

Chapter 2

Dynamical Model of Lorentz-Augmented Orbital Motion

Dynamics analysis and controller design of Lorentz-augmented space missions rely highly on the precise modeling of Lorentz-augmented orbital motion. Current models for Lorentz-augmented absolute or relative orbital motion, especially the relative ones, are mostly developed based on the assumption of a nontilted magnetic dipole. However, the Earth's magnetic dipole is tilted by about 11.3° with respect to the rotation axis of Earth, which is not small enough to be neglected. In view of this fact, this chapter develops dynamical models for Lorentz-augmented absolute and relative orbital motion that explicitly include the dipole tilt angle in both two-body and J_2 -perturbed environment.

2.1 Model of Absolute Orbital Motion

2.1.1 Geomagnetic Field

The potential function of the geomagnetic field can be expressed as [1]

$$V = R_E \sum_{n=1}^{\infty} \sum_{m=0}^n \left\{ P_n^m(\cos \theta) [g_n^m(t) \cos(m\lambda) + h_n^m(t) \sin(m\lambda)] (R_E/r)^{n+1} \right\} \quad (2.1)$$

where $R_E = 6371.2$ km is the magnetic reference spherical radius of Earth, and r is the orbital radius. θ and λ refer to the geocentric colatitude and east longitude, respectively. $g_n^m(t)$ and $h_n^m(t)$ are time-dependent Gauss coefficients, which can be derived by referring to the International Geomagnetic Reference Field (IGRF) [1].

$P_n^m(\cos \theta)$ refers to the Schmidt quasi-normalized Legendre function of degree n and order m , given by [2]

$$P_n^m(\cos \theta) = \frac{1}{2^n \cdot n!} \left[\frac{\varepsilon_m (n-m)! (1 - \cos^2 \theta)^m}{(n+m)!} \right]^{1/2} \frac{d^{n+m}}{d(\cos \theta)^{n+m}} (\cos^2 \theta - 1)^n \quad (2.2)$$

where

$$\varepsilon_m = \begin{cases} 1, & m = 0 \\ 2, & m \geq 1 \end{cases}, n \geq m \geq 0 \quad (2.3)$$

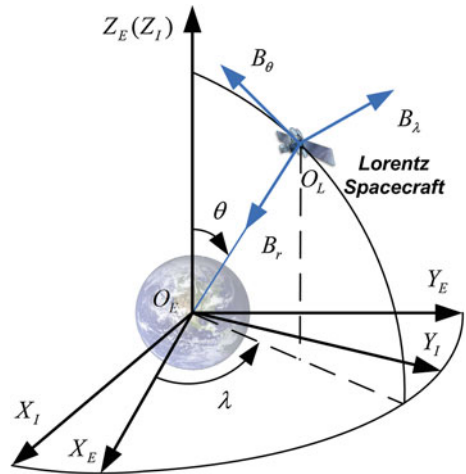
The magnetic field $\mathbf{B}$ can then be derived as the negative gradient of the potential, given by

$$\mathbf{B} = -\nabla V \quad (2.4)$$

As depicted in Fig. 2.1, $O_E X_I Y_I Z_I$ is an Earth-centered inertial (ECI) frame, and $O_E X_E Y_E Z_E$ is another Earth-fixed frame that is corotating with Earth. O_E is the center of Earth. O_L refers to the center of mass (c.m.) of the Lorentz spacecraft, where the local geocentric longitude and colatitude are λ and θ , respectively. Define B_θ , B_λ , and B_r as the components of the local magnetic field $\mathbf{B}$ in the northward, eastward, and radially inward directions, respectively. By using Eq. (2.4), it can be derived that [2]

$$\begin{aligned} B_\theta &= -\frac{1}{r} \frac{\partial V}{\partial \theta} \\ &= \sum_{n=1}^N (R_E/r)^{n+2} \sum_{m=0}^n [g_n^m \cos(m\lambda) + h_n^m \sin(m\lambda)] \frac{\partial P_n^m(\cos \theta)}{\partial \theta} \end{aligned} \quad (2.5)$$

Fig. 2.1 Illustration of the decomposition of the geomagnetic field



$$\begin{aligned}
B_\lambda &= -\frac{1}{r \sin \theta} \frac{\partial V}{\partial \lambda_L} \\
&= \sum_{n=1}^N (R_E/r)^{n+2} \sum_{m=0}^n m [g_n^m \sin(m\lambda) - h_n^m \cos(m\lambda)] \frac{P_n^m(\cos \theta)}{\sin \theta}
\end{aligned} \tag{2.6}$$

$$\begin{aligned}
B_r &= \frac{\partial V}{\partial r} \\
&= -\sum_{n=1}^N (R_E/r)^{n+2} (n+1) \sum_{m=0}^n [g_n^m \cos(m\lambda) + h_n^m \sin(m\lambda)] P_n^m(\cos \theta)
\end{aligned} \tag{2.7}$$

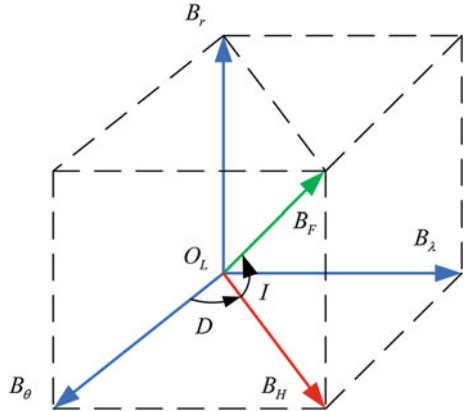
As shown Fig. 2.2, B_F is the total intensity, B_H is the horizontal intensity, D is the declination angle, and I is the inclination angle. Based on the definitions in Fig. 2.2, these variables can be calculated via [1]

$$\begin{aligned}
B_F &= \sqrt{B_\theta^2 + B_\lambda^2 + B_r^2}, \quad B_H = \sqrt{B_\theta^2 + B_\lambda^2} \\
D &= \arctan(B_\lambda/B_\theta), \quad I = \arctan(B_r/B_H)
\end{aligned} \tag{2.8}$$

According to the 11th generation of IGRF (IGRF-11), the magnetic field information at the orbital altitude of 566.9 km in 2010 is shown from Figs. 2.3, 2.4 and 2.5.

If the series in Eq. (2.1) is truncated at $n = 1$ that only g_1^0 , g_1^1 , and h_1^1 remain, then the geomagnetic field reduces to a tilted dipole model. Each of the three Gauss coefficients corresponds to a dipole located at the center of Earth. Notably, these three dipoles are perpendicular with respect to each other. Thus, the composed dipole is still located at the center of Earth, but is tilted by an angle with respect to the rotation axis of Earth.

Fig. 2.2 Definitions of the related variables



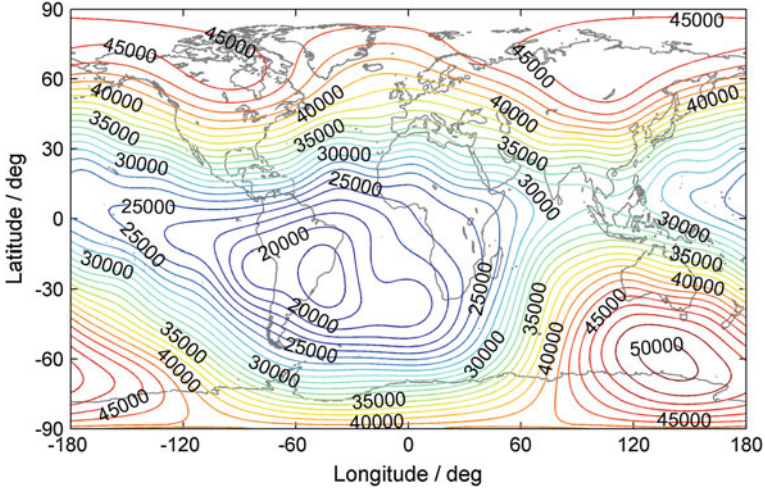


Fig. 2.3 Equivalence graph of the total intensity (unit: nT)

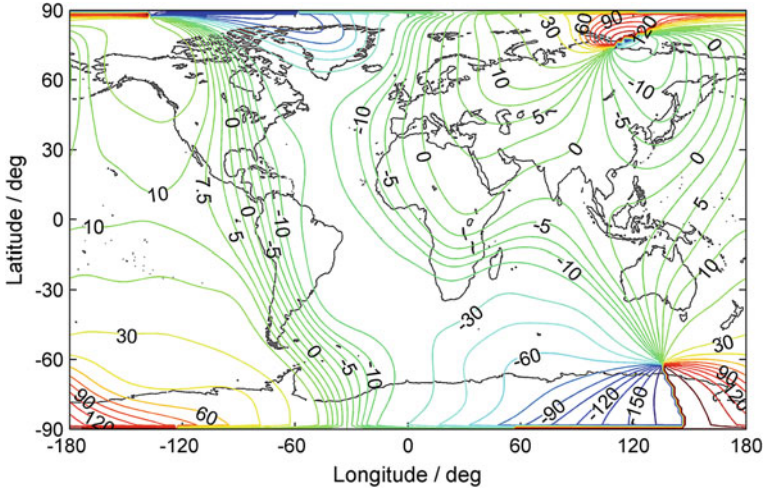


Fig. 2.4 Equivalence graph of the declination angle (unit: deg)

By modeling the geomagnetic field as a tilted dipole corotating with Earth, the magnetic vector potential can be represented as [3]

$$A = \frac{B_0}{r^2} (\hat{n} \times \hat{r}) \quad (2.9)$$

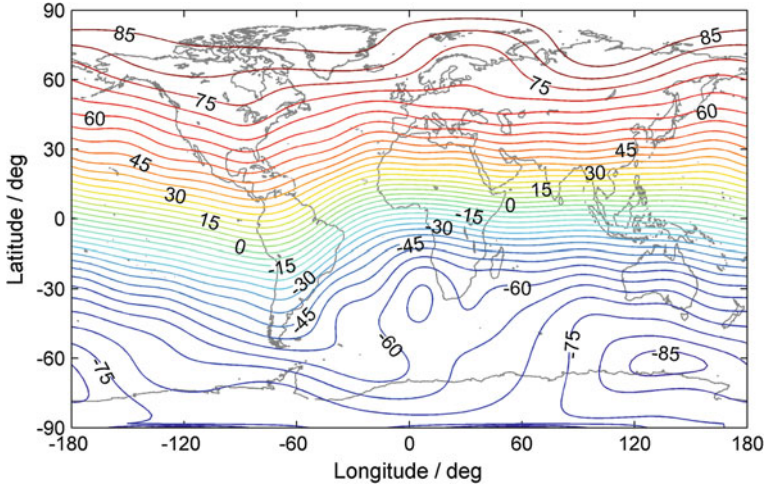


Fig. 2.5 Equivalence graph of the inclination angle (unit: deg)

where $B_0 = 8 \times 10^{15} \text{ T m}^3$ is the magnetic dipole moment of Earth, and r is the orbital radius. The superscript $\hat{\cdot}$ refers to a unit vector in that direction. For example, $\hat{n}$ is the unit vector in the direction of the magnetic dipole moment and $\hat{r}$ is the unit orbital radius vector.

The magnetic field located at r can be then calculated via [3]

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{B_0}{r^3} [3(\hat{n} \cdot \hat{r})\hat{r} - \hat{n}] \quad (2.10)$$

2.1.2 Two-Body Model

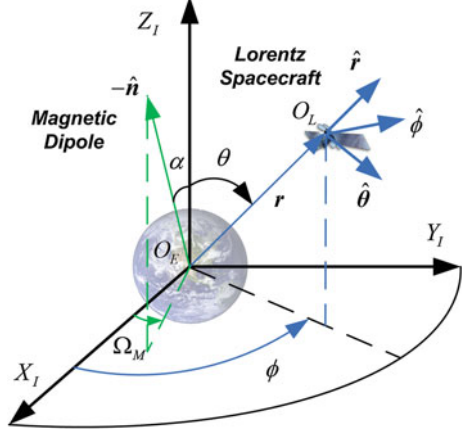
As shown in Fig. 2.6, $O_E X_I Y_I Z_I$ is the ECI frame, and the angles α and Ω_M define the orientation of the magnetic dipole in the ECI frame, where α is the dipole tilt angle with respect to the rotation axis of Earth, Ω_M is the inertial rotation angle of the magnetic dipole, given by

$$\Omega_M = \omega_E t + \Omega_0 \quad (2.11)$$

where ω_E is the rotation rate of Earth, t denotes the time, and Ω_0 is the initial phase angle of the magnetic dipole.

Based on the above definitions, the unit vector $\hat{n}$ can be expressed in the ECI frame as

Fig. 2.6 Definitions of the spherical coordinate frame and the dipole orientation. Reprinted from Ref. [11], Copyright 2015, with permission from SAGE



$$\hat{n} = -\cos \Omega_M \sin \alpha \hat{X}_I - \sin \Omega_M \sin \alpha \hat{Y}_I - \cos \alpha \hat{Z}_I \quad (2.12)$$

where $\hat{X}_I$, $\hat{Y}_I$, and $\hat{Z}_I$ refer to the unit vector along the three axes of ECI frame, respectively.

In Fig. 2.6, O_L is the c.m. of the Lorentz spacecraft, the spherical inertial coordinates r , ϕ , and θ give the position of O_L in ECI frame. Thus, the orbital radius vector of the Lorentz spacecraft is given by

$$\mathbf{r} = r \cos \phi \sin \theta \hat{X}_I + r \sin \phi \sin \theta \hat{Y}_I + r \cos \theta \hat{Z}_I \quad (2.13)$$

Also, the unit orbital radius vector $\hat{\mathbf{r}}$ is

$$\hat{\mathbf{r}} = \cos \phi \sin \theta \hat{X}_I + \sin \phi \sin \theta \hat{Y}_I + \cos \theta \hat{Z}_I \quad (2.14)$$

Taking the time derivative of Eq. (2.13) yields the velocity of Lorentz spacecraft, given by

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \begin{bmatrix} \dot{r} \cos \phi \sin \theta - r \dot{\phi} \sin \phi \sin \theta + r \dot{\theta} \cos \phi \cos \theta \\ \dot{r} \sin \phi \sin \theta + r \dot{\phi} \cos \phi \sin \theta + r \dot{\theta} \sin \phi \cos \theta \\ \dot{r} \cos \theta - r \dot{\theta} \sin \theta \end{bmatrix} \quad (2.15)$$

Then, the kinetic energy per unit mass of Lorentz spacecraft is derived as

$$T = \frac{1}{2} \mathbf{v}^T \mathbf{v} = \frac{1}{2} (\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \dot{\phi}^2 \sin^2 \theta) \quad (2.16)$$

The velocity-dependent magnetic potential of unit mass of Lorentz spacecraft is [4]

$$U_L = -\lambda_L \mathbf{V}_{\text{rel}}^T \mathbf{A} \quad (2.17)$$

where $\lambda_L = q_L/m_L$ is the specific charge of Lorentz spacecraft (i.e., charge-to-mass ratio), with q_L and m_L being the charge and mass of Lorentz spacecraft, respectively. $\mathbf{V}_{\text{rel}}$ is the velocity of Lorentz spacecraft with respect to the local magnetic field, given by

$$\mathbf{V}_{\text{rel}} = \mathbf{v} - \boldsymbol{\omega}_E \times \mathbf{r} \quad (2.18)$$

where $\boldsymbol{\omega}_E = [0 \ 0 \ \omega_E]^T$ is the angular velocity vector of Earth in ECI frame.

In a two-body model, the gravitational potential per unit mass of Lorentz spacecraft is

$$U_g = -\frac{\mu}{r} \quad (2.19)$$

where μ is the gravitational parameter of Earth.

Therefore, the Lagrange function per unit mass of Lorentz spacecraft can be summarized as

$$L = T - U_g - U_L \quad (2.20)$$

Substitution of Eqs. (2.16), (2.17), and (2.19) into Eq. (2.20) yields that [3]

$$\begin{aligned} L = & \frac{1}{2}(\dot{r}^2 + \dot{\theta}^2 r^2 + \dot{\phi}^2 r^2 \sin^2 \theta) + \frac{\mu}{r} + \frac{q_L B_0}{m_L r} \{ \dot{\theta} \sin(\phi - \Omega_M) \sin \alpha \\ & + (\dot{\phi} - \omega_E) \sin \theta [\cos \theta \cos(\phi - \Omega_M) \sin \alpha - \sin \theta \cos \alpha] \} \end{aligned} \quad (2.21)$$

By solving the Lagrangian equation

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\boldsymbol{\chi}}} \right) - \frac{\partial L}{\partial \boldsymbol{\chi}} = 0 \quad (2.22)$$

where $\boldsymbol{\chi} = [r \ \phi \ \theta]^T$ denotes the generalized coordinates, the dynamic equations of the two-body absolute orbital motion of Lorentz spacecraft can be obtained as [3]

$$\begin{aligned} \ddot{r} = & -\frac{q_L B_0}{m_L r^2} \{ \dot{\theta} \sin(\phi - \Omega_M) \sin \alpha + (\dot{\phi} - \omega_E) \sin \theta [-\sin \theta \cos \alpha \\ & + \cos \theta \cos(\phi - \Omega_M) \sin \alpha] \} + r \dot{\theta}^2 + r \dot{\phi}^2 \sin^2 \theta - \frac{\mu}{r^2} \end{aligned} \quad (2.23)$$

$$\begin{aligned} \ddot{\phi} = & -\frac{q_L B_0}{m_L r^4} [\cos \alpha (\dot{r} - 2r \dot{\theta} \cot \theta) - \cos(\phi - \Omega_M) \sin \alpha (\dot{r} \cot \theta + 2r \dot{\theta})] \\ & - 2\dot{\phi} \frac{\dot{r}}{r} - 2\dot{\phi} \dot{\theta} \cot \theta \end{aligned} \quad (2.24)$$

$$\ddot{\theta} = \frac{q_L B_0}{m_L r^3} \{ (\dot{\phi} - \omega_E) [(\cos 2\theta - 1) \cos(\phi - \Omega_M) \sin \alpha - \sin 2\theta \cos \alpha] + [\dot{r} \sin(\phi - \Omega_M)]/r \} - 2\dot{\theta} \frac{\dot{r}}{r} + \dot{\phi}^2 \sin \theta \cos \theta \quad (2.25)$$

Notably, Eq. (2.24) rectifies the typos in Eq. (2.15) of Ref. [3].

2.1.3 J_2 -Perturbed Model

A Lorentz spacecraft is more effective and efficient in low Earth orbits (LEOs) where the magnetic field is intenser and the spacecraft travels faster than high Earth orbits. Moreover, J_2 perturbation, arising from the oblateness and nonhomogeneity of Earth, is one of the most dominant disturbances in LEOs. Therefore, to capture the environment in LEOs more precisely, a dynamical model that characterizes the J_2 -perturbed orbital motion of Lorentz spacecraft is developed as follows.

In consideration of the J_2 perturbation, the gravitational potential per unit mass of Lorentz spacecraft is revised as [5]

$$U_{g,J_2} = -\frac{\mu}{r} \left[1 - J_2 (R_E/r)^2 (3 \cos^2 \theta - 1)/2 \right] \quad (2.26)$$

where is $J_2 = 1.0826 \times 10^{-3}$ the second zonal harmonic coefficient of Earth.

Correspondingly, the Lagrange function is revised as

$$L = T - U_{g,J_2} - U_L \quad (2.27)$$

where the kinetic energy and magnetic potential per unit mass are the same as those in Eqs. (2.16) and (2.17).

Similarly, by solving the Lagrangian equation, the dynamic equations of J_2 -perturbed orbital motion of Lorentz spacecraft can be derived as

$$\ddot{r} = -\frac{q_L B_0}{m_L r^2} \left\{ \dot{\theta} \sin(\phi - \Omega_M) \sin \alpha + (\dot{\phi} - \omega_E) \sin \theta [\cos \theta \cos(\phi - \Omega_M) \sin \alpha - \sin \theta \cos \alpha] \right\} + r\dot{\theta}^2 + r\dot{\phi}^2 \sin^2 \theta - \frac{\mu}{r^2} + \frac{3}{2} \mu J_2 \frac{R_E^2}{r^4} (3 \cos^2 \theta - 1) \quad (2.28)$$

$$\ddot{\phi} = -\frac{q_L B_0}{m_L r^4} \left[\cos \alpha (\dot{r} - 2r\dot{\theta} \cot \theta) - \cos(\phi - \Omega_M) \sin \alpha (\dot{r} \cot \theta + 2r\dot{\theta}) \right] - 2\dot{\phi} \frac{\dot{r}}{r} - 2\dot{\phi} \dot{\theta} \cot \theta \quad (2.29)$$

$$\ddot{\theta} = \frac{q_L B_0}{m_L r^3} \left\{ (\dot{\phi} - \omega_E) [(\cos 2\theta - 1) \cos(\phi - \Omega_M) \sin \alpha - \sin 2\theta \cos \alpha] \right. \\ \left. + [\dot{r} \sin(\phi - \Omega_M) \sin \alpha] / r \right\} - 2\dot{\theta} \frac{\dot{r}}{r} + \dot{\phi}^2 \sin \theta \cos \theta + \frac{3}{2} \mu J_2 \frac{R_E^2}{r^5} \sin 2\theta \quad (2.30)$$

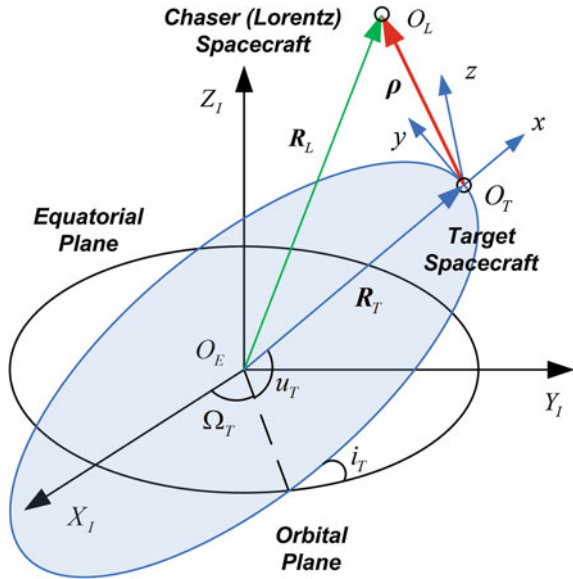
2.2 Model of Relative Orbital Motion

2.2.1 Two-Body Model

2.2.1.1 Equations of Relative Motion

Consider a chaser spacecraft and another target spacecraft flying in Earth orbits. The chaser is assumed to be a charged Lorentz spacecraft, but the target is uncharged. As shown in Fig. 2.7, $O_E X_I Y_I Z_I$ is the ECI frame. $O_T xyz$ is a local-vertical-local-horizontal (LVLH) frame located at the c.m. of the target, O_T , where x axis is along the radial direction, z axis is normal to the target's orbital plane, and y axis completes the right-handed Cartesian frame. $\mathbf{R}_L$ and $\mathbf{R}_T$ refer to the orbital radius vector of the chaser and the target, respectively. The relative position vector between the chaser and the target is thus $\boldsymbol{\rho} = \mathbf{R}_L - \mathbf{R}_T = [x \ y \ z]^T$. i_T ,

Fig. 2.7 Definitions of the coordinate frames. Reprinted from Ref. [11], Copyright 2015, with permission from SAGE



Ω_T , and u_T are, respectively, the inclination, right ascension of the ascending node, and argument of latitude of the target.

The equations of orbital motion of the Lorentz and the target spacecraft are, respectively, given by

$$\frac{d^2 \mathbf{R}_L}{dt^2} = -\frac{\mu}{R_L^3} \mathbf{R}_L + \mathbf{a}_L + \mathbf{a}_C \quad (2.31)$$

$$\frac{d^2 \mathbf{R}_T}{dt^2} = -\frac{\mu}{R_T^3} \mathbf{R}_T \quad (2.32)$$

where $\mathbf{a}_L = [a_x \ a_y \ a_z]^T$ and $\mathbf{a}_C = [a_R \ a_S \ a_W]^T$ refer to the Lorentz acceleration and control acceleration acting on the Lorentz spacecraft, respectively.

Given that $\boldsymbol{\rho} = \mathbf{R}_L - \mathbf{R}_T$, taking the second order time derivative of this equation yields that

$$\frac{d^2 \boldsymbol{\rho}}{dt^2} = \ddot{\boldsymbol{\rho}} + \dot{\mathbf{u}}_T \times (\dot{\mathbf{u}}_T \times \boldsymbol{\rho}) + 2\dot{\mathbf{u}}_T \times \dot{\boldsymbol{\rho}} + \ddot{\mathbf{u}}_T \times \boldsymbol{\rho} = \frac{d^2 \mathbf{R}_L}{dt^2} - \frac{d^2 \mathbf{R}_T}{dt^2} \quad (2.33)$$

where $\dot{\mathbf{u}}_T = [0 \ 0 \ \dot{u}_T]^T$ and $\ddot{\mathbf{u}}_T = [0 \ 0 \ \ddot{u}_T]^T$ refer to orbital angular velocity and acceleration of the target, respectively.

Substitution of Eqs. (2.31) and (2.32) into Eq. (2.33) yields the nonlinear equations of relative orbital motion of Lorentz spacecraft expressed in the LVLH frame, given by

$$\begin{aligned} \ddot{x} &= 2\dot{u}_T \dot{y} + \dot{u}_T^2 x + \ddot{u}_T y + n_T^2 R_T - n_L^2 (R_T + x) + a_x + a_R \\ \ddot{y} &= -2\dot{u}_T \dot{x} + \dot{u}_T^2 y - \ddot{u}_T x - n_L^2 y + a_y + a_S \\ \ddot{z} &= -n_L^2 z + a_z + a_W \end{aligned} \quad (2.34)$$

where $n_T = \sqrt{\mu/R_T^3}$ and $n_L = \sqrt{\mu/R_L^3}$. $R_L = [(R_T + x)^2 + y^2 + z^2]^{1/2}$ is the orbital radius of Lorentz spacecraft.

Provided that the relative distance between spacecraft is negligibly small compared to the orbital radius of either spacecraft (i.e., $\|\boldsymbol{\rho}\| \ll R_L, R_T$), the above nonlinear equations can be further linearized. Based on this precondition, it holds that

$$\left(\frac{R_T}{R_L}\right)^3 = \left(\frac{R_T^2}{R_L^2}\right)^{3/2} = \left(1 + \frac{2x}{R_T} + \frac{\|\boldsymbol{\rho}\|^2}{R_T^2}\right)^{-3/2} \approx 1 - \frac{3x}{R_T} \quad (2.35)$$

By substituting Eq. (2.35) into Eq. (2.34) and neglecting the second order small item, the linearized model can be derived as

$$\begin{aligned}
\ddot{x} &= 2\dot{u}_T\dot{y} + \dot{u}_T^2x + \ddot{u}_Ty + 2n_T^2x + a_x + a_R \\
\ddot{y} &= -2\dot{u}_T\dot{x} + \dot{u}_T^2y - \ddot{u}_Tx - n_T^2y + a_y + a_S \\
\ddot{z} &= -n_T^2z + a_z + a_W
\end{aligned} \tag{2.36}$$

Without the Lorentz acceleration, Eq. (2.36) is the well-known Tschanuer–Hempel (TH) equation [6], a dynamical model that describes the linearized relative orbital motion about an elliptic reference orbit.

Moreover, if the target is flying in a circular orbit, that is, $\dot{u}_T = \mu/R_T^3 = n_T$ and $\ddot{u}_T = 0$, then the linearized model further reduces to

$$\begin{aligned}
\ddot{x} &= 2n_T\dot{y} + 3n_T^2x + a_x + a_R \\
\ddot{y} &= -2n_T\dot{x} + a_y + a_S \\
\ddot{z} &= -n_T^2z + a_z + a_W
\end{aligned} \tag{2.37}$$

Also, without the Lorentz acceleration, Eq. (2.37) is the well-known Hill–Clohessy–Wiltshire (HCW) equation [7] that describes the linearized relative orbital motion about a circular reference orbit.

2.2.1.2 The Lorentz Force

A particle with a net charge of q_L is subject to the Lorentz force $\mathbf{F}_L$ when moving in a magnetic field $\mathbf{B}$ with a relative velocity of $\mathbf{V}_{\text{rel}}$, given by [8]

$$\mathbf{F}_L = q_L \mathbf{V}_{\text{rel}} \times \mathbf{B} \tag{2.38}$$

If the mass of the charged particle is m_L , the resulting Lorentz acceleration will be

$$\mathbf{a}_L = (q_L/m_L) \mathbf{V}_{\text{rel}} \times \mathbf{B} \tag{2.39}$$

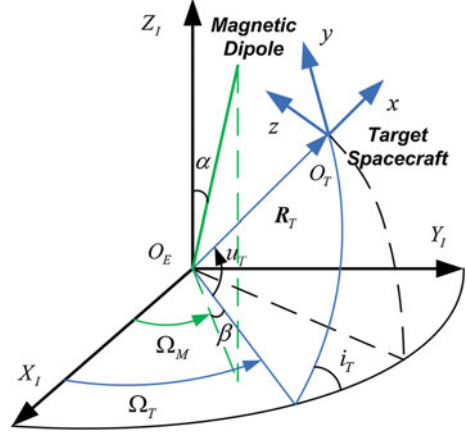
To derive the expressions of the Lorentz acceleration in the LVLH frame, following assumptions are proposed: (1) the Earth magnetic field is modeled as a perfect dipole located at the Earth’s center that is corotating with Earth; (2) the dipole is tilted by angle α with respect to the rotation axis of Earth, as shown in Fig. 2.8; (3) the Lorentz spacecraft could be regarded as a charged point mass [3].

As shown in Fig. 2.8, the transformation matrix from the ECI frame to the LVLH frame can be represented as

$$\mathbf{M}_{LI} = \mathbf{M}_3[u_T] \mathbf{M}_1[i_T] \mathbf{M}_3[\Omega_T] \tag{2.40}$$

where $\mathbf{M}_3[\cdot]$ and $\mathbf{M}_1[\cdot]$ are the basic transformation matrix, given by

Fig. 2.8 Definitions of related angles. Reprinted from Ref. [10], Copyright 2014, with permission from SAGE



$$\mathbf{M}_3[\cdot] = \begin{bmatrix} \cos(\cdot) & \sin(\cdot) & 0 \\ -\sin(\cdot) & \cos(\cdot) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{M}_1[\cdot] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\cdot) & \sin(\cdot) \\ 0 & -\sin(\cdot) & \cos(\cdot) \end{bmatrix} \quad (2.41)$$

Equation (2.12) has given the expression of $\hat{\mathbf{n}}$ in the ECI frame. Then, its expression in the LVLH frame can be obtained as $\hat{\mathbf{n}}^L = \mathbf{M}_L \hat{\mathbf{n}}^I$, where the superscript L or I denotes the LVLH or ECI frame. By using Eqs. (2.12) and (2.40), the unit magnetic dipole vector can be expressed in the LVLH frame as

$$\begin{aligned} \hat{\mathbf{n}} &= [n_x \quad n_y \quad n_z]^T \\ &= \begin{bmatrix} -(\cos \beta \cos u_T + \sin \beta \cos i_T \sin u_T) \sin \alpha - \sin i_T \sin u_T \cos \alpha \\ (\cos \beta \sin u_T - \sin \beta \cos i_T \cos u_T) \sin \alpha - \sin i_T \cos u_T \cos \alpha \\ \sin \beta \sin i_T \sin \alpha - \cos i_T \cos \alpha \end{bmatrix} \end{aligned} \quad (2.42)$$

where the angle β is defined as $\beta = \Omega_M - \Omega_T$, as shown in Fig. 2.7.

Furthermore, the unit orbital radius vector of the Lorentz spacecraft can be expressed in the LVLH frame as

$$\hat{\mathbf{R}}_L = (1/R_L)[R_T + x \quad y \quad z]^T \quad (2.43)$$

Then, by substituting Eqs. (2.42) and (2.43) into Eq. (2.10), the magnetic field local at the Lorentz spacecraft is expressed in the LVLH frame as [9]

$$\mathbf{B} = [B_x \quad B_y \quad B_z]^T = (B_0/R_L^3)[3(\hat{\mathbf{n}} \cdot \hat{\mathbf{R}}_L)\hat{\mathbf{R}}_L - \hat{\mathbf{n}}] \quad (2.44)$$

Given that the geomagnetic field is corotating with Earth, the velocity of Lorentz spacecraft with respect to the local magnetic field is thus given by

$$\mathbf{V}_{\text{rel}} = \frac{d\mathbf{R}_L}{dt} - \boldsymbol{\omega}_E \times \mathbf{R}_L = \dot{\mathbf{R}}_T + \dot{\boldsymbol{\rho}} + (\dot{\mathbf{u}}_T - \boldsymbol{\omega}_E) \times (\mathbf{R}_T + \boldsymbol{\rho}) \quad (2.45)$$

or in the LVLH coordinates

$$\mathbf{V}_{\text{rel}} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} \dot{R}_T + \dot{x} - y(\dot{u}_T - \omega_E \cos i_T) - z\omega_E \sin i_T \cos u_T \\ \dot{y} + (R_T + x)(\dot{u}_T - \omega_E \cos i_T) + z\omega_E \sin i_T \sin u_T \\ \dot{z} + (R_T + x)\omega_E \sin i_T \cos u_T - y\omega_E \sin i_T \sin u_T \end{bmatrix} \quad (2.46)$$

Now, the expressions of the Lorentz acceleration in LVLH frame can be derived via substitution of Eqs. (2.44) and (2.46) into Eq. (2.39). Furthermore, the dynamical model of two-body Lorentz spacecraft relative motion can then be derived by substituting Eq. (2.39) into Eq. (2.34).

2.2.2 Analytical Solutions to Two-Body Model

2.2.2.1 Approximate Analytical Solutions

In this section, approximate analytical solutions are derived for a special case where the target is flying in an inclined circular Earth orbit. Since the reference orbit is circular, the linearized equations of Lorentz spacecraft relative motion have been given in Eq. (2.37).

To linearize the Lorentz acceleration and thereafter derive approximate analytical solutions to Eq. (2.37), following assumptions are proposed: (1) the relative coordinates between spacecraft are negligibly small as compared to the orbital radius of the target, that is, $|x|, |y|, |z| \ll R_T$; (2) the relative velocity between spacecraft are negligibly small as compared to the orbital translational velocity of the target, that is, $|\dot{x}|, |\dot{y}|, |\dot{z}| \ll \dot{u}_T R_T$ [9]; (3) except for the Lorentz acceleration, the control acceleration and other perturbations are neglected, that is, $\mathbf{a}_C = 0$. Based on these assumptions, the Lorentz acceleration can be linearized in the LVLH frame as [10]

$$a_x \approx (q_L/m_L) [R_T(n_T - \omega_E \cos i_T)B_z - (R_T\omega_E \sin i_T \cos u_T + \dot{z})B_y] \quad (2.47)$$

$$a_y \approx (q_L/m_L)(R_T\omega_E \sin i_T \cos u_T + \dot{z})B_x \quad (2.48)$$

$$a_z \approx c_1 \cos(u_T + \beta) + c_2 \cos(u_T - \beta) + c_3 \sin u_T \quad (2.49)$$

where the constant parameters c_1 , c_2 , and c_3 are given by

$$\begin{aligned}
c_1 &= q_L B_0 (n_T - \omega_E \cos i_T) \sin \alpha (1 - \cos i_T) / (m_L R_T^2) \\
c_2 &= q_L B_0 (n_T - \omega_E \cos i_T) \sin \alpha (1 + \cos i_T) / (m_L R_T^2) \\
c_3 &= 2q_L B_0 (n_T - \omega_E \cos i_T) \cos \alpha \sin i_T / (m_L R_T^2)
\end{aligned} \tag{2.50}$$

As can be seen, the out-of-plane relative orbital motion is decoupled of the in-plane one. Thus, the equation of out-of-plane relative motion is solved at first. Denote z_0 and $\dot{z}_0$ as the initial relative normal position and velocity, respectively, and u_{T0} as the initial argument of latitude of the target, then the solution can be expressed as [10]

$$\begin{bmatrix} z \\ \dot{z} \end{bmatrix} = \Phi^{\text{out}} \begin{bmatrix} z_0 \\ \dot{z}_0 \end{bmatrix} + \begin{bmatrix} \lambda_z(t) \\ \dot{\lambda}_z(t) \end{bmatrix} \tag{2.51}$$

with

$$\Phi^{\text{out}} = \begin{bmatrix} \cos n_T t & (\sin n_T t)/n_T \\ -n_T \sin n_T t & \cos n_T t \end{bmatrix} \tag{2.52}$$

$$\begin{aligned}
\lambda_z(t) &= k_{z1} \cos n_T t + \frac{1}{n_T} k_{z2} \sin n_T t - \frac{c_1 \cos[(n_T + \omega_E)t + \varphi_1]}{\omega_E(\omega_E + 2n_T)} \\
&\quad - \frac{c_2 \cos[(n_T - \omega_E)t + \varphi_2]}{\omega_E(\omega_E - 2n_T)} - \frac{c_3 t}{2n_T} \cos(n_T t + \varphi_3) + \frac{c_3}{4n_T^2} \sin(n_T t + \varphi_3)
\end{aligned} \tag{2.53}$$

where the coefficients k_{z1} , k_{z2} , φ_1 , φ_2 and φ_3 are, respectively, given by

$$\begin{aligned}
k_{z1} &= \frac{c_1 \cos \varphi_1}{\omega_E(\omega_E + 2n_T)} + \frac{c_2 \cos \varphi_2}{\omega_E(\omega_E - 2n_T)} - \frac{c_3}{4n_T^2} \sin \varphi_3 \\
k_{z2} &= -\frac{c_1(n_T + \omega_E)}{\omega_E(\omega_E + 2n_T)} \sin \varphi_1 - \frac{c_2(n_T - \omega_E)}{\omega_E(\omega_E - 2n_T)} \sin \varphi_2 + \frac{c_3}{4n_T} \cos \varphi_3
\end{aligned} \tag{2.54}$$

$$\begin{aligned}
\varphi_1 &= u_{T0} + \Omega_0 - \Omega_T \\
\varphi_2 &= u_{T0} - \Omega_0 + \Omega_T \\
\varphi_3 &= u_{T0}
\end{aligned} \tag{2.55}$$

Substitution of the expression of $\dot{z}(t)$ into Eqs. (2.47) and (2.48) yields that [10]

$$a_x = \kappa_{t,0}^x + \sum_{m=1}^2 t^{m-1} \left[\sum_{i=1}^9 C_{m,i}^x \sin(A_{m,i}^x t + \theta_{m,i}^x) + D_{m,i}^x \cos(B_{m,i}^x t + \phi_{m,i}^x) \right] \tag{2.56}$$

$$a_y = \kappa_{t,0}^y + \kappa_{t,1}^y t + \sum_{m=1}^2 t^{m-1} \left[\sum_{i=1}^9 C_{m,i}^y \sin(A_{m,i}^y t + \theta_{m,i}^y) + D_{m,i}^y \cos(B_{m,i}^y t + \phi_{m,i}^y) \right] \quad (2.57)$$

where the related coefficients are summarized in the Appendix.

By substituting Eqs. (2.56) and (2.57) into Eq. (2.37), the approximate analytical expressions of the in-plane relative motion are derived as [10]

$$\begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} = \Phi^{in} \begin{bmatrix} x_0 \\ y_0 \\ \dot{x}_0 \\ \dot{y}_0 \end{bmatrix} + \begin{bmatrix} \lambda_x(t) \\ \lambda_y(t) \\ \dot{\lambda}_x(t) \\ \dot{\lambda}_y(t) \end{bmatrix} \quad (2.58)$$

with

$$\Phi^{in} = \begin{bmatrix} 4 - 3 \cos n_T t & 0 & (\sin n_T t)/n_T & 2(1 - \cos n_T t)n_T \\ 6(\sin n_T t - n_T t) & 1 & 2(\cos n_T t - 1)/n_T & -3t + 4(\sin n_T t)/n_T \\ 3n_T \sin n_T t & 0 & \cos n_T t & 2 \sin n_T t \\ 6n_T(\cos n_T t - 1) & 0 & -2 \sin n_T t & 4 \cos n_T t - 3 \end{bmatrix} \quad (2.59)$$

where x_0 and y_0 are initial relative radial and in-track position. $\dot{x}_0$ and $\dot{y}_0$ are initial relative radial and in-track velocity. The expressions of $\lambda_x(t)$, $\lambda_y(t)$, and related coefficients are summarized in the Appendix.

Heretofore, the approximate analytical expressions of the relative orbital motion of a Lorentz spacecraft about an inclined circular orbit have been derived as Eqs. (2.51) and (2.58). Without numerical integration, the relative states at an arbitrary time t can be determined analytically.

2.2.2.2 Numerical Simulations

In this section, a typical scenario in LEO is simulated to assess the accuracy of the analytical solutions. The initial orbit elements of the target spacecraft are listed in Table 2.1. The initial phase angle of the magnetic dipole is 40° .

In this scenario, the Lorentz spacecraft with a constant specific charge of 10^{-4} C/kg starts from the origin of the LVLH frame. The total simulation time is 15 orbital periods, that is, about 24 h. To evaluate the accuracy of the derived analytical solutions, the exact numerical solutions are introduced to make comparisons. Notably, the numerical solutions are derived via integration of the nonlinear equations of relative motion in Eq. (2.34). Also, to evaluate the effect of the dipole tilt angle on model accuracy, the analytical solutions for inclined orbit with non-tilted dipole proposed in [9] are also introduced to make comparisons. Define Δx , Δy , and Δz as the radial, in-track, and normal relative position error between the

Table 2.1 Initial orbit elements of the target (Reprinted from Ref. [10], Copyright 2014, with permission from SAGE)

Orbit element	Value
Semi-major axis (km)	6945.034
Eccentricity	0
Inclination (deg)	30
Right ascension of ascending node (deg)	30
Argument of latitude (deg)	20

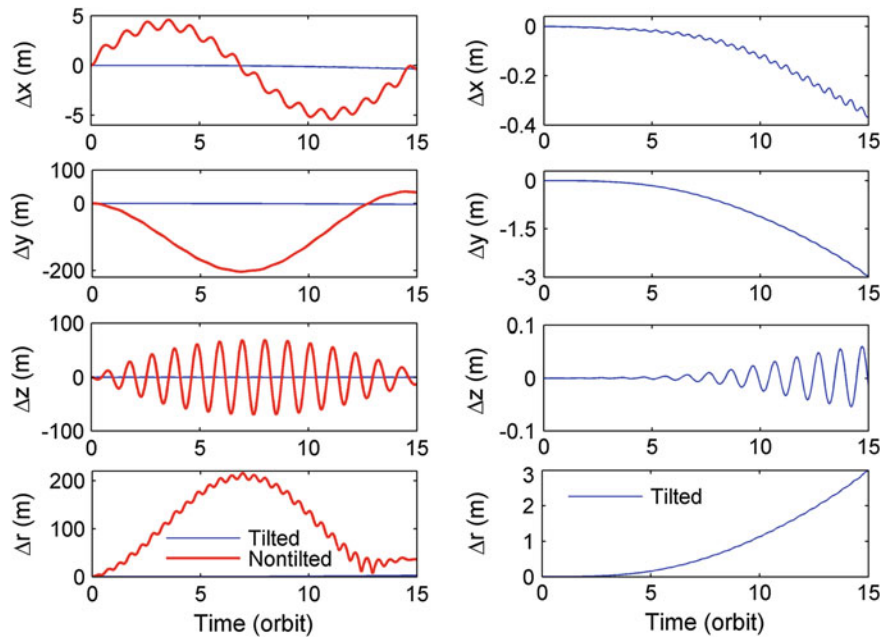


Fig. 2.9 Time histories of model errors for the tilted and nontilted dipole models. Reprinted from Ref. [10], Copyright 2014, with permission from SAGE

analytical and numerical solutions, respectively. Thus, the error relative distance is defined as $\Delta r = (\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2}$. Time histories of the model errors for the tilted and nontilted dipole models are then shown in Fig. 2.9. Also, details about the titled one are depicted separately in the right side of Fig. 2.9 to show it more distinctively. As can be seen, the tilted dipole model proposed in this book presents much more precise prediction of relative position than the nontilted one. For the tilted dipole model, the maximal relative position error is about 0.37, 2.97 and 0.06 m in each axis. However, for the nontilted dipole model, the model errors increase to 5.43, 203.95 and 69.69 m in each axis, which are about two orders of magnitude larger than those of the tilted model in y and z axes. This enhancement is

due to the reason that the tilted dipole model explicitly includes the dipole tilt angle, which is more representative of the Earth's magnetic field. Thus, it presents enhanced accuracy. Meanwhile, it is notable that the model accuracy decreases with increasing simulation time and increasing specific charge. By using this numerical example, it has been proved that the dipole tile angle should be taken into account when analyzing the Lorentz-augmented relative motion about inclined LEOs.

2.2.3 J_2 -Perturbed Model

2.2.3.1 Equations of J_2 -Perturbed Relative Motion

J_2 perturbation, arising from the oblateness and nonhomogeneity of Earth, is one of the most dominant perturbations in LEOs. To capture the environment in LEOs more precisely, a nonlinear dynamical model of J_2 -perturbed Lorentz spacecraft relative motion is developed in this subsection to analyze the effect of both Lorentz acceleration and J_2 perturbation on spacecraft relative motion. Notably, the reference orbit is a J_2 -perturbed one. Therefore, a brief review of J_2 -perturbed orbital dynamics of the target spacecraft is given at first. In the LVLH frame, the governing dynamics of the J_2 -perturbed target's orbit can be described by the following equations developed by Xu and Wang, given by [5]

$$\dot{R}_T = V_r \quad (2.60)$$

$$\dot{V}_r = -\frac{\mu}{R_T^2} + \frac{h_T^2}{R_T^3} - \frac{k_J}{R_T^4}(1 - 3 \sin^2 i_T \sin^2 u_T) \quad (2.61)$$

$$\dot{h}_T = -\frac{k_J \sin^2 i_T \sin 2u_T}{R_T^3} \quad (2.62)$$

$$\dot{i}_T = -\frac{k_J \sin 2i_T \sin 2u_T}{2h_T R_T^3} \quad (2.63)$$

$$\dot{u}_T = \frac{h_T}{R_T^2} + \frac{2k_J \cos^2 i_T \sin^2 u_T}{h_T R_T^3} \quad (2.64)$$

$$\dot{\Omega}_T = -\frac{2k_J \cos i_T \sin^2 u_T}{h_T R_T^3} \quad (2.65)$$

where the six orbital variables ($R_T, V_r, h_T, i_T, u_T, \Omega_T$) are, respectively, the orbital radius, radial velocity, orbital angular momentum, inclination, argument of latitude,

and right ascension of the ascending node of the target spacecraft. The constant k_J is defined as

$$k_J = 3J_2\mu R_E^2/2 \quad (2.66)$$

In the presence of J_2 perturbations, the angular velocity of the LVLH frame $\boldsymbol{\omega}$ is given by [5]

$$\boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -(k_J \sin 2i_T \sin u_T)/(h_T R_T^3) \\ 0 \\ h_T/R_T^2 \end{bmatrix} \quad (2.67)$$

Correspondingly, the angular acceleration of the LVLH frame can be derived as [5]

$$\boldsymbol{\varepsilon} = \dot{\boldsymbol{\omega}} = [\varepsilon_x \quad 0 \quad \varepsilon_z]^T \quad (2.68)$$

with

$$\begin{aligned} \varepsilon_x = & \frac{3V_r k_J \sin 2i_T \sin u_T}{R_T^4 h_T} - \frac{k_J \sin 2i_T \cos u_T}{R_T^5} \\ & - \frac{8k_J^2 \sin^3 i_T \cos i_T \sin^2 u_T \cos u_T}{R_T^6 h_T^2} \end{aligned} \quad (2.69)$$

$$\varepsilon_z = -\frac{2h_T V_r}{R_T^3} - \frac{k_J \sin^2 i_T \sin 2u_T}{R_T^5} \quad (2.70)$$

It is notable that both the angular velocity and acceleration are described in the LVLH frame.

Similarly, to derive the dynamical model of J_2 -perturbed relative motion, following assumptions are proposed: (1) the Earth's magnetic field can be modeled as a tilted dipole located at the Earth's center corotating with Earth; (2) the dipole is tilted by angle α with respect to the rotation axis of Earth, as shown in Fig. 2.8; (3) the Lorentz spacecraft can be regarded as a charged point mass.

Based on these assumptions, the Lagrange dynamics method is introduced to develop the dynamical model in the LVLH frame. In this approach, the generalized coordinate vector $\boldsymbol{\chi}$ is chosen as the relative position between the Lorentz spacecraft and the target, given by $\boldsymbol{\chi} = [x \quad y \quad z]^T$. Then, it remains to represent the Lagrange function by the generalized coordinate and its first order time derivative.

Under the effect of J_2 perturbations, the kinetic energy (per unit mass) of the Lorentz spacecraft is

$$T_{J_2} = \frac{1}{2} \mathbf{V}_{J_2}^T \mathbf{V}_{J_2} \quad (2.71)$$

where $\mathbf{V}_{J_2}$ is the absolute velocity of the Lorentz spacecraft, given by

$$\mathbf{V}_{J_2} = \frac{d\mathbf{R}_L}{dt} = \dot{\mathbf{R}}_L + \boldsymbol{\omega} \times \mathbf{R}_L = \begin{bmatrix} \dot{R}_T + \dot{x} - \omega_z y \\ \dot{y} + \omega_z (R_T + x) - \omega_x z \\ \dot{z} + \omega_x y \end{bmatrix} \quad (2.72)$$

Also, the magnetic potential (per unit mass) of the Lorentz spacecraft is

$$U_{L,J_2} = -\lambda_L \mathbf{V}_{rel,J_2}^T \mathbf{A} = -\frac{q_L}{m_L} \mathbf{V}_{rel,J_2}^T [(B_0/R_L^2)(\hat{\mathbf{n}} \times \hat{\mathbf{R}}_L)] \quad (2.73)$$

where the unit magnetic dipole momentum vector is given in Eq. (2.42), and the unit orbital radius vector is given in Eq. (2.43). In the presence of J_2 perturbation, the velocity of the Lorentz spacecraft with respect to the local magnetic field is revised as

$$\begin{aligned} \mathbf{V}_{rel,J_2} &= \mathbf{V}_{J_2} - \boldsymbol{\omega}_E \times \mathbf{R}_L = [V_{x,J_2} \quad V_{y,J_2} \quad V_{z,J_2}]^T \\ &= \begin{bmatrix} \dot{R}_T + \dot{x} - y(\omega_z - \omega_E \cos i_T) - z\omega_E \sin i_T \cos u_T \\ \dot{y} + (R_T + x)(\omega_z - \omega_E \cos i_T) - z(\omega_x - \omega_E \sin i_T \sin u_T) \\ \dot{z} + (R_T + x)\omega_E \sin i_T \cos u_T + y(\omega_x - \omega_E \sin i_T \sin u_T) \end{bmatrix} \end{aligned} \quad (2.74)$$

Furthermore, the gravitational potential (per unit mass) of the Lorentz spacecraft, including J_2 perturbation, is [5]

$$U_{g,J_2} = -\frac{\mu}{R_L} - \frac{k_J}{3R_L^3} (1 - 3 \cos^2 \theta) \quad (2.75)$$

with

$$\cos \theta = R_{Lz}/R_L \quad (2.76)$$

where R_{Lz} is the projection of $\mathbf{R}_L$ on the Z_I axis of the ECI frame, given by

$$R_{Lz} = (R_T + x) \sin i_T \sin u_T + y \sin i_T \cos u_T + z \cos i_T \quad (2.77)$$

Substitution of Eqs. (2.76) and (2.77) into Eq. (2.75) yields the gravitational potential in terms of the generalized coordinates, given by

$$U_{g,J_2} = -\frac{k_J}{3R_L^3} \left\{ 1 - 3 \frac{[(R_T + x) \sin i_T \sin u_T + y \sin i_T \cos u_T + z \cos i_T]^2}{R_L^2} \right\} - \frac{\mu}{R_L} \quad (2.78)$$

Thus, the Lagrange function can be summarized as

$$L = T_{J_2} - U_{L,J_2} - U_{g,J_2} \quad (2.79)$$

where the expressions of T_{J_2} , U_{L,J_2} , and U_{g,J_2} are, respectively, shown in Eqs. (2.71), (2.73), and (2.78).

In consideration of the control acceleration $\mathbf{a}_C$, the Lagrangian equation for Lorentz spacecraft relative motion is

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\boldsymbol{\chi}}} \right) - \frac{\partial L}{\partial \boldsymbol{\chi}} = \mathbf{a}_C \quad (2.80)$$

where $\dot{\boldsymbol{\chi}} = [\dot{x} \ \dot{y} \ \dot{z}]^T$, $\mathbf{a}_C = [a_R \ a_S \ a_W]^T$ is the control acceleration acting on the Lorentz spacecraft.

Solving the above Lagrange function yields that [11]

$$\begin{aligned} \ddot{x} = & 2\dot{y}\omega_z - x(\eta_L^2 - \omega_z^2) + y\dot{e}_z - z\omega_x\omega_z - (\xi_L - \xi) \sin i_T \sin u_T \\ & - R_T(\eta_L^2 - \eta^2) + a_x + a_R \end{aligned} \quad (2.81)$$

$$\begin{aligned} \ddot{y} = & -2\dot{x}\omega_z + 2\dot{z}\omega_x - x\dot{e}_z - y(\eta_L^2 - \omega_z^2 - \omega_x^2) + z\dot{e}_x \\ & - (\xi_L - \xi) \sin i_T \cos u_T + a_y + a_S \end{aligned} \quad (2.82)$$

$$\ddot{z} = -2\dot{y}\omega_x - x\omega_x\omega_z - y\dot{e}_x - z(\eta_L^2 - \omega_x^2) - (\xi_L - \xi) \cos i_T + a_z + a_W \quad (2.83)$$

with

$$\eta_L^2 = \frac{\mu}{R_L^3} + \frac{k_J}{R_L^5} - \frac{5k_J R_{Lz}^2}{R_L^7} \quad (2.84)$$

$$\eta^2 = \frac{\mu}{R_T^3} + \frac{k_J}{R_T^5} - \frac{5k_J \sin^2 i_T \sin^2 u_T}{R_T^5} \quad (2.85)$$

$$\xi_L = \frac{2k_J R_{Lz}}{R_L^5} \quad (2.86)$$

$$\zeta = \frac{2k_J \sin i_T \sin u_T}{R_T^4} \quad (2.87)$$

where $\mathbf{a}_L = [a_x \ a_y \ a_z]^T$ is the Lorentz acceleration acting on the Lorentz spacecraft, given by

$$\mathbf{a}_L = \lambda \mathbf{l} = \lambda [l_x \ l_y \ l_z]^T \quad (2.88)$$

with

$$\begin{aligned} l_x = & \frac{B_0}{R_L^5} V_{y,J_2} \{ (3z^2 - R_L^2) n_z + 3z[(R_T + x)n_x + yn_y] \} \\ & - \frac{B_0}{R_L^5} V_{z,J_2} \{ (3y^2 - R_L^2) n_y + 3y[(R_T + x)n_x + zn_z] \} \end{aligned} \quad (2.89)$$

$$\begin{aligned} l_y = & \frac{B_0}{R_L^5} V_{z,J_2} \{ [3(R_T + x)^2 - R_L^2] n_z + 3(R_T + x)(yn_y + zn_z) \} \\ & - \frac{B_0}{R_L^5} V_{x,J_2} \{ (3z^2 - R_L^2) n_z + 3z[(R_T + x)n_x + zn_z] \} \end{aligned} \quad (2.90)$$

$$\begin{aligned} l_z = & \frac{B_0}{R_L^5} V_{x,J_2} \{ (3y^2 - R_L^2) n_y + 3y[(R_T + x)n_x + zn_z] \} \\ & - \frac{B_0}{R_L^5} V_{y,J_2} \{ [3(R_T + x)^2 - R_L^2] n_z + 3(R_T + x)(yn_y + zn_z) \} \end{aligned} \quad (2.91)$$

where $\hat{\mathbf{n}} = [n_x \ n_y \ n_z]^T$ and $\mathbf{V}_{\text{rel},J_2} = [V_{x,J_2} \ V_{y,J_2} \ V_{z,J_2}]^T$ are, respectively, given in Eqs. (2.42) and (2.74). $(R_T, V_r, h_T, i_T, u_T, \Omega_T)$ are the solutions to Eqs. (2.60)–(2.65).

Heretofore, the dynamical model that describes the relative orbital motion of a Lorentz spacecraft about a J_2 -perturbed reference orbit has been derived as Eqs. (2.81)–(2.91).

2.2.3.2 Numerical Simulations

A typical scenario in LEO is simulated to evaluate effect of J_2 perturbation on the Lorentz-augmented relative motion. The initial orbit elements of the target are chosen the same as those given in Table 2.1. Also, other simulation parameters are chosen the same as those given in Sect. 2.2.2.2. The only difference is the inclusion of J_2 perturbation here.

The exact trajectories of the relative position are generated by numerical integrations of the nonlinear equations of J_2 -perturbed relative motion from Eqs. (2.81) to (2.83). For the tilted and nontilted dipole models, time histories of model errors

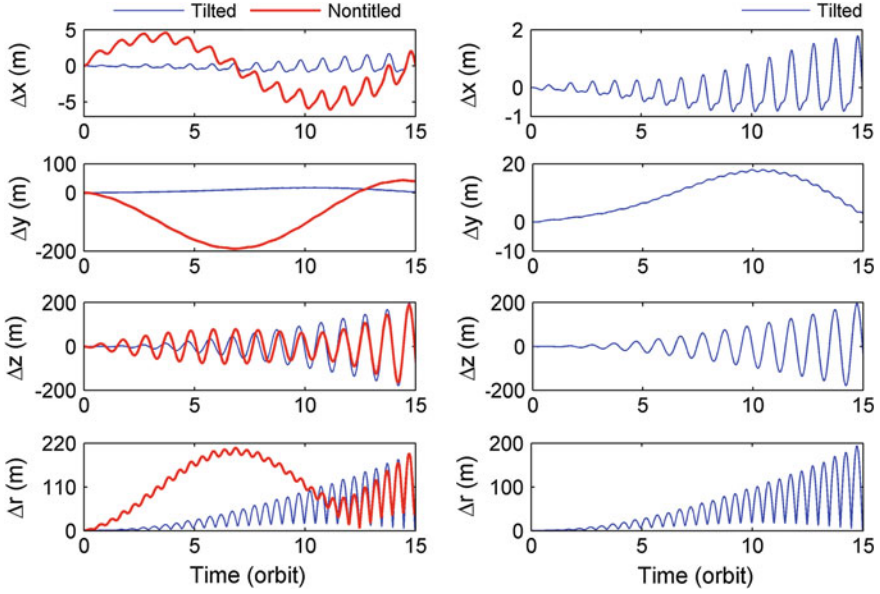


Fig. 2.10 Time histories of model errors for the titled and nontitled dipole modes in J_2 -perturbed environment. Reprinted from Ref. [10], Copyright 2014, with permission from SAGE

are compared in Fig. 2.10. Also, details about the titled dipole model are depicted separately in the right side of Fig. 2.10 to show it more distinctively. As can be seen, because both models do not take J_2 perturbation into account, the prediction accuracies of both models decrease distinctly after several periods in J_2 -perturbed environment, especially the relative position accuracy in z axis. Thus, it can be concluded that J_2 perturbation should be taken into account when analyzing the long-term Lorentz-augmented relative motion about LEOs.

2.3 Conclusions

Based on the assumption that the Earth's magnetic field can be approximated by a tilted dipole located at the Earth's center corotating with Earth, dynamical models of absolute and relative orbital motion of Lorentz spacecraft are developed by the Lagrange dynamics method in both two-body and J_2 -perturbed environment. For the absolute orbital motion, the generalized coordinate is selected as the absolute position of the Lorentz spacecraft in space. For the relative one, the generalized coordinate is the relative position. All models in this chapter explicitly incorporate the dipole tilt angle, and approximate analytical solutions are derived for Lorentz-augmented spacecraft relative motion about circular inclined LEOs. Due to the inclusion of dipole tilt angle, which is more representative of the Earth's

magnetic field, the newly derived analytical solutions present enhanced accuracy than previous nontilted one. Notably, numerical simulation results indicate that J_2 perturbation, one of the most dominant disturbances in LEOs, should be taken into account when analyzing the long-term Lorentz-augmented relative orbital motion.

References

1. Finlay CC, Maus S, Beggan CD et al (2010) International geomagnetic reference field: the eleventh generation. *Geophys J Int* 183:1216–1230
2. Wang X, Wang B, Li H (2012) An autonomous navigation scheme based on geomagnetic and starlight for small satellites. *Acta Astronaut* 81:40–50
3. Pollock GE, Gangestad JW, Longuski JM (2010) Inclination change in low-Earth orbit via the geomagnetic Lorentz force. *J Guid Control Dyn* 33:1387–1395
4. Greenwood DT (1997) *Classical dynamics*. Dover, New York
5. Xu G, Wang D (2008) Nonlinear dynamic equations of satellite relative motion around an oblate Earth. *J Guid Control Dyn* 31:1521–1524
6. Tschauner J, Hempel P (1965) Rendezvous with a target in an elliptical orbit. *Acta Astronaut* 11:104–109
7. Clohessy W, Wiltshire R (1960) Terminal guidance system for satellite rendezvous. *J Aerosp Eng* 27:653–658
8. Griffiths DJ (1999) *Introduction to electrodynamics*. Prentice-Hall, New Jersey
9. Pollock GE, Gangestad JW, Longuski JM (2011) Analytical solutions for the relative motion of spacecraft subject to Lorentz-force perturbations. *Acta Astronaut* 68:204–217
10. Huang X, Yan Y, Zhou Y et al (2014) Improved analytical solutions for relative motion of Lorentz spacecraft with application to relative navigation in low Earth orbit. *Proc Inst Mech Eng Part G J Aerosp Eng* 228:2138–2154. doi:[10.1177/0954410013511426](https://doi.org/10.1177/0954410013511426)
11. Huang X, Yan Y, Zhou Y et al (2015) Nonlinear relative dynamics of Lorentz spacecraft about J_2 -perturbed orbit. *Proc Inst Mech Eng Part G J Aerosp Eng* 229:467–478. doi:[10.1177/0954410014537231](https://doi.org/10.1177/0954410014537231)

Dynamics and Control of Lorentz-Augmented Spacecraft
Relative Motion

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2017, XI, 148 p. 70 illus. in color., Hardcover

ISBN: 978-981-10-2602-7