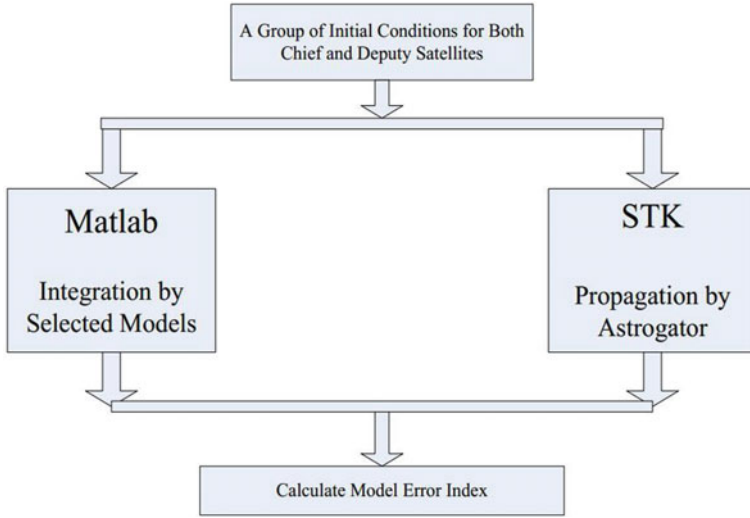


Fig. 2.11 Function chart of model comparison method

including relative positions and relative velocities. In the second step, the STK–MATLAB interface is utilized to transfer data into STK/Astrogator. Then, the initial conditions are propagated in STK which includes various orbital disturbances. Meanwhile, the same initial conditions are integrated numerically by the selected relative dynamics in MATLAB. In the third step, through the STK–MATLAB interface again, the propagated satellite orbit data in STK are collected in MATLAB. Finally, the propagated data from STK and the integrated data by the selected dynamics in MATLAB are synthesized, and the model error index is calculated.

In this comparison method, the key technique is the calculation of the model error index, which is comparable between different dynamic models. Alfriend and Yan (2005) proposed a nonlinear index for comparing the accuracy of various dynamic models. Their method is extended from the linear index of Junkins et al. (1996). Here, a simple index is proposed for the comparison by means of the numerical simulation. The proposed index σ is described as follows:

$$\sigma(n) = \frac{1}{n} \sum_{i=1}^n \log_2(1 + P_i^{(e)})(1 + V_i^{(e)})^w \quad (2.71)$$

where

$$V_i^{(e)} = \cos^{-1} \left(\frac{\mathbf{v}_i^{STK} \cdot \mathbf{v}_i^M}{\|\mathbf{v}_i^{STK}\| \|\mathbf{v}_i^M\|} \right) \quad (2.72)$$

$$P_i^{(e)} = \frac{\|\mathbf{r}_i^{STK} - \mathbf{r}_i^M\|}{\rho} \quad (2.73)$$

$\mathbf{v}_i^{STK}$, $\mathbf{v}_i^M$ are velocity vectors and $\mathbf{r}_i^{STK}$, $\mathbf{r}_i^M$ are position vectors. The data propagated in STK and the data integrated in MATLAB are with superscript STK and M, respectively. n denotes the total steps. i denotes the i -th step. $P_i^{(e)}$ and $V_i^{(e)}$ represent the position and velocity differences between the results of STK and dynamic model. In detail, $V_i^{(e)}$ is an adjusted radian value between two velocity vectors. It can effectively indicate the model error caused by the relative orbit rotation such as the tumbling. The exponent w is a weight to adjust $V_i^{(e)}$ in line with $P_i^{(e)}$. Increasing w would lead to magnifying rotation effect in the model error index and vice versa. In the simulation, $w = 2$ is carried out to balance the errors caused by the relative orbit rotation and the drift.

Note that the above model error index is computed at every sampling step. Moreover, this model error index is proportional to the model error. So, a smaller model error index implies that the model is more accurate.

2.5.2 Selected Dynamic Models for Comparison Study

This comparison study focuses on the direct ODE models. Thus, the equations, which describe the relative motion between satellites, are generally in the following form:

$$f(\ddot{\mathbf{q}}, \dot{\mathbf{q}}, \mathbf{q}, \mathbf{e}) = 0 \quad (2.74)$$

where $\mathbf{q} = [x \ y \ z]^T$ represents the vector of relative position of the deputy satellite in LVLH coordinate frame. $\mathbf{e}$ is the vector of orbit parameters of the chief satellite in ECI frame. It is constant for time-invariant models and is variable for time-varying models.

Five widely used models, including Clohessy-Wiltshire (CW) model, Tschauner-Hempel (TH) model, Unperturbed Nonlinear (UN) model, Schweighart-Sedwick (SS) model, and Xu-Wang model, are compared and evaluated. The assumptions about reference (chief) satellite orbit, perturbation, and formation size are different for each model and are summarized in Table 2.1. The five models are introduced as follows:

2.5.2.1 Clohessy-Wiltshire Model

CW model (Hill 1878) (Hill's equations) is established in the LVLH coordinate by making the assumption of a circular chief orbit, spherical Earth, linearizing the

Table 2.1 Comparison of assumptions for direct ODE models

Models		CW	TH	SS	NU	XW
Assumption	Chief orbit	C	E	C	E	E
	Perturbation	No	No	J_2	No	J_2
	Formation size	S	S	S	L	L

Note S and L represent small and large, respectively. C and E denote circle and ellipse, respectively

differential gravitational forces, and no other perturbations included. It was used initially to solve the satellite rendezvous problem and later to study the satellite formation flying. CW model is a fundamental model. Other direct ODE models are benchmarked on this model.

$$\ddot{x} - 2n\dot{y} - 3n^2x = 0 \quad (2.75)$$

$$\ddot{y} + 2n\dot{x} = 0 \quad (2.76)$$

$$\ddot{z} + n^2z = 0 \quad (2.77)$$

where x, y and z are the LVLH Cartesian coordinates, and $\dot{x}, \dot{y}$ and $\dot{z}$ are the relative velocity components, $n = \sqrt{\mu/r^3}$ is the mean motion, μ denotes gravitational coefficient, and r denotes radius of satellite orbit.

2.5.2.2 Tschauner-Hempel Model

Inalhan and How (Tillerson and How 2002; Clohessy and Wiltshire 1960) proved that the eccentricity has great effects on the relative motion. Using CW model, even a small eccentricity of $e = 0.005$ may result in a large fuel consumption to maintain a specified formation. Tschauner and Hempel (1965) presented a method to express the linearized relative dynamics around an eccentric orbit. TH model can be taken as an extension of CW model.

$$\ddot{x} = \frac{2\mu}{r^3}x + 2\omega y + \omega^2x \quad (2.78)$$

$$\ddot{y} = \frac{-\mu}{r^3}y - 2\omega\dot{x} - \omega^2y \quad (2.79)$$

$$\ddot{z} = \frac{-\mu z}{r^3} \quad (2.80)$$

where $\omega = \dot{f} = \frac{n(1+e\cos f)^2}{(1-e^2)^{3/2}}$, $\dot{\omega} = \ddot{f} = \frac{-2n^2e\sin f(1+e\cos f)^3}{(1-e^2)^3}$, $r = \frac{a(1-e^2)}{1+e\cos f}$.

Since this linear model will be used in Sect. 5.3 for decentralized formation keeping, this model needs to include the control input and be rewritten into a new form. Although (2.78)–(2.80) is expressed in the time domain, monotonically increasing true anomaly f of the chief orbit provides a natural basis for parameterizing the fleet time and motion. This observation is based on the fact that the angular velocity and the radius describing the orbital motion are functions of the true anomaly. When f is used as the free variable, the equations of motion can be transformed using the following relationships between derivation over time (represented by $(\dot{\cdot})$) and derivation over θ (represented by $(\cdot)'$)

$$(\dot{\cdot}) = (\cdot)' \dot{f}, \quad (\ddot{\cdot}) = (\cdot)'' \dot{f}^2 + \dot{f} \dot{f}' (\cdot)' \quad (2.81)$$

where the time rate of change of the true anomaly $\dot{f}$ can be written as

$$\dot{f} = \frac{n(1 + e \cos f)^2}{(1 + e^2)^{3/2}} \quad (2.82)$$

With these transformations, the set of LTV equations describing the relative motion of the satellite with respect to an elliptic reference orbit can be written as (Inalhan et al. 2002).

$$\mathbf{x}'(f) = A(f)\mathbf{x}(f) + B(f)\mathbf{u}(f) \quad (2.83)$$

where

$$A(f) = \begin{bmatrix} \frac{2e \sin f}{1 + e \cos f} & \frac{3 + e \cos f}{1 + e \cos f} & 2 & \frac{-2e \sin f}{1 + e \cos f} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -2 & \frac{2e \sin f}{1 + e \cos f} & \frac{2e \sin f}{1 + e \cos f} & \frac{e \cos f}{1 + e \cos f} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2e \sin f}{1 + e \cos f} & \frac{-1}{1 + e \cos f} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (2.84)$$

$$B(f) = \frac{(1 - e^2)^3}{(1 + e \cos f)^4 n^2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T \quad (2.85)$$

where $\mathbf{x}(f) = [x', x, y', y, z', z]^T$ represent relative positions and velocities of the satellite with respect to the LVLH frame expressed in f domain, $\mathbf{u}(f) = [u_x \ u_y \ u_z]^T$ represent the vector of control acceleration, n is the natural frequency of the reference orbit, and e is the eccentricity.

2.5.2.3 Unperturbed Nonlinear Model

TH model can be further expanded to a system of nonlinear differential equations accounting for both effects of nonlinearity and eccentricity (Gurfil 2005), which is called the unperturbed nonlinear model. The parameter $\mathbf{e}$ of the reference satellite in (2.47) is described by three augmented first-order differential equations in the unperturbed nonlinear model.

$$\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2x = -\frac{\mu(r_c + x)}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} + \frac{\mu}{r_c^2} \quad (2.86)$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2y = -\frac{\mu y}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} \quad (2.87)$$

$$\ddot{z} = -\frac{\mu z}{\left[(r_c + x)^2 + y^2 + z^2\right]^{3/2}} \quad (2.88)$$

$$\ddot{r}_c = r_c \dot{\theta}^2 - \frac{\mu}{r_c^2} \quad (2.89)$$

$$\ddot{\theta} = -\frac{2\dot{r}_c \dot{\theta}}{r_c} \quad (2.90)$$

where x, y and z are the relative motion coordinates of the deputy with respect to the chief in the LVLH frame. r_c refers to the radius of the chief satellite from the center of the Earth, θ refers to the latitude angle of the chief, and μ is the gravitational parameter.

2.5.2.4 Schweighart-Sedwick Model

Schweighart and Sedwick (2002) developed a set of linearized differential equations to capture the relative motion between satellites under J_2 effect around a circular orbit. This model can be taken as another extension of CW model.

$$\ddot{x} - 2(nc)\dot{y} - (5c^2 - 2)n^2x = 0 \quad (2.91)$$

$$\ddot{y} + 2(nc)\dot{x} = 0 \quad (2.92)$$

$$\dot{\gamma} - nb \cos \gamma = 0 \quad (2.93)$$

$$\dot{\Phi} - nb\Phi \cos \gamma \sin \gamma = 0 \quad (2.94)$$

where $z = r_{\text{ref}}\Phi \sin(knt - \gamma)$. γ is angular distance between the equator and the intersection of two orbital planes (chief and deputy orbit). Φ represents maximum angular cross-track separation. $r_{\text{ref}}, i_{\text{ref}}$ are radius and inclination of reference satellite orbit.

2.5.2.5 Xu-Wang Model

Xu and Wang (2008a) recently developed a satellite relative dynamic model which includes eccentricity, nonlinearity, and J_2 perturbation. This model is developed based on the essential fact that the precise relative dynamics highly depends on the accurate information of the reference orbit. The parameter $\mathbf{e}$ of the chief satellite in (2.74) is described by five augmented first-order differential equations to describe reference orbit dynamics. It is stated that this dynamic model does not have model error in arbitrary eccentric orbits under J_2 perturbation. So, Xu-Wang model can be used to propagate the satellite relative motion from arbitrary initial conditions. Xu-Wang model is introduced in Sect. 2.2.2.

2.5.3 Case Studies

2.5.3.1 Simulation Scenario

For setting up comparison scenario, the idea of the projected circular orbit (PCO) in the LVLH frame centered at the chief satellite is utilized. It can be described by

$$x = 0.5\rho \sin(\theta + \alpha_0) \quad (2.95)$$

$$y = \rho \cos(\theta + \alpha_0) \quad (2.96)$$

$$z = \rho \sin(\theta + \alpha_0) \quad (2.97)$$

where ρ is the radial of PCO; θ is the true latitude angle of the chief satellite; α_0 is the initial phase angle of relative orbit. It is known that an exact PCO is only possible in circular orbits. When eccentricity and perturbations are present, the relative orbit may be distort. However, if the eccentricity is small, the relative orbit will still be close to PCO. Actually, an exact PCO is not necessary in the comparison study, and a distorted PCO is acceptable.

In all test cases, the energy-matching initial conditions derived by Xu and Wang (Xu et al. 2009) are used to prevent a fast drift of the formation. It should be kept in mind that both the distorted PCO formation and the energy-matching conditions do not change the evaluation and comparison results. In the simulation, no matter what

Table 2.3 Orbit parameters of deputy satellite

Deputy satellite orbit	Value
x_0	0.5 km
y_0	0 km
z_0	ρ km
$\dot{x}_0$	0 km/s
$\dot{y}_0$	Adjusted
$\dot{z}_0$	0 km/s
ρ	0.10–20.0 km

Table 2.2 Orbit elements of chief satellite

Chief satellite orbit	Value
a	6600–8000 km
Ω	0°
i	0° – 90°
e	0–0.01
ω	0°
v	0°

Table 2.4 Physical parameters of both satellites

Satellite parameters	Value
Dry mass	150 kg
Drag coefficient	2.2
Drag area	5 m^2

kind of formation is used, the selected dynamic model should predict the relative motion which is close to the result in the STK propagator as long as the same set of initial conditions are applied. Nevertheless, using a stable formation may improve the reliability of the model error index.

Four parameters i , e , a , and ρ are selected as variables for comparison in the simulation. Later on, it will be proved that these four parameters have important effects on dynamic model accuracy. The objective was to calculate the model error index σ of each selected model due to each of the variables e , a , i , and ρ . In the simulation, the testing cases are assigned in the following Tables 2.2, 2.3, and 2.4:

Within the calculation of the model error index by (2.71), the scenario duration is 24 h, and the sampling interval is 1 min, which correspond that the total steps are $n = 24 \times 60 = 1440$. The astrogator in STK is customized to incorporate the Earth zonal harmonic perturbations up to J_{21} and the atmospheric drag.

2.5.3.2 Case 1: Error Index Versus Formation Size

Figure 2.12 shows the index comparison varying with the PCO radius. It is clear that the model error indexes of linear models tend to be larger than those of nonlinear models (unperturbed nonlinear model and Xu-Wang model) as the

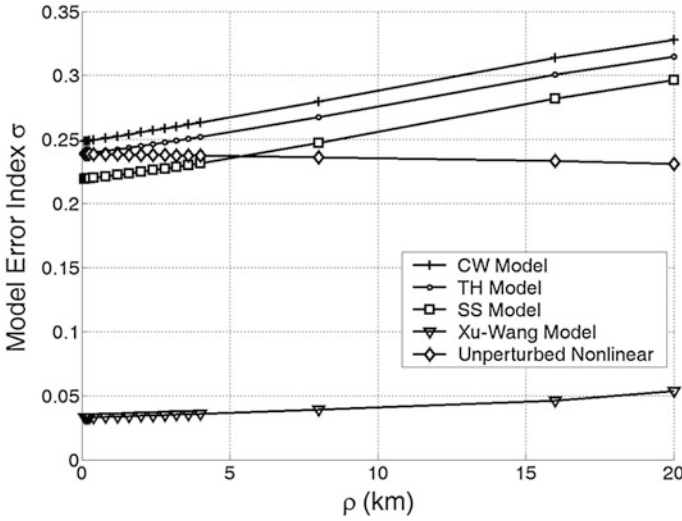


Fig. 2.12 Index comparison for $e = 0.0001$, $i = 45^\circ$, $a = 6600$ km

formation size increases. In this simulation, since the eccentricity is almost zero, the indexes of CW model and TH model are close to each other, while SS model that include J_2 effects performs better than other linear models. The model error difference between TH model and unperturbed nonlinear model in the figure shows the impact of nonlinearity. It is a function of formation size and increases as formation size getting large. The model error difference between unperturbed nonlinear model and Xu-Wang model shows the impact of J_2 effects. It is found that the influence of J_2 effects is bigger than the nonlinearity effect.

2.5.3.3 Case 2: Error Index Versus Eccentricity

Figure 2.13 shows the error index as a function of the eccentricity. It can be seen that errors of the models which exclude eccentricity consideration (CW model and SS model) grow larger and larger as the eccentricity gets larger. Since the formation size is small, the performance of TH model is almost the same as unperturbed nonlinear model, because both of them compensate the effect of eccentricity. In contrast to error due to formation size variation, eccentricity changes lead to more noticeable error growth. The model error difference between CW model and TH model shows the impact of eccentricity. In contrast to the error difference between the unperturbed nonlinear model and Xu-Wang model, which shows impact of J_2 effects, it is found that the eccentricity leads to much larger error growth. Thus, eccentricity is the dominant error in formation flying design and needs to be considered with high priority.

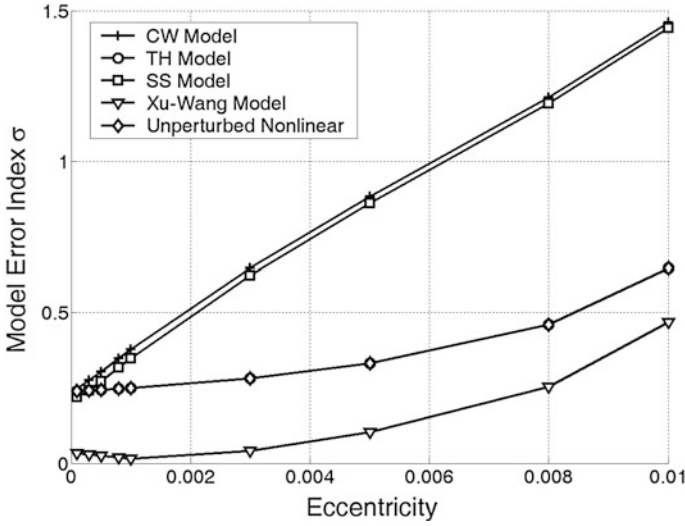


Fig. 2.13 Index comparison for $\rho = 0.1$ km, $i = 45^\circ$, $a = 6600$ km

2.5.3.4 Case 3: Error Index Versus Inclination

Figure 2.14 shows error index comparison as a function of the inclination. The simulation results show that SS model has distinct error trend. It is interesting that the SS model even does not perform as good as the CW model in a larger inclination. The error indexes of all other models decrease as inclination increases. Nevertheless, the models, which consider J_2 effects, have better performance than other models (the change in magnitude is relatively small.). This demonstrates that the J_2 effect has a tight relationship with the inclination.

2.5.3.5 Case 4: Error Index Versus Semimajor Axis

Figure 2.15 shows error index comparison as a function of the chief satellite perigee. Since none of the selected dynamic models considers the atmospheric drag, their model errors increase exponentially when the chief satellite perigee is smaller than a certain value. It can be seen that the inflexion of model error indexes appears when the perigee is around 300 km, which corresponds to a satellite semimajor axis of about 6750 km. Therefore, if the satellite altitude is lower than 300 km, the inclusion of atmospheric drag in the model is necessary. On the other hand, if the satellite altitude is higher than 400 km, the effect of atmospheric drag is small.

In summary, the index comparison shows every model has distinct characteristic and performance, which is summarized in Table 2.5.

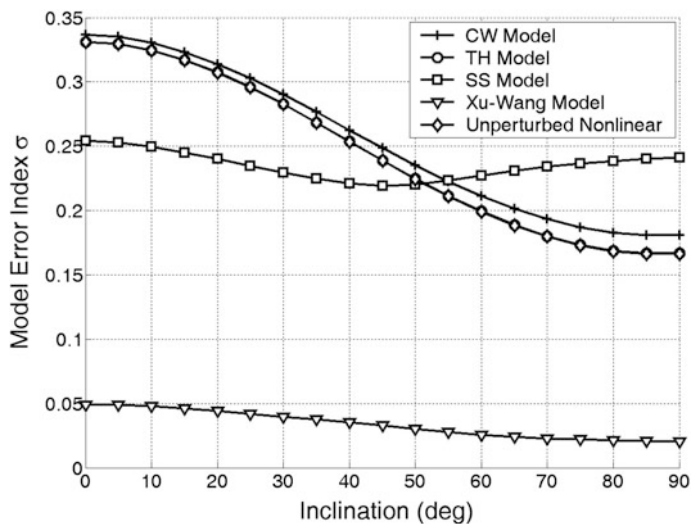


Fig. 2.14 Index comparison for $\rho = 0.1$ km, $e = 0.0001$, $a = 6600$ km

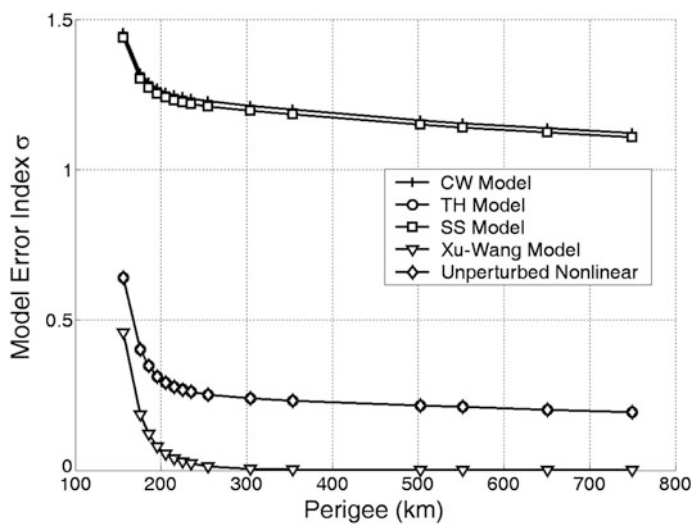


Fig. 2.15 Index comparison for $\rho = 0.1$ km, $e = 0.01$, $i = 45^\circ$

Table 2.5 Comparison of model error sources for direct ODE models

Models		CW	TH	SS	NU	XW
Model error caused by	Eccentricity	L	S	L	S	S
	J_2	L	L	L	L	S
	Nonlinearity	L	L	L	S	S

Note “S” and “L” represent small and large, respectively

2.6 Summary

Firstly, the exact J_2 dynamics for a single satellite is presented based on RSV in this chapter. The expression of this dynamics itself is rather simple. Three variables of RSV, i.e., angular momentum h , inclination i , and right ascension of ascending node Ω , are slowly time varying under perturbations. The most important benefit of this dynamics is that the rotation properties of LVLH frame can be expressed in terms of RSV explicitly and simply. So, it is suitable to describe the motion of a rotating LVLH frame.

Secondly, the exact J_2 nonlinear relative model is derived. Since no approximation is applied in the derivation, this model does not have error if only J_2 perturbation is present. The model depends on 5 parameters of RSV, so that the satellite relative motion is equivalently described by 11 simple first-order differential equations. It is a good candidate to solve the precise control problems of long-term flying satellite formation such as formation maintenance. It can also be used in the study of formation design or even as a propagator of satellite relative motion.

Thirdly, two J_2 linear relative models are developed by removing the nonlinear effect and the second-order J_2 effect from the exact J_2 nonlinear relative model. The numerical study demonstrates that the removed second-order J_2 effect compensates some of the removed nonlinear effect at the beginning stage. Moreover, the first-order J_2 linear relative model is much simpler than the other two models. It is expected to have potential application on formation maneuver and keeping.

Lastly, a simulation-based evaluation method is presented to compare various satellite formation flying dynamic models in this chapter. With the formulation of a model error index, five existing direct ODE models have been evaluated and compared in the proposed simulation platform. The simulation results provide insights and guidance for the selection of suitable relative dynamic model taking into account the presence of various perturbation effects. The numerical results show that eccentricity, J_2 perturbation, nonlinearity, and atmospheric drag play different roles to affect the model error. The decision on model selection should be made through the balanced consideration of every factor. If the satellite formation flies in orbits higher than 400 km, the eccentricity dominates the disturbances; the next crucial perturbation is J_2 effect followed by nonlinearity. However, if the satellite formation flies in orbits lower than 300 km, the atmospheric drag affects the model accuracy significantly. In contrast to other factors, the nonlinearity has a

smaller effect for a small size formation. Among the five selected direct ODE models, the developed, exact nonlinear relative dynamics Xu-Wang model, introduced in Sect. 2.2.2, which takes into account of both J_2 and chief (reference) orbit eccentricity, performs best in every simulated scenario.

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