Chapter 2
Describing the Mathematical Intentions of Early Learning Childhood Experiences

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Abstract This chapter is written to inform the subsequent design of intentional experiences for young children, especially in family settings. There is clearly a world of mathematical possibilities for young children but it will assist in ensuring that children have experiences that can assist them in interpreting the world mathematically and in adapting to the demands of schooling. Based on analysis of research and critique of similar documents, the chapter presents a set of key foci that can inform the design of suggestions in which parents (and educators) can engage with children.

Keywords Early years mathematics · Measurement · Number · Space and location · Early years curriculum

Introduction: The Purpose of Defining Mathematics Learning Goals

Governments and local communities increasingly recognise that productive family-based experiences and effective pre-school education can position children favourably to participate fully in the learning opportunities that school will offer them subsequently. Most children arrive at school having had a wide variety of educative activities that shape their subsequent learning and dispositions for learning. While formal education settings contribute to children's learning prior to school, family-based experiences are also critical for young children's learning and

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development. This chapter describes, for parents and educators, the potential focus of their engagements with their children around their learning of mathematics. The process followed in the development of the Australian Curriculum: Mathematics (AC:M) was used to develop the descriptions as sets of statements. That process included the articulation of the overall goals and principles informing the subsequent statements, the summary of evidence that informed expectations for learning, and the preparation of statements that describe the mathematical experiences from which it is anticipated that all children would benefit.

Some assumptions that underpin our intentions, and the analysis that follows, are that:

- productive mathematics learning experiences that children have prior to school enhance their effective participation in schooling;
- parents and educators welcome suggestions of the type of intentional experiences in which they can engage with children in ways that enhance children's mathematics learning, thinking and dispositions;
- productive experiences can be the result not only of observation and exploration in response to child initiated mathematics but also of educator and parent initiated activities and
- parents and educators sometimes can be unaware of the impact of their own interactions on their children's learning and enhancing this awareness can improve intentional experiences that can be initiated to provoke learning.

A further assumption is that the first step in the design and analysis of illustrative experiences and engagements is the articulation of mathematical learning goals. In the Numeracy@Home project, an Australian Research Council funded project in partnership with the Victorian Department of Education and Training and Catholic Education Melbourne, this articulation takes the form of a small number of statements. Some principles that guided the development of the statements are that they should:

- be few in number and succinctly written, focusing on the key mathematics learning goals;
- be consistent in form with the content descriptions and processes of the relevant school curricula to facilitate the transition to school learning—in this case this is the Australian Curriculum: Mathematics (ACARA 2015);
- be informative and use language appropriate to a non professional audience;
- inform the design of inclusive experiences appropriate for all children; and
- be focused on children aged 3–5 years (meaning the years just before attending school).

Similar principles were articulated by Clements (2014) in describing standards that apply in the US context. He argued:

> The most important standards for early childhood are standards for programs, for teaching, and for assessment. These should be built on flexible, developmental guidelines for young children's mathematical learning. Guidelines should be based on available research and expert practice, focus on and elaborate the big ideas of mathematics, and represent a range of expectations for child outcomes that are developmentally appropriate (p. 15).

The following section seeks to synthesise and critique some established sources that have described mathematical learning goals for children prior to school and to pose illustrative statements that can guide the design of specific experiences that promote mathematics learning for young children.

The Process and Data Used to Define the Learning Goals

Essentially the analysis reported in this section is intended to inform the creation of learning experiences that are described as tasks in the literature on primary and secondary mathematics teaching and learning. As argued by Hiebert and Wearne (1997), "what students learn is largely defined by the tasks they are given" (p. 395). Anthony and Walshaw (2009), in a research synthesis, concluded that "it is through tasks, more than in any other way, that opportunities to learn are made available to the students" (p. 96). Further, Sullivan et al. (2014) presented results that indicated that teachers welcome suggestions not only of specific tasks but also the structure of lessons in which those tasks might be posed. Sullivan et al. found that teachers appreciated the presentation of theory, the connection of the theory to practice through specific exemplars and the explication of the pedagogies associated with those exemplars. It seems logical that similar thinking can be applied to the design of learning goals and intentional experiences for children prior to school, although the language used to describe the experiences may be different from that used in schools.

We used an adaptation of the theory of didactic situations (TDS) and didactic engineering (DE) developed by Brousseau (1997) in articulating the set of learning goals. DE is a process that incorporates design and implementation and emphasises both the importance of considering the mathematical goals and pedagogical opportunities at the design stage, and the importance of describing the complexity of informal and prompted mathematical experiences at the implementation stage.

The first phase is the preliminary analysis of the mathematical goals of projected learning experiences. The following is intended to represent this preliminary analysis. The subsequent phase is design and a priori analysis though which illustrative experiences that exemplify the mathematical goals are developed. Even though some suggestions of experiences are presented below, the following is intended to elaborate the preliminary analysis rather than define the experiences that will be prepared subsequent to this phase.

The process is that, for different domains within mathematics, we draw on some literature that defines the key ideas, available data and analysis of various sources to articulate the mathematical focus of experiences in early childhood.

The first set of data presented is from the Early Numeracy Research Project (ENRP) (Clarke et al. 2002) in which the first two authors participated. The project involved one on one interviews with trained interviewers using a structured format and supplied materials. The project report presents data from the 868 students who were interviewed six times from the start of the first year of school to the end of the third year of school, although only data from the first interview are presented here. This analysis also includes data from subsequent interviews prescribed to explore emergent mathematical ideas for those children who were not yet able to count 20 small plastic teddies in the initial interview (described in the following as the "detour").

This analysis also includes data from the *Let's Count* Longitudinal Study (Gervasoni and Perry 2015) that assessed the mathematical knowledge of children at the beginning and end of their preschool year. *Let's Count* is an early mathematics program designed to assist educators in early childhood contexts to work in partnership with parents and other family members to promote positive mathematical experiences for young children (3–5 years). The program aims to foster opportunities for children to engage with the mathematics encountered as part of their everyday lives, talk about it, document it, and explore it in ways that are fun and relevant to them.

The children in the *Let's Count* project were assessed using the Mathematics Assessment Interview (MAI) that is a revised version of the instrument used during the ENRP. Overall, 125 children were assessed in December 2012 to form the comparison group, 142 children were assessed in 2013 (March and December), and 172 children in 2014 (March and December). Children were assessed by members of the research team who were experienced in working with young children and received training about using the MAI script and record sheet. The MAI record sheets were analysed independently by research assistants who entered responses and strategies for each task into a database, and also used an established algorithm to code overall performance in each domain to determine the ENRP growth point reached by each child.

Other documents from which we present relevant extracts are:

- the Australian Curriculum: Mathematics (AC:M) (2015) which as used as a form of backward mapping of the respective concepts;
- the Early Years Learning Framework (EYLF) (DET 2015) which "describes the principles, practices and outcomes that support and enhance young children's learning from birth to 5 years of age, as well as their transition to school" (DET 2015, p. 1);
- a curriculum analysis by Clements (2014) focusing on the early years that was intended to complement the standards approach in the United States; and
- the International Baccalaureate Primary Years Program (PYP) (IBO 2012) for children aged 3–5 which is an internationally recognised curriculum standard.

The first two of these documents were written for the Australian context and were extensively consulted with relevant stakeholders. The third source is a widely quoted analysis on early mathematics learning. The fourth was written for an international context and is used in hundreds of schools worldwide. Where other sources are used, they are referenced subsequently.

Analysis and Synthesis of Particular Content Domains

The following presents detailed analyses of the sources and recommendations on the topic of measurement, time, and number. There is also a less detailed description of the domains of shape, location and patterns. It is possible that some experiences related to chance are suitable for young children but there has been limited research on chance and little attempt to describe the foci of chance experiences. In each discussed domain, the intention is to synthesise the respective sources and to distill statements that can be used as the basis of subsequent resources development.

Measurement

Even though many texts on early years numeracy learning do not include this domain, learning about measures and measuring begins from birth and the curriculum in the first years of school builds on these early experiences. McDonough and Sullivan (2011), for example, in re-analysing ENRP data related to the Length items concluded "… the key targets for the learning of length in the first 3 years of school are, respectively, learning to compare, learning to use a unit iteratively, and measuring using formal units" (p. 27). They argued that learning to compare is the first step, and it can be assumed that comparisons are also important in other aspects of measurement such as mass and capacity, although time is conceptualised differently. Interestingly, McDonough and Sullivan found very little relationship between responses to length comparison items and facility with counting, for example, suggesting that learning of measurement is dependent on experiences that are specific to the measurement concepts being learned and not an indicator of some general mathematical capacity.

McDonough and Sullivan (2011) drawing on earlier work by Piaget, argued that there are three key concepts associated with measurement comparisons:

Visual comparisons, which do not require the ends to be aligned in the case of length or about which judgements can be made without testing for mass and capacity;

Conservation (in which size of an object is irrespective of its arrangement) is associated with direct comparisons, even if more than one object is being so compared; and

Transitivity (if John is taller (or heavier) than Sally, and Sally is taller (or heavier) than Ben, then John is taller (or heavier) than Ben) is associated with indirect comparisons in which a third object is needed to make the comparisons.

In the early years, experiences connected to both visual and direct comparisons are desirable for all children whereas in schools students move towards considering transitivity. At the start of the first year of school, the ENRP reports that around half of the students could compare a string and a stick, and another 20 % could say how many paper clips long is a straw. Both of these tasks involve direct comparisons. In the "detour", 60 % of the students could order three candles smallest to largest, and 50 % could order 4 candles. These candle tasks rely on visual comparisons because there is no need to align starting points. Two thirds of students at the start of school could compare the mass of two objects by hand and decide which was heavier. There was no ENRP assessment of capacity.

The *Let's Count* research reported that at the beginning of preschool in 2014, 53 % of 100 children (4 year olds) could compare a string and a stick, and 5 % could say how many paper clips long is a straw. In the "detour", 52 % of 194 children could order 3 candles smallest to largest, and 28 % could order 4 candles.

Both sets of results indicate that while some children have progressed beyond the earliest theoretical indicators, there are others who have not. Given that comparing the string and stick, or comparing which is heavier out of two small containers, seems more or less intuitive, and would arguably develop without any specific or directed experiences at all, it seems possible that it is the language of the task that may explain the numbers of students at the start of school who experienced difficulty.

The Australian Curriculum (AC) Foundation level curriculum includes the following statements, which are written in terms of student actions:

> Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language.

Based on the ENRP and *Let's Count* data, this statement describes important learning for all students, noting that substantial numbers of students can perform such comparisons at the start of school. The statement gives some prominence to language in the expectation that students will explain reasoning, although this could be more explicit. It is arguable that it might be reasonable to expect that all students do more than two way direct comparisons (and so the description could use longest, heaviest, and holds most) in the first year of school.

It is interesting to compare this statement with both the general statements and the details within the PYP documentation (2012) for children aged 3–5 years. The covering statement proposes that "Students will identify and compare attributes of real objects …". The details, under the stem "When constructing meaning, learners …" include the following:

> *Understand that attributes of real objects can be compared and described, for example, longer, shorter, empty, full, heavier, hotter, colder.*

This is a broader set of measures, and in particular the inclusion of "shorter" is useful. The EYLF addresses measurement with the following statement:

> *Children demonstrate an increasing understanding of measurement ... using vocabulary to describe size, length, volume, capacity ....*

This statement does not communicate to the reader that these measures are relative (at least at this level) in that it treats the measures as absolutes in the use of the term "measurement". It is also not clear what is meant by "increasing", "understanding", and “size”. Further, the distinction between volume and capacity is nuanced and complex and arguably not appropriate for very young children. The EYLF statement is unlikely to inform the design of productive intentional education measurement experiences.

Clements (2014) in articulating the “big ideas” of measurement wrote:

> Comparing and measuring can be used to specify ‘how much’ of an attribute (e.g., length) objects possess. Measurement is giving a number to an attribute of objects, such as length, area, capacity or weight (p. 50).

The other measurement comments relate to iteration of a unit, which arguably comes later than the focus of this analysis.

Noting that there is no discussion of learning measurement even though the monograph is titled mathematics, Sousa (2008) includes the following statement of what pre-schoolers should learn:

> Children compare the height of a block tower with the height of a chair or table. They measure each other’s height and distance from the desk to a wall. They learn that a block is too long or too short to complete a project (p. 79).

Although this only focuses on length, it emphasises comparisons and provides an interesting prompt as to potential experiences for children.

It is interesting that there are various publications describing early childhood mathematics, like Sousa (2008), that do not include statements related to measurement. For example, Anders et al. (2013) do not include measurement as part of their assessment of child development.

It is noted that the measurement domain is unique in that the concepts being developed are relative in the early years (e.g., longer, heavier) whereas the absolute concepts require use of tools, which are part of subsequent learning. It also seems that the term “attribute” is not appropriate in that objects do not have attributes in isolation.

Comparisons and the use of language seem to be common characteristics of all of the above statements. Indeed, the skills of comparisons (such as aligning ends) seem relatively natural and it can be concluded that nearly all of the experiences of comparisons in the early years should focus on words used to describe the aspects of objects being compared. The notion of visual comparisons (which do not require aligning ends) and direct comparisons (which do) are both important. In summary, the following statement is proposed with the intention of informing subsequent task design, following the stem “children learn maths when they …”

> *Compare objects and describe, in everyday language, which is longer, shorter, heavier, lighter, holds more, hold less.*

The type of formal experiences suggested by Sousa and indicated above are illustrative of what is possible. In addition, other illustrative formal experiences are tasks such as comparing two strings to decide which is longer, or deciding who is taller if one person is standing on a step. In both cases, examples which can be compared visually, followed by experiences that require direct comparison are desirable. It is possible that experiences that require indirect comparisons (such as deciding which is taller: the table in this room; or the table in that room) may help to consolidate the direct comparison experiences.

Time

Although best described as measurement, it seems that learning about time is different from other aspects. The most significant research on children's learning about time was reported by Piaget et al. (1960). In their research drawing on extensive individual task based interviews, they found that children first learn about intuitive time that involves sequencing of events (seriation) and comparing the duration of events. Connected to this is the naming of events such as the names of days, parts of the day, and even "5 min" as code for a short time. As with the other aspects of measurement it seems that the concepts involved are intuitive and the key focus for learning is the relevant language.

In the ENRP interview, children were asked to draw a clock, and prompted to say what clocks are used for, and what are the numbers. There were also questions about specific times on clocks. Clarke et al. (2002) reported "When the children arrive at school at the beginning of their Prep year, 84 % of the children are aware of time, and a further 16 % know some clock times and days, and can relate events to these" (p. 84). While their assessment items address only some aspects of time, at least for such elements many students are aware of time and its use. Gervasoni and Perry (2015) found that 73 % of children at the beginning of preschool were also aware of time and 5 % knew some clock times and days, and could relate events to these.

The AC:M says:

- Compare and order the duration of events using the everyday language of time
- Connect days of the week to familiar events and actions

These statements articulate key time concepts although it may have been helpful to include parts of days, parts of the year (seasons), and even months. The order of the words in the first sentence is not quite right (it would be better as "compare the duration of, and order, events …"). Nevertheless this is a useful description of the various elements of time at this level. The PYP described time concepts as follows:

- Understand that events in daily routines can be described and sequenced, for example, before, after, bedtime, story-time, today, tomorrow

The emphasis on routines is useful although the statement represents some aspects and not others. Some of the terms are about comparisons and some are about events and it would be helpful to delineate these since they represent quite different learning.

The EYLF included:

- Children notice and predict patterns of routines and the passing of time.

This is clearer than other statements in the EYLF although the required concepts are broader than what is described in that statement. The following is an attempt to synthesise this information into statements using the stem "children learn maths … when they …":

- use words that describe points in time, events and routines;
- compare the duration of everyday events using mathematical language; and
- describe and arrange connected events in the usual sequence that they occur.

Illustrative experiences could involve conversations about time, and in particular using descriptive words like day, night, early, late, morning, every day, today, tomorrow, yesterday, sleeps, seasons; and comparative words such as earlier, later, longer, shorter, faster, slower, days, months, before, after; and developing familiarity with clocks and the use of the term o'clock.

Numbers

Learning to use numbers is fundamental to the learning of mathematics. As Sousa (2008) and Clements (2014) explained, children can distinguish between quantities from a very early age. This immediate recognition of numbers, termed subitising (from the Latin for sudden), is described as pre attentive, meaning it does not require conscious activity. Sousa described perceptual subitising as when a number is assigned to a collection without deliberate counting, which for most young children applies to very small numbers (specifically 1, 2, 3, 4).

Sousa (2008) explained that conceptual subitising involves pattern recognition (such as the patterns on dice), as distinct from assigning numbers to objects one by one. Sousa argued that such conceptual pattern recognition is important for more abstract use of numbers. It is also arguable that conceptual subitising happens after other counting concepts and is less central for children prior to school than perceptual subitising.

It goes without saying that the various concepts associated with counting are also critical for learning to use numbers. The ENRP (2002) argued that experiences with counting objects assists with the development of number concepts. A key pre-requisite to learning to count is being able to say the sequence of numbers. While there are many descriptions of the next steps in counting collections, the list by Fuson (1990) is indicative of most lists and proposes that the key counting ideas to be learned, in order, are:

- one to one correspondence (count each object once and only once);
- conservation of number (how many objects there are does not depend on how the objects are laid out); and
- cardinality (the last number counted is the number of the collection).

Margolinas and Wozniak (2014) emphasise similar elements in describing pre-school learning of number in Chap. 10 of this volume.

In the *Let's Count* Study, 22 % of the children at the start of their preschool year could say the numbers names to 20, 79 % could say the numbers to 10, and 22 % could count a collection of around 20 teddies. In the "detour", over 60 % could recognise 2, 0, and 3 dots without counting but accuracy was much lower for higher numbers. Over 60 % could match the symbols for 2 and 3 to sets of dots, and more than 45 % could match 0, 4 and 5. Nearly 50 % could show 6 on their fingers, and 24 % could order numeral cards 1–9. While most children have progressed well on these skills and understandings, there is still about half of the group (aged 3 years 8 months to 4 years 8 months) who have less familiarity at the start of preschool.

In the ENRP interview, 35 % of the children at the start of school could not say the numbers names to 20, and 51 % could count a collection of around 20 teddies. In the "detour", over 80 % could recognise 2, 0, and 3 dots without counting but accuracy was much lower for higher numbers. Around 80 % could match the symbols for 2 and 3 to sets of dots, and more than 60 % could match 0, 4 and 5 to such sets. Nearly 80 % could show 6 on their fingers, and nearly half could order numeral cards 1–9. Eighty per cent could say the number after 4, and 50 % the number before 3. This suggests that specific experiences focused on saying the number names in sequence, working out the totals of collections and connecting these with symbols, and even partitioning numbers (that a 6 can be seen as a 5 and a 1) are not only possible with all pre school children but also desirable. While most students have progressed well on these skills and understandings, there is still close to a fifth of the age group who have developed less familiarity by the start of school. It is argued that purposeful experiences are likely to help those students.

The relevant content descriptions from the Foundation level (the first school year) of the AC:M are:

- Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point
- Connect number names, numerals and quantities, including zero, initially up to 10 and then beyond
- Subitise small collections of objects
- Compare, order and make correspondences between collections, initially to 20, and explain reasoning
- Represent practical situations to model addition and sharing

Given the Let's Count and ENRP data, it seems that these statements are somewhat basic, and the challenge for teachers in the first year of school is to ensure that all students can achieve these while creating opportunities for those students who need greater challenge than these statements infer. It is possible that the statement on subitising lacks specificity in that it presumably does not refer to perceptual subitising (which is innate human ability available to most) and is intended to move students to conceptual subitising.

The PYP, for children aged 3–5 years, has a covering statement that is:

> *Students will read, write, count, compare and order numbers to 20. They will model number relationships to 10, develop a sense of 1:1 correspondence and conservation of number. They will select and explain an appropriate method for solving a problem.*

Also in the PYP, under the stem "when constructing meaning, learners…" are the statements:

- Understand one-to-one correspondence
- Understand that, for a set of objects, the number name of the last object counted describes the quantity of the whole set
- Understand conservation of number
- Understand the relative magnitude of whole numbers
- Recognise groups of zero to five objects without counting (subitising)
- Subitise in real-life situations

The term "understand" is not ideal in that it is hard to know how this could be assessed. It also seems that the list is a more or less random collection of aspects that may assist with early learning. It is also an eclectic set of language. For example, the list uses "conservation" but not "cardinality". The idea of saying sequences is not mentioned.

The EYLF includes two statements:

- Children demonstrate an increasing understanding of … number using vocabulary to describe size … and names for numbers.
- Children use language to communicate thinking about quantities … and to explain mathematical ideas.

While the attention to language and vocabulary is helpful, otherwise this is a limited and unclear statement of experiences for pre school children and is unlikely to inform intentional activities. Clements (2014), in describing learning associated with number for these levels as general statements explained:

> Number can be used to tell us how many, describe order, and measure; they involve numerous relations and can be represented in various ways (p. 16).

> Another key element of object counting readiness is learning standard sequences of number words, learning that is facilitated by discovering patterns (p. 27).

More specifically, for this level, he defined the following aspects of number learning:

> Object counting involves creating a one-to-one correspondence between a number word in a verbal counting sequence and each item of a collection, using some action indicating each action as you say a number word (p. 28).
> Use counting or matching (one-to-one correspondence) to determine the equivalence or order (smaller or larger) of two collections, despite distracting appearances, and uses words equal, more, less, fewer (p. 30).

These statements are helpful in their clarity and specificity. Interestingly, and unusually in the Australian context, Clements (2014) also included:

> Representing collections and numerical relations with written symbols is a key step towards abstract mathematical thinking (p. 29).

We agree that recognising and even writing symbols can be part of the preschool experiences of children. Anders et al. (2013) described an assessment of pre-school children that sought to measure children's knowledge of counting, and recognising numbers. In the first level of their instrument, children are asked to count objects, and identify numerals up to 10. After that, their focus is on operations.

The following presents some statements that could be used to inform advice on experiences for parents, and EC educators, following the prompt "children learn maths … when they …":

> *Say number names forward in sequence to 10 (and then to 20 and beyond)*
> *Use numbers to describe collections*
> *Describe small quantities without counting, or by counting and matching to compare one collection or part with another*
> *Match number names, symbols and quantities up to 10 and beyond*
> *Show different ways to make or organise a total (with small numbers)*

One of the purposes of this chapter is to inform the creation or identification of illustrative experiences. Interestingly, Sousa (2008), in the middle of an outstanding discussion of insights from cognitive science, in describing experiences for pre school children, wrote:

> Children learn about numbers by counting objects and talking about the results. "You gave Billy five cards. How many more does Mary need?" Children count spaces on broad games "You are now on space three. How many more spaces do you need to get to space seven?" They count days until their birthdays. The teacher might say, "Yesterday there were nine days until your birthday. How many days are there now? Children read counting books and recited nursery rhymes with numbers (p. 79).

This excerpt highlights a lack of clarity in identifying aspects in the number domain and emphasises the importance of connecting the mathematical goals with suggested experiences. In the above case, the use of addition does not match with the other statements made by this author.Some examples of illustrative experiences that might be prompted in family contexts are:

- Saying and acting rhymes and reading stories that focus on number
- Playing games that focus on spatial patterns such as dominoes
- Having conversations about comparisons such as "who has more grapes?" Or how many more grapes do you need?

In more formal settings, educators can plan purposeful experiences such as:

- Construction or threading beads projects using number as a describing word (make a three-two pattern)
- Comparison tasks that can be estimated (for example, with one number 2 and the other much larger)

- Comparison tasks that require moving objects (arrange that blocks so that the groups are the same or some that one group has more, etc.)
- Finding a given amount (find me four of something)
- Specific matching tasks that require children to connect representations, including symbols

Shape

Two aspects that lead to the domain of the curriculum described a geometry are shape and location. The *Let's Count* data indicated that two thirds of children could recognise and match similar shapes at the start of pre-school. This was a greater percentage than that identified by the ENRP for school entrants.

The PYP suggests that students aged 3–5 years can "sort, describe and compare 3-D shapes" and "understand that 2D and 3D shapes have characteristics that can be described and compared". Similarly the AC includes the following for the first year of school:

> *Sort, describe and name familiar two-dimensional shapes and three-dimensional objects in the environment.*

We argue that the following statement can be used to inform advice on experiences for parents, and EC educators, following the prompt "children learn maths when they …":

> Play with, name, describe, and organise 2D shapes and 3D objects.

This statement implies the nature of suggested experiences, with the emphasis on handling shapes and objects while talking about what they are doing.

Location (Visualising)

Other experiences leading to geometry are those related to location, including visualising. More than 70 % of children participating in *Let's Count* at the beginning of pre-school demonstrated understanding of the positional words 'beside', 'behind' and 'in front of'. The ENRP found that around two thirds of students could identify shapes in the room that matched ("same shape") with a given rectangle.

In terms of describing experiences, the EYLF proposed:

> *Children demonstrate spatial awareness and orient themselves, moving around and through their environments, confidently and safely*

This statement again lacks the specificity that would inform purposeful experiences. The AC:M also could be more descriptive than it is stating only “Describe position and movement”. More helpfully, the PYP described the dual foci of this aspect as:

- explore the paths, regions and boundaries of their immediate environment and their position.
- understand that a common language can be used to describe position and direction, for example, inside, outside, above, below, next to, behind, in front of, up, down

On balance, it seems that the more important aspect for students prior to school is developing familiarity with the language and meaning of location.

The following statement can be used to inform advice on experiences for parents, and EC educators, following the prompt “children learn maths when they …”:

> *Use words and ideas to describe where things are, for example, inside, outside, above, below, next to, behind, in front of, up, down, here, there, north, west, middle, across, opposite*

This statement implies the nature of suggested experiences, with the emphasis on conversations about the placement of objects and people with respect to others.

Patterns

More or less all mathematics involves identifying and//or describing patterns in some way. In fact pattern recognition is central to all of the mathematics concepts discussed above. Nevertheless, there can be experiences created that focus the attention of children onto aspects of patterns and structure.

The AC:M includes the following statement on patterns:

- Sort and classify familiar objects and explain the basis for these classifications. Copy, continue and create patterns with objects and drawings.
- The PYP documents include the following statements for pre school children:
- Describes patterns found in everyday situations, for example, sounds, actions, objects, nature
- Describes patterns in various ways, for example, using words, drawings, symbols, materials, actions, numbers
- Copies, extends and creates repeating patterns
- The following statement that can be used to inform advice on experiences for parents, and EC educators, following the prompt “children learns math … when they …”:
- Describe, copy, represent and extend patterns found in everyday situations, including sounds, objects, actions and images.

Summary

The analysis presented in this chapter is a first step towards articulating some experiences that can inform parents and educators of some possible foci for mathematics learning of children prior to attending school. The analysis drew on data on achievement of young children on assessments of the mathematics knowledge and also on some similar statements in common use. The resulting statements are presented in Fig. 2.1.

Mathematics for young children

Children learn mathematics when they …

Compare objects and describe, in everyday language, which is longer, shorter, heavier, lighter, or holds more, hold less.

Play with, name, describe, and organise 2D shapes and 3D objects.

Use words and ideas to describe where things are positioned, for example, inside, outside, above, below, next to, behind, in front of, up, down, here, there, north, middle, across, opposite.

Describe, copy, represent and extend patterns found in everyday situations.

Use time words that describe points in time, events and routines (including days, months, seasons and celebrations).

Compare the duration of everyday events using mathematical language and arrange connected events in the usual sequence that they occur.

Say number names forward in sequence to 10 (and eventually to 20 and beyond).

Use numbers to describe and compare collections.

Use, progressively, perceptual and conceptual subitising, counting and matching to compare the number of items in one collection with another.

Show different ways to make a total (at first with models and small numbers).

Match number names, symbols and quantities up to 10.

Fig. 2.1 Resulting statements

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