Chapter 2
Analysis of Systemic Risk: A Dynamic Vine Copula-Based ARMA-EGARCH Model

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2.1 Introduction

The definition of systemic risk from the report to G20 Finance Ministers and Governors agreed upon among the International Monetary Fund (IMF), Bank for International Settlements (BIS), and Financial Stability Board (FSB) [1] is that "(i) caused by an impairment of all or parts of the financial system and (ii) has the potential to have serious negative consequences for the real economy". Furthermore, "G-20 members consider an institution, market or instrument as systemic if its failure or malfunction causes widespread distress, either as a direct impact or as a trigger for broader contagion." A common factor from the various definitions of systemic risk is that a trigger event causes a chain of adverse economic consequences, referred to as a "domino effect". Given the definition of systemic risk quoted above, measuring systemic risk is done by estimating the probability of failure of an institution that is the cause of a distress for the financial system. Therefore, we only consider the Value-at-Risk (VaR), the potential loss in value of an asset or portfolio for a given time period and probability, as the risk measurement. In addition, the VaR ratio of a sector to the system (S&P 500 Index), which interprets that the sector risk provides to the entire system, is present.

Girardi and Ergün [2] modified the CoVaR methodology that is proposed by Adrian and Brunnermeier [3] which used the dynamic conditional correlation GARCH, while Hakwa et al. [4] modified the methodology based on copula

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modeling. We present dynamic vine Copula-based ARMA-EGARCH (1, 1) VaR measure into a high dimensional analysis in systemic risk.

Sklar [5] introduced the copula to describe the dependence structure between variables. Patton [6] defined the conditional version of Sklar's theorem, which extends the copula applications to the time series analysis. Otani and Imai [7] presented a basket credit default swaps (CDSs) pricing model with nested Archimedean copulas. However, multivariate Archimedean copulas are limited in that there are only one or two parameters to capture the dependence structure. Joe [8] introduced a construction of multivariate distribution based on pair-copula construction (PCC), while Aas et al. were the first to recognize that the pair-copula construction (PCC) principle can be used with arbitrary pair-copulas, referred to as the graphical structure of R-vines [9]. Furthermore, Dissmann et al. [10] developed an automated algorithm of jointly searching for an appropriate R-vines tree structures, the pair-copula families and their parameters. Accordingly, a high dimensional joint distribution can be decomposed to bivariate and conditional bivariate copulas arranged together according to the graphical structure of a regular vine. Besides, Rockinger and Jondeau [11] was the first to introduce the copula-based GARCH modeling. Afterwards, Lee and Long [12] concluded that copula-based GARCH models outperform the dynamic conditional correlation model, the varying correlation model and the BEKK model. In addition, Fang et al. [13] investigated that using Akaike Information Criterion (AIC) as a tool for choosing copula from a couple of candidates is more efficient and accurate than the multiplier goodness-of-fit test method.

During 2008, the subprime mortgage crisis was a systemic collapse triggered by the financial industry. The purpose of this paper is to present an application of the estimation of systemic risk in terms of the VaR/ES ratio by using the dynamic vine copula-based ARMA-EGARCH (1, 1) model. To compute systemic risk for our system, we use S&P 500 sector indices and S&P 500 index to be our components and system, respectively. Since the parameters change over time, we calibrate the parameters every ten steps to capture a change of the structure. This scalable prototype of US financial system with limited dimensionality can be easily tailored to any underlying sector, country or financial market.

This paper has four sections. The first section briefly introduces existing research regarding systemic risk. The second section describes the definition of the VaR/ES ratio, and outlines the methodology of vine Copula-based EGARCH (1, 1) modeling. The third section describes the data and explains the results of VaR/ES ratio. The fourth section concludes our findings.

2.2 Methodology

2.2.1 Risk Methodology

The definition of Value-at-Risk (VaR) is that the maximum loss at most is (1-α) probability given by a period [14]. People usually determine α as 95 %, 99 %, or 99.9 % to be their confidence level. In this study, we use the Copula-based ARMA-EGARCH (1, 1) methodology to obtain the dynamic VaR from each sector. We denote $VaR_{t,1-\alpha}^{i\to j}$ ratio, the sector i's risk contribution to the system j (S&P 500 index) at the confidence level α, by

$$VaR_{t,1-\alpha}^{i\to j}\,Ratio = \frac{VaR_{t,1-\alpha}^{i}}{VaR_{t,1-\alpha}^{j}}$$

The higher $VaR_{t,1-\alpha}^{i\to j}$ ratio indicates that the sector is the risk provider to the system. In addition, the methodology can be easily extended from the VaR ratio to the expected shortfall (ES) ratio.

2.2.2 Univariate ARMA-EGARCH Model

Engle is the first researcher to introduce the ARCH model, which deals with volatility clustering, usually referred to as conditional heteroskedasticity. Bollerslev [15] extended the ARCH model to the generalized ARCH (GARCH) model. Chen and Khashanah [16] implemented ARMA (p, q)-GARCH (1, 1) with the Student's t distributed innovations for the marginal to account for the time-varying volatility, whereas the Student's t distributed innovations cannot explain the skewness. In addition, to overcome the leverage effect in financial time series, we use the exponential GARCH (EGARCH) model in handling asymmetric effects between positive and negative asset returns proposed by Nelson [17]. According to the augmented Dickey–Fuller (ADF) test, all the series are stationary. Therefore, ARMA (p, q)-EGARCH (1, 1) with the skewed Student's t distributed innovation can then be written as [18]

$$\begin{aligned}
r_t &= \mu_t + \sum_{i=1}^{p} \vartheta_i r_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t, \\
\varepsilon_t &= \sigma_t z_t, \\
\ln(\sigma_t^2) &= \kappa_t + \alpha_t z_{t-1} + \xi_t(|z_{t-1}| - \mathrm{E}\{|z_{t-1}|\}) + \beta_t \ln(\sigma_{t-1}^2)
\end{aligned}$$

where r_t is the log return, μ_t is the drift term, ε_t is the error term, ξ_t capture the size effect, and the standardized innovation term z_t is the skewed Student's t distribution. The skewed student's t density function can be expressed as [19]

$$p(z|\eta,f) = \frac{2}{\eta+\eta^{-1}}\left\{f(\frac{z}{\eta})I_{[0,\infty)}(z) + f(\eta z)I_{(-\infty,0]}(z)\right\}$$

where f is a univariate pdf that is symmetric around 0, I_S is the indicator function on S, η is the asymmetric parameter, and $\eta = 1$ for the symmetric Student's t distribution. In addition, the correlated random variables can be flexible and easily estimated under an overwhelming feature of Copula-based ARMA-EGARCH model.

2.2.3 Sklar's Theory

Sklar's Theorem [5] states that given random variables $X_1, X_2, \ldots, X_n$ with continuous distribution functions $F_1, F_2, \ldots, F_n$ and joint distribution function H, and there exists a unique copula C such that for all $x = (x_1, x_2, \ldots, x_n) \in \mathfrak{R}^n$

$$H(x) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n))$$

If the joint distribution function is n-times differentiable, then taking the nth cross-partial derivative of the equation:

$$\begin{aligned} f(x_1, x_2, \ldots, x_n) &= \frac{\partial^n}{\partial x_1 \ldots \partial x_n} H(x) \\ &= \frac{\partial^n}{\partial u_1 \ldots \partial u_n} C(F_1(x_1), \ldots, F_n(x_n)) \cdot \prod_{i=1}^{n} f_i(x_i) \\ &= c(F_1(x_1), \ldots, F_n(x_n)) \cdot \prod_{i=1}^{n} f_i(x_i) \end{aligned}$$

where u_i is the probability integral transform of x_i.

For the purpose of estimating the VaR or ES based on time series data, Patton [6] defined the conditional version of Sklar's theorem. Let $F_{1,t}$ and $F_{2,t}$ be the continuous conditional distributions of $x_1|\mathfrak{I}_{t-1}$ and $x_2|\mathfrak{I}_{t-1}$, given the conditioning set $\mathfrak{I}_{t-1}$, and let H_t be the joint conditional bivariate distribution of $(X_1, X_2|\mathfrak{I}_{t-1})$. Then, there exists a unique conditional copula C_t such that

$$H_t(x_1, x_2|\mathfrak{I}_{t-1}) = C_t(F_{1,t}(x_1|\mathfrak{I}_{t-1}), F_{2,t}(x_2|\mathfrak{I}_{t-1})|\mathfrak{I}_{t-1})$$

2.2.4 Parametric Copulas

Joe [9] and Nelsen [20] gave comprehensive copula definitions for each family.

(1) The bivariate Gaussian copula is defined as:

$$C(u_1, u_2; \rho) = \Phi_\rho(\Phi^{-1}(u_1), \Phi^{-1}(u_2))$$

where Φ_ρ is the bivariate joint normal distribution with linear correlation coefficient ρ and Φ is the standard normal marginal distribution.

(2) The bivariate student's t copula is defined by the following:

$$C(u_1, u_2; \rho, \nu) = t_{\rho,\nu}(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2))$$

where ρ is the linear correlation coefficient and ν is the degree of freedom.

(3) The Clayton generator is given by $\varphi(u) = u^{-\theta} - 1$ with $\theta \in (0, \infty)$, its copula is defined by

$$C(u_1, u_2; \theta) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta}$$

(4) The Gumbel generator is given by $\varphi(u) = (-\ln u)^\theta$ with $\theta \in [1, \infty)$, and the bivariate Gumbel copula is given by

$$C(u_1, u_2; \theta) = \exp(-\left\{(-\ln u_1)^\theta + (-\ln u_2)^\theta\right\}^{1/\theta})$$

(5) The Frank generator is given by $\varphi(u) = \ln[(e^{-\theta u} - 1)/(e^{-\theta} - 1)]$ with $\theta \in (-\infty, 0) \cup (0, \infty)$, and the bivariate Frank copula is defined by

$$C(u_1, u_2; \theta) = -\frac{1}{\theta}\ln(1 + \frac{\left(e^{-\theta u_1} - 1\right)\left(e^{-\theta u_2} - 1\right)}{e^{-\theta} - 1})$$

(6) The Joe generator is $\varphi(u) = u^{-\theta} - 1$, and the Joe copula is given by

$$C(u_1, u_2; \theta) = 1 - (\overline{u_1}^\theta + \overline{u_2}^\theta - \overline{u_1}^\theta \overline{u_2}^\theta)^{1/\theta}, \text{with } \theta \in [1, \infty)$$

(7) The BB1 (Clayton-Gumbel) copula with $\theta \in (0, \infty) \cap \delta \in [1, \infty)$ is

$$C(u_1, u_2; \theta, \delta) = \left(1 + \left[\left(u_1^{-\theta} - 1\right)^{\delta} + \left(u_2^{-\theta} - 1\right)^{\delta}\right]^{1/\delta}\right)^{-1/\theta}$$

(8) The BB6 (Joe-Gumbel) copula with $\theta \in [1, \infty) \cap \delta \in [0, \infty)$ is

$$C(u_1, u_2; \theta, \delta) = 1 - \left(1 - \exp\left\{-\left[\left(-\ln\left(1 - \bar{u}_1^{\theta}\right)\right)^{\delta} + \left(-\ln\left(1 - \bar{u}_2^{\theta}\right)\right)^{\delta}\right]^{1/\delta}\right\}\right)^{1/\theta}$$

(9) The BB7 (Joe-Clayton) copula with $\theta \in [1, \infty) \cap \delta \in [0, \infty)$ is

$$C(u_1, u_2; \theta, \delta) = 1 - \left(1 - \left[\left(1 - \bar{u}_1^{\theta}\right)^{-\delta} + \left(1 - \bar{u}_2^{\theta}\right)^{-\delta} - 1\right]^{-1/\delta}\right)^{1/\theta}$$

(10) The BB8 (Frank-Joe) copula with $\theta \in [1, \infty) \cap \delta \in (0, 1]$ is

$$C(u_1, u_1; \theta, \delta) = \frac{1}{\delta}(1 - \left[1 - \frac{1}{1 - (1 - \delta)^{\theta}}(1 - (1 - \delta u_1)^{\theta})(1 - (1 - \delta u_2)^{\theta})\right]^{1/\theta})$$

2.2.5 Vine Copulas

Even though it is simple to generate multivariate Archimedean copulas, they are limited in that there are only one or two parameters to capture the dependence structure. Vine copula method allows a joint distribution to be built from bivariate and conditional bivariate copulas arranged together according to the graphical structure of a regular vine, which is a more flexible measure to capture the dependence structure among assets. It is well known that any multivariate density function can be decomposed as

$$f(x_1, \ldots, x_n) = f(x_n | x_1, \ldots, x_{n-1}) \ldots f(x_3 | x_1, x_2) f(x_2 | x_1) f(x_1)$$

Moreover, the conditional densities can be written as copula functions. For instance, the first and second conditional density can be decomposed as

$$\begin{aligned} f(x_2|x_1) &= c_{1,2}(F_1(x_1), F_2(x_2)) \cdot f_2(x_2), \\ f(x_3|x_1, x_2) &= c_{2,3|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) \cdot f_3(x_3|x_1) \\ &= c_{2,3|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) \cdot c_{1,3}(F_1(x_1), F_3(x_3)) \cdot f_3(x_3) \end{aligned}$$

After rearranging the terms, the joint density can be written as

$$\begin{aligned} f(x_1, x_2, x_3) = c_{2,3|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) \cdot c_{1,2}(F_1(x_1), F_2(x_2)) \\ \cdot c_{1,3}(F_1(x_1), F_3(x_3)) \cdot f_1(x_1) \cdot f_2(x_2) \cdot f_3(x_3) \end{aligned}$$

The summary of vine copulas is given by Kurowicka and Joe [21], and the general n-dimensional canonical vine copula, in which one variable plays a pivotal role, can be written as

$$f(x_1, \ldots, x_n) = \prod_{k=1}^{n} f(x_k) \times \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1,\ldots,j-1}$$

Similarly, D-vines are constructed by choosing a specific order for the variables, and the general n-dimensional D-vine copula can be written as

$$f(x_1, \ldots, x_n) = \prod_{k=1}^{n} f(x_k) \times \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i,i+j|i+1,\ldots,i+j-1}$$

Dissmann et al. [10] proposed that the automated algorithm involves searching for an appropriate R-vine tree structure, the pair-copula families, and the parameter values of the chosen pair-copula families based on AIC, which is summarized in Table 2.1.

Table 2.1 Sequential method to select an R-Vine model

Algorithm. Sequential method to select an R-Vine model1. Calculate the empirical Kendall's tau for all possible variable pairs.
2. Select the tree that maximizes the sum of absolute values of Kendall's taus.
3. Select a copula for each pair and fit the corresponding parameters based on AIC.
4. Transform the observations using the copula and parameters from Step 3. To obtain the transformed values.
5. Use transformed observations to calculate empirical Kendall's taus for all possible pairs.
6. Proceed with Step 2. Repeat until the R-Vine is fully specified.

2.2.6 Tail Dependence

Tail dependence looks at the concordance and discordance in the tail, or extreme values of u_1 and u_2. It concentrates on the upper and lower quadrant tails of the joint distribution function. Given two random variables $u_1 \sim F_1$ and $u_2 \sim F_2$ with copula C, the coefficients of tail dependency are given by [22, 20]

$$\lambda_L \equiv \lim_{u \to 0^+} P[F_1(u_1) < u | F_2(u_2) < u] = \lim_{u \to 0^+} \frac{C(u,u)}{u},$$
$$\lambda_U \equiv \lim_{u \to 1^-} P[F_1(u_1) > u | F_2(u_2) > u] = \lim_{u \to 1^-} \frac{C(u,u)}{u}$$

where C is said to have lower (upper) tail dependency *iff* $\lambda_L \neq 0\,(\lambda_U \neq 0)$. The interpretation of the tail dependency is that it measures the probability of two random variables both taking extreme values as shown in Table 2.2 [22, 20].

2.2.7 Estimation Method

Generally, the two-step separation procedure is called the Inference functions for the margin method (IFM) [9]. It implies that the joint log-likelihood is simply the sum of univariate log-likelihoods and the copula log-likelihood is shown as below.

$$\log f(x) = \sum_{i=1}^{n} \log f_i(x_i) + \log c(F_1(x_1), \ldots, F_n(x_n))$$

Table 2.2 The coefficients of tail dependency

Family	Lower tail dependence	Upper tail dependence
Gaussian	–	–
Student's t	$2t_{\nu+1}\left(-\sqrt{\frac{(\nu+1)(1-\theta)}{1+\theta}}\right)$	$2t_{\nu+1}\left(-\sqrt{\frac{(\nu+1)(1-\theta)}{1+\theta}}\right)$
Clayton	$2^{-1/\theta}$	–
Gumbel	–	$2-2^{1/\theta}$
Frank	–	–
Joe	–	$2-2^{1/\theta}$
BB1	$2^{-1/\theta\delta}$	$2-2^{1/\delta}$
BB6	–	$2-2^{1/\theta\delta}$
BB7	$2^{-1/\delta}$	$2-2^{1/\theta}$
BB8	–	$2-2^{1/\theta}$ if $\delta=1$, otherwise 0

Note:—represents that there is no tail dependency

Therefore, it is convenient to use this two-step procedure to estimate the parameters by maximum log-likelihood, where marginal distributions and copulas are estimated separately.

2.3 Data and Empirical Findings

2.3.1 Data Representation

We use indices prices instead of other financial instruments or financial accounting numbers. One of the main reasons is that an index price could reflect a timely financial environment in contrast to financial accounting numbers that are published quarterly. Furthermore, indices can easily be constructed and tell us which sector contributes more risk to the entire market. Standard and Poor separates the 500 members in the S&P 500 index into 10 different sector indices based on the Global Industrial Classification Standard (GICS). All data is acquired from Bloomberg, sampled at daily frequency from January 1, 1995 to June 5, 2009. We separate the sample into two parts, the in-sample estimation period is from January 1, 1995 to December 31, 2007 (3271 observations) and the out-of-sample forecast validation period is from January 1, 2008 to June 5, 2009 (360 observations). Since the parameters change over time, we recalibrate the parameters every ten steps in the out-sample dataset. In other words, we calibrate the parameters 36 times. The following tables only show the results for the first calibration.

The summary statistics of these indices are listed in Table 2.3 as well as the statistical hypothesis testing. The statistical hypothesis testing for the unit-root based on Augmented Dickey-Fuller (ADF) test, and the results show that the values of 1 in ADF test rejects the null hypothesis of a unit root in a univariate time series. The results of Jarque-Bera (J-B) test reject that the distributions of returns are normality, and the results of Engle's ARCH test show that the indices' returns present conditional heteroscedasticity at the 5 % significance level. In addition, we assign an identification number to each sector.

2.3.2 Results for the Marginal Models

We estimate the parameters of p and q by minimizing Akaike information criterion (AIC) values for possible values ranging from zero to five. Table 2.4 lists the parameters which are estimated by minimized AIC values, and statistical hypothesis tests for the residuals are based on the Jarque-Bera (J-B) test and the Engle's ARCH

Table 2.3 Summary statistics of the in-sample dataset and statistical hypothesis testings

ID	Sector	Mean (%)	Sigma (%)	Skew	Kurt	ADF test	J-B test	ARCH test	Min/Max
1	S5FINL Index Financials	0.04	1.41	0.073	6.078	1	1	1	−8.04 %/8.39 %
2	S5INFT Index Technology	0.04	1.99	0.183	6.775	1	1	1	−10.01 %/ 16.08 %
3	S5COND Index Consumer Discretionary	0.03	1.24	−0.147	8.231	1	1	1	−10.33 %/8.47 %
4	S5ENRS Index Energy	0.06	1.39	−0.089	4.648	1	1	1	−7.21 %/7.94 %
5	S5HLTH Index Health Care	0.04	1.21	−0.180	7.097	1	1	1	−9.17 %/7.66 %
6	S5INDU Index Industrials	0.04	1.18	−0.227	7.410	1	1	1	−9.60 %/7.21 %
7	S5UTIL Index Utilities	0.02	1.12	−0.409	9.608	1	1	1	−9.00 %/8.48 %
8	S5CONS Index Consumer Staples	0.03	0.97	−0.233	9.905	1	1	1	−9.30 %/7.59 %
9	S5MATR Index Materials	0.03	1.31	0.036	5.929	1	1	1	−9.12 %/6.98 %
10	S5TELS Index Telecommunication Services	0.02	1.44	−0.100	6.674	1	1	1	−10.32 %/8.03 %
11	S&P 500 Index	0.04	1.07	−0.136	6.438	1	1	1	−7.11 %/5.57 %

Table 2.4 Estimation of in-sample parameters and statistical hypothesis testings for each marginal for the first calibration

	1	2	3	4	5	6	7	8	9	10	11
p	5	5	4	4	3	4	4	5	3	4	4
q	4	2	2	3	5	4	2	4	3	3	4
φ_1	0.4510	1.8223	1.8961	1.6080	−0.8015	0.8777	1.5697	0.1674	−0.7647	0.7181	−0.1260
φ_2	0.4420	−1.0559	−1.0770	−1.5377	0.2459	0.1199	−1.0570	0.6270	−0.3414	0.7243	1.3587
φ_3	0.5891	0.1041	0.1001	0.6024	0.1156	0.8492	0.0708	−0.6045	−0.0104	−0.6112	0.2708
φ_4	−0.8651	−0.0666	−0.0445	−0.0671	*	−0.9740	−0.0332	−0.4271	*	0.0066	−0.5033
φ_5	0.0305	0.0110	*	*	*	*	*	−0.0677	*	*	*
θ_1	−0.4268	−1.8192	−1.8833	−1.6313	0.8216	−0.8771	−1.5463	−0.1991	0.7991	−0.7441	0.1028
θ_2	−0.4789	0.9966	1.0042	1.5122	−0.2823	−0.1379	0.9896	−0.6501	0.3417	−0.7061	−1.4048
θ_3	−0.6191	*	*	−0.5585	−0.2107	−0.8330	*	0.6227	0.0391	0.5958	−0.2610
θ_4	0.8570	*	*	*	−0.0439	0.9744	*	0.4464	*	*	0.5640
θ_5	*	*	*	*	−0.0197	*	*	*	*	*	*
μ	0.0004	0.0005	0.0002	0.0006	0.0004	0.0004	0.0003	0.0004	0.0004	0.0004	0.0006
κ	−0.0974	−0.0597	−0.1056	−0.1281	−0.0825	−0.1404	−0.1566	−0.1136	−0.1083	−0.0655	−0.1502
α	−0.0723	−0.0539	−0.0907	−0.0383	−0.0479	−0.0891	−0.0325	−0.0552	−0.0480	−0.0315	−0.1062
β	0.9890	0.9928	0.9884	0.9853	0.9909	0.9847	0.9832	0.9882	0.9878	0.9925	0.9841
ξ	0.1335	0.1253	0.1159	0.1115	0.1184	0.1064	0.1730	0.1256	0.1264	0.1133	0.1016
η	0.9681	0.9080	0.9179	0.9289	0.9475	0.9357	0.8963	0.9677	1.0031	1.0001	0.8934
ν	9.9165	14.5997	10.4272	15.8222	7.9741	10.5557	9.2773	8.0843	9.3139	8.4905	9.8285
LLH	9897.7	8810.4	10304.2	9606.4	10281.9	10397	10658.2	11036.6	9903.6	9721.9	10734
AIC	−19763	−17593	−20582	−19185	−20534	−20764	−21290	−22041	−19781	−19415	−21438
J-B test	1	1	1	1	1	1	1	1	1	1	1
ARCH test	0	0	0	0	0	0	0	0	0	0	0

test. The results show that using the skewed Student's t innovation distribution for the residual terms is appropriately fitted to the return data because the degree of freedom is usually smaller than 15 and Jarque-Bera test rejects the null hypothesis of normality. In addition, the asymmetric parameter is not equal to one. Using EGARCH (1, 1) model is appropriate because the result of the Engle's ARCH test of residuals shows no conditional heteroscedasticity, and parameter β is usually larger than 0.9, which indicates the conditional volatility is time-dependent.

2.3.3 Results for the Copula Models

After the estimation of each marginal, we consider the set of standardized residuals from the ARMA-EGARCH (1, 1) model and transform them into a set of uniform variables. All the transformed residuals of the Kolmogorov-Smirnov test are 0, indicating that the distribution of transformed residuals and the uniform distribution are from the same continuous distribution, which null hypothesis cannot be rejected at the 5 % significance level. Using the Student's t copula as our benchmark with a degree of freedom of 10.1289, AIC values in Table 2.5 show that vine copula-based model outperforms Student's t copula-based model in high-dimensional modeling.

The catalogue of pair-copula families includes elliptical copulas such as Gaussian and Student's t, single parameter Archimedean copulas such as Clayton, Frank, and Gumbel, alongside two parameter families such as BB1, BB6, BB7, and BB8. All various copulas implemented are in the VineCopula library in R [23].

2.3.4 Results for the Copula VaR and Copula VaR Ratio

We empirically examine which sector dominates more risk contributions on systemic risk with 10,000 Monte Carlo simulations using vine Copula-based ARMA-EGARCH (1, 1) modeling. The results of residuals, fitted by ARMA-EGARCH (1, 1) with the skewed student's t innovations, are shown in Fig. 2.1. Using the vine copula, the tree structure in tree level one is shown in Fig. 2.2. The results of the worst 5 % return loss, shown in Fig. 2.3, are not surprising that the financial industry has the largest risk during 2008 financial crisis. As

Table 2.5 Estimation for the copula models from in-sample data for the first calibration

	Number of parameters	Log-likelihood	AIC
Gaussian copula	55	15259	−30408.4
Student's t copula	56	16074	−32038
Vine copula	96	16242	−32292.6

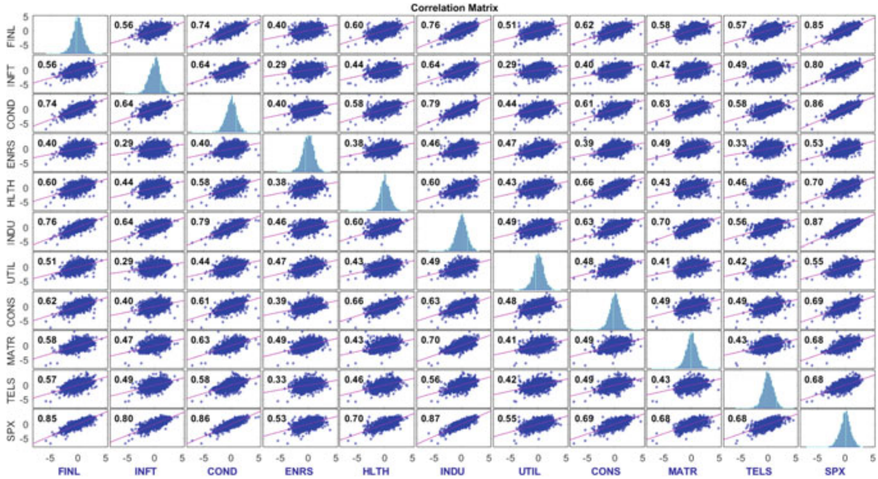

Fig. 2.1 The scatter plot of residuals for the first calibration

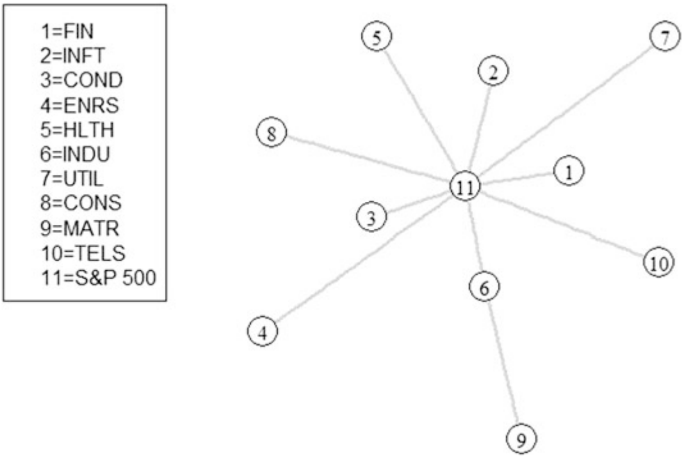

Fig. 2.2 The plot of vine copula structure in tree one for the first calibration

seen in Fig. 2.4 and Fig. 2.5, we realize that the financial sector contributed more risk during the subprime crisis from 2008 to 2009, while the consumer staples sector is the major risk receiver. Hence, this measure is a simplified and efficient methodology to analyze systemic risk.

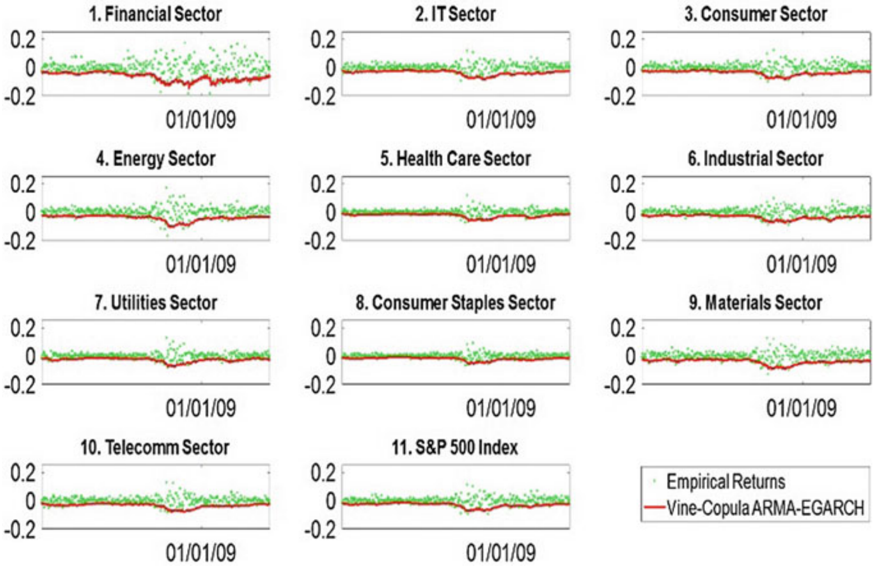

Fig. 2.3 The one-day ahead worst 5 % return loss for each sector index

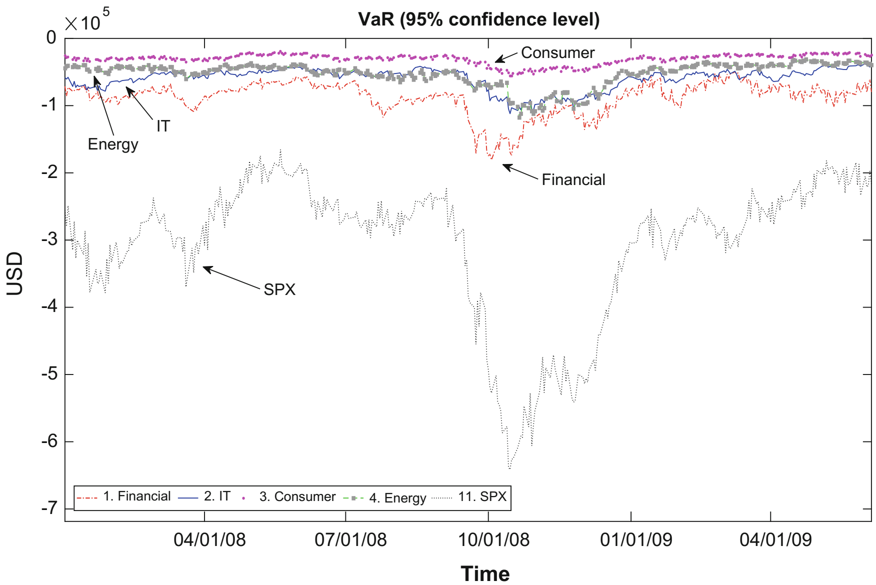

Fig. 2.4 The one-day ahead VaR at the 95 % confidence level

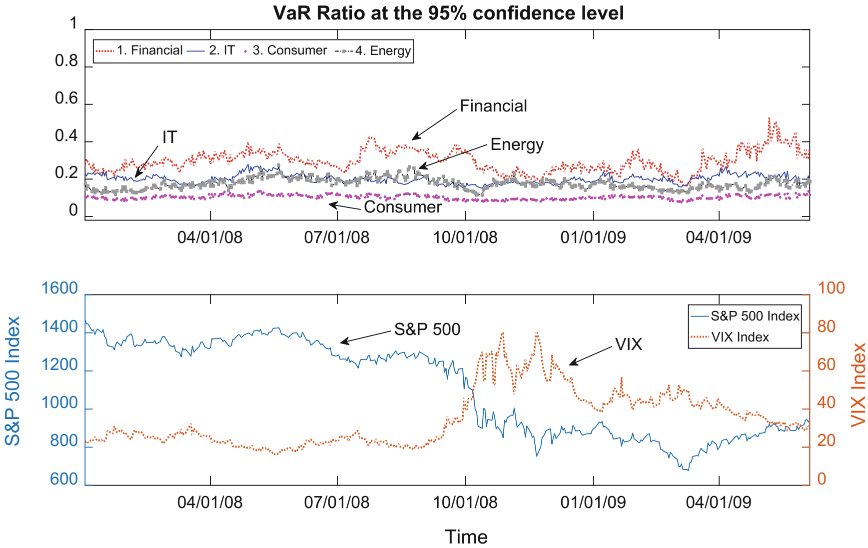

Fig. 2.5 The one-day ahead VaR ratio at the 95 % confidence level

2.4 Conclusion

The evidence in our paper shows that not only does vine Copula-based ARMA-EGARCH (1, 1) outperform the Gaussian and the Student's t copula-based ARMA-EGARCH (1, 1) based on AIC values, but also is the skewed student's t innovations more appropriate than using the student's t innovations.

In addition, using vine Copula-based ARMA-EGARCH model to forecast Copula VaR and Copula VaR ratio, we develop a real-time and useful way to evaluate systemic risk. Moreover, the VaR ratio provides the information of the risk contribution from each sector. This approach is very general and can be tailored to any underlying country and financial market easily.

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