

Chapter 2

Torsional Dynamics of Gear-Rotor System in Wind Turbine Gearbox

A torsional dynamics modeling for planetary gear transmission system in wind turbine is established after comprehensively considering these nonlinear factors associated with time-varying mesh stiffness, dynamic transmission error, gear mesh impact, and input-varying load. Load factors of planetary gear transmission system are induced from the coupled nonlinear dynamics modeling. Then, the load-sharing performance is studied and the load balance method suited for this model which is validated by the test is obtained from the perspective of load-sharing structure. These results provide fundamental basis for multistage planetary gear transmission system design.

2.1 Lumped Mass Model for Torsional Dynamics

2.1.1 Mechanical Schematic

A two-stage planetary transmission gearbox system in the wind turbine is shown in Fig. 2.1, and its dynamic model is shown in Fig. 2.2. The equivalent dynamics model based on the lumped-parameters and Lagrange general function is shown in Fig. 2.3 [1, 2]. The 1st stage has four planet gears, and the 2nd has three ones. Since the relative position between the axial and meshing line direction is simple (there is a helix angle between them and the planetary location has no effect on it), there is no axial diagram for illustration [3, 4].

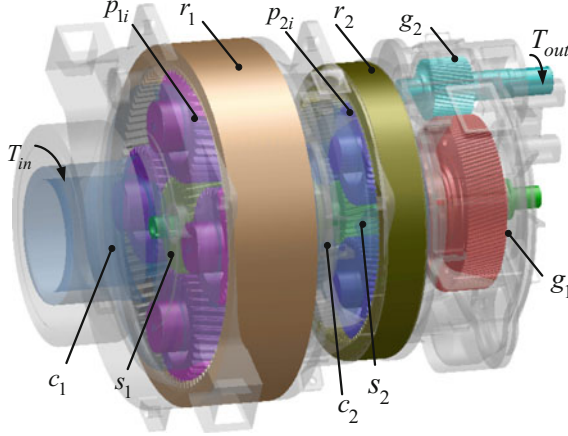


Fig. 2.1 A two-stage planetary gearbox used in wind turbine

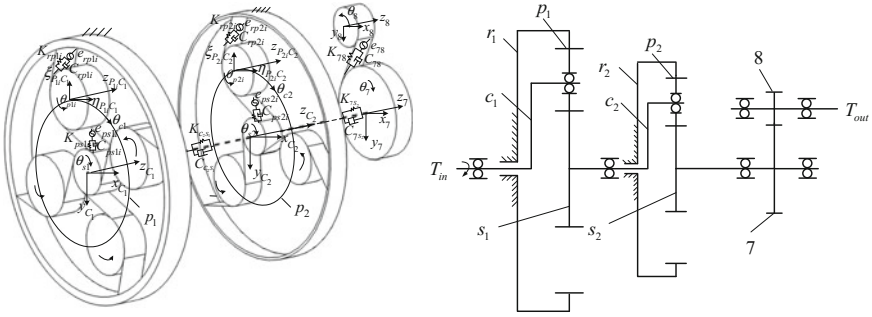


Fig. 2.2 Dynamic model for torsional dynamics

2.1.2 Definition of DOFs

The generalized coordinates of the planetary transmission in two stages are established in rotary coordinate systems which are rotating with each carrier, respectively. Because of the carriers in two stages rotating in plane, rotary coordinate system has no influence on DOF along z -axis. The definition of every variable and parameter of the planetary system is also shown in Fig. 2.3.

The displacement vector of the gear-rotor system of the two-stage planetary gearbox is defined as

$$\delta = (u_{c1}, x_{c1}, y_{c1}, z_{c1}, u_{p1i}, \xi_{p1i}, \eta_{p1i}, z_{p1i}, u_{r1}, x_{r1}, y_{r1}, z_{r1}, u_{s1}, x_{s1}, y_{s1}, z_{s1}, u_{c2}, x_{c2}, y_{c2}, z_{c2}, u_{p2i}, \xi_{p2i}, \eta_{p2i}, z_{p2i}, u_{r2}, x_{r2}, y_{r2}, z_{r2}, u_{s2}, x_{s2}, y_{s2}, z_{s2}, \theta_7, x_7, y_7, z_7, \theta_8, x_8, y_8, z_8)^T \quad (2.1)$$

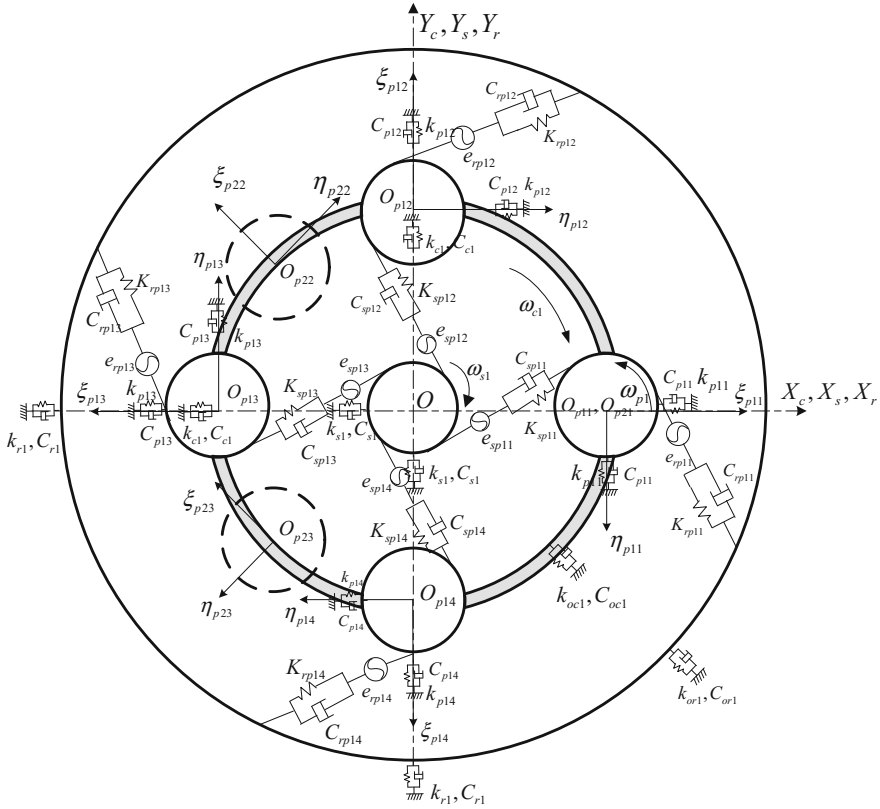


Fig. 2.3 Dynamics model of the planetary gear train

The vector is 60 degrees of freedom, in which x, y, z represents the deflection along the respective coordinate axes, u represents torsional deflection presented as $u = r_b \omega$, r_b represents base circle radius of gears or carriers, the subscripts of s, p, r , and c , respectively, represent sun gear, planet, ring gear, and carrier.

2.1.3 Mesh Stiffness Excitation

Considering the helical gear tooth meshing begins at one end and gradually extends to the entire tooth surface, time-varying mesh stiffness is proposed as the first-order harmonic instead of step waveforms to simulate helical gear mesh condition [5].

Supposing that the variation range of a meshing period is 2π and initial phase of Planet 1 is 0, the phase of Planet i should be $(Z_r, Z_s)\psi_i$. The inner and external time-varying mesh stiffness are presented in Eq. (2.2) [6].

$$\begin{aligned} k_{rp}(t) &= \bar{k}_{rp}[1 + S \sin(\omega_m t + Z_r \psi_i + \gamma_{sr})] \\ k_{sp}(t) &= \bar{k}_{sp}[1 + S \sin(\omega_m t + Z_s \psi_i)] \end{aligned} \quad (2.2)$$

$$\omega_{mi} = \frac{(\omega_{si} - \omega_{ci})Z_{si}\pi}{30} \quad (2.3)$$

In Eq. (2.2), $\bar{k}_{rp}, \bar{k}_{sp}$ are the average values of inner and external mesh stiffness. S is the amplitude ratio valuing 0.2. γ_{sr} is phase difference between inner mesh and external mesh in planetary transmission, which values 0 in case that planet teeth number is even and values π in case of odd. ω_{mi} is mesh frequency in the i th stage. The expression in case of fixed ring gear is Eq. (2.3). ω_{si} and ω_{ci} are rotational speeds of sun gear and carrier in the i th stage. Z_s and Z_r are the numbers of sun gear teeth and ring gear teeth. ψ_i is planetary phase represented as

$$\psi_i = 2\pi(i - 1)/N \quad (2.4)$$

where $N = 4$ for the 1st stage; $N = 3$ for the 2nd stage; $i = 1, \dots, N$.

2.1.4 Transmission Error Excitation

The component errors affecting load-sharing performance considered in this paper can be classified as tooth frequency error, shaft-frequency error, and constant error by frequencies which are shown in Table 2.1. θ_{ri} and θ_{si} are presented as Eq. (2.5). α_i is transverse pressure angle of the ring and sun.

$$\theta_{r,si} = \alpha_i \pm \psi_i \quad (2.5)$$

Equivalent accumulated errors are presented in Eqs. (2.6) and (2.7)

$$e_{spi} = e_{Es} + e_{As} + e_{sEpi} + e_{sApi} + e_{\mu sp} + e_{es} + e_{epi} \quad (2.6)$$

$$e_{rpi} = e_{Er} + e_{Ar} + e_{rEpi} + e_{rApi} + e_{\mu rp} + e_{er} + e_{epi} \quad (2.7)$$

where the errors are defined and listed in Table 2.1.

Table 2.1 Error definitions

Part name	Error name	Symbol	Equivalent error
Carrier	Eccentricity error	(E_c, β_c)	$e_{Ec} = -E_c \cos(y_i - b_c)$ (ξ_p -axis component) $e_{Ec} = -E_c \sin(b_c - y_i)$ (η_p -axis component)
	Planet shaft hole tangential error	(E_τ)	$e_{Et} = -E_\tau$
Sun	Manufacturing error	(E_s, β_s)	$e_{Es} = E_s \sin[\theta_{si} + \beta_s - \omega_s t]$
	Fixing error	(A_s, γ_s)	$e_{As} = A_s \sin(\theta_{si} + \gamma_s)$
	Tooth profile error of external mesh	(μ_{sp})	$e_{\mu sp} = \mu_{sp} \sin(\omega_m t + Z_s \psi_i)$
	Thickness deviation	(ε_s)	$e_{\varepsilon s} = -\varepsilon_s$
Planet	Manufacturing error	(E_{pi}, β_{pi})	$e_{sEpi} = -E_{pi} \sin[\beta_{pi} + \alpha_{sp} + \omega_p t]$ (external mesh)
			$e_{rEpi} = -E_{pi} \sin[\beta_{pi} - \alpha_{rp} + \omega_p t]$ (inner mesh)
	Fixing error	(A_{pi}, γ_{pi})	$e_{sApi} = -A_{pi} \sin(\gamma_{pi} + \alpha_{sp})$ (external mesh)
			$e_{rApi} = -A_{pi} \sin(\gamma_{pi} - \alpha_{rp})$ (inner mesh)
	Thickness deviation	(ε_{pi})	$e_{\varepsilon pi} = -\varepsilon_{pi}$
Ring	Manufacturing error	(E_r, β_r)	$e_{Er} = -E_r \sin(\theta_{ri} - \beta_r)$
	Fixing error	(A_r, γ_r)	$e_{Ar} = -A_r \sin(\theta_{ri} - \beta_r)$
	Tooth profile error of inner mesh	(ε_r)	$e_{\mu rp} = \mu_{rp} \sin(\omega_m t + Z_r \psi_i + \gamma_{sp})$
	Thickness deviation	(ε_r)	$e_{\varepsilon r} = -\varepsilon_r$

2.1.5 Mesh Damping in the Planetary Train

The damping coefficient of the gear tooth mesh is as follows [7]

$$c_{rp,sp} = 2\xi_g \sqrt{\frac{\bar{k}_{rp,sp} m_1 m_2}{m_1 + m_2}} \quad (2.8)$$

where ξ_g is damping ratio of tooth mesh, which is between 0.03 and 0.17. $\bar{k}_{rp,sp}$ is the average values of mesh stiffness. m_1 and m_2 represent the mass of driving and driven gears, respectively.

2.1.6 Governing Equations

Due to the use of helical gears in the transmission system, torsional deflection, radial deflection, and axial deflection of each gear should be taken into account in the model. Considering the influence of torsional deflection, radial deflection, and axial deflection of planet carrier, it totally has 60 degrees of freedom. The dynamics equations of the system are as follows.

Dynamic equations in the 1st stage are represented in Eqs. (2.9)–(2.12).

$$\left\{ \begin{array}{l} (I_{c1}/r_{c1}^2)\ddot{u}_{c1} + C_{oc1}\dot{u}_{c1} + k_{oc1}u_{c1} + \sum_{i=1}^4 (k_{p1}\delta_{c\eta 1i} + C_{p1}\dot{\delta}_{c\eta 1i}) = T_{c1}/r_{c1} \\ m_{c1}(\ddot{x}_{c1} - 2\omega_{c1}\dot{y}_{c1} - \omega_{c1}^2 x_{c1}) + C_{c1}(\dot{x}_{c1} - \omega_{c1}y_{c1}) + k_{c1}x_{c1} \\ - k_{p1} \sum_{i=1}^4 (\delta_{c\xi 1i} \cos \psi_{1i} + \delta_{c\eta 1i} \sin \psi_{1i}) \\ - C_{p1} \sum_{i=1}^4 (\dot{\delta}_{c\xi 1i} \cos \psi_{1i} + \dot{\delta}_{c\eta 1i} \sin \psi_{1i}) = 0 \\ m_{c1}(\ddot{y}_{c1} + 2\omega_{c1}\dot{x}_{c1} - \omega_{c1}^2 y_{c1}) + C_{c1}(\dot{y}_{c1} + \omega_{c1}x_{c1}) + k_{c1}y_{c1} \\ - k_{p1} \sum_{i=1}^4 (\delta_{c\xi 1i} \sin \psi_{1i} + \delta_{c\eta 1i} \cos \psi_{1i}) \\ - C_{p1} \sum_{i=1}^4 (\dot{\delta}_{c\xi 1i} \sin \psi_{1i} + \dot{\delta}_{c\eta 1i} \cos \psi_{1i}) = 0 \\ m_{c1}\ddot{z}_{c1} + C_{c1}\dot{z}_{c1} + k_{c1}z_{c1} - k_{p1} \sum_{i=1}^4 \delta_{c\xi 1i} - C_{p1} \sum_{i=1}^4 \dot{\delta}_{c\xi 1i} = 0 \end{array} \right. \quad (2.9)$$

$$\left\{ \begin{array}{l} (I_{p1}/r_{bp1}^2)\ddot{u}_{p1i} - [k_{rp1i}(t)f(\delta_{rp1i}, b_{r1}) + C_{rp1i}\dot{\delta}_{rp1i}] \cos \beta_{b1} \\ + [k_{sp1i}(t)f(\delta_{sp1i}, b_{s1}) + C_{sp1i}\dot{\delta}_{sp1i}] \cos \beta_{b1} = 0 \\ m_{p1}(\ddot{\xi}_{p1i} - 2\omega_{c1}\dot{\eta}_{p1i} - \omega_{c1}^2 \xi_{p1i}) + C_{p1}(\dot{\delta}_{c\xi 1i} - \omega_{c1}\delta_{c\xi 1i}) + k_{p1}\delta_{c\xi 1i} \\ + [k_{rp1i}(t)f(\delta_{rp1i}, b_{r1}) + C_{rp1i}\dot{\delta}_{rp1i}] \sin \alpha_{t1} \cos \beta_{b1} - [k_{sp1i}(t)f(\delta_{sp1i}, b_{s1}) \\ + C_{sp1i}\dot{\delta}_{sp1i}] \sin \alpha_{t1} \cos \beta_{b1} = 0 \\ m_{p1}(\ddot{\eta}_{p1i} + 2\omega_{c1}\dot{\xi}_{p1i} - \omega_{c1}^2 \eta_{p1i}) + C_{p1}(\dot{\delta}_{c\eta 1i} + \omega_{c1}\delta_{c\eta 1i}) + k_{p1}\delta_{c\eta 1i} \\ - [k_{rp1i}(t)f(\delta_{rp1i}, b_{r1}) + C_{rp1i}\dot{\delta}_{rp1i}] \cos \alpha_{t1} \cos \beta_{b1} - [k_{sp1i}(t)f(\delta_{sp1i}, b_{s1}) \\ + C_{sp1i}\dot{\delta}_{sp1i}] \cos \alpha_{t1} \cos \beta_{b1} = 0 \\ m_{p1}\ddot{\xi}_{p1i} + C_{p1}\dot{\delta}_{c\xi 1i} + k_{p1}\delta_{c\xi 1i} + [k_{rp1i}(t)f(\delta_{rp1i}, b_{r1}) + C_{rp1i}\dot{\delta}_{rp1i}] \sin \beta_{b1} \\ - [k_{sp1i}(t)f(\delta_{sp1i}, b_{s1}) + C_{sp1i}\dot{\delta}_{sp1i}] \sin \beta_{b1} = 0 \end{array} \right. \quad (2.10)$$

$$\left\{ \begin{array}{l} (I_{r1}/r_{br1}^2)\ddot{u}_{r1} + C_{or1}\dot{u}_{r1} + k_{or1}u_{r1} \\ + \sum_{i=1}^4 [k_{rp1i}(t)f(\delta_{rp1i}, b_{r1}) + C_{rp1i}\dot{\delta}_{rp1i}] \cos \beta_{b1} = 0 \\ m_{r1}(\ddot{x}_{r1} - 2\omega_{c1}\dot{y}_{r1} - \omega_{c1}^2 x_{r1}) + C_{r1}(\dot{x}_{r1} - \omega_{c1}y_{r1}) + k_{r1}x_{r1} \\ - \sum_{i=1}^4 [k_{rp1i}(t)f(\delta_{rp1i}, b_{r1}) + C_{rp1i}\dot{\delta}_{rp1i}] \sin \theta_{r1i} \cos \beta_{b1} = 0 \\ m_{r1}(\ddot{y}_{r1} + 2\omega_{c1}\dot{x}_{r1} - \omega_{c1}^2 y_{r1}) + C_{r1}(\dot{y}_{r1} + \omega_{c1}x_{r1}) + k_{r1}y_{r1} \\ + \sum_{i=1}^4 [k_{rp1i}(t)f(\delta_{rp1i}, b_{r1}) + C_{rp1i}\dot{\delta}_{rp1i}] \cos \theta_{r1i} \cos \beta_{b1} = 0 \\ m_{r1}\ddot{z}_{r1} + C_{r1}\dot{z}_{r1} + k_{r1}z_{r1} \\ - \sum_{i=1}^4 [k_{rp1i}(t)f(\delta_{rp1i}, b_{r1}) + C_{rp1i}\dot{\delta}_{rp1i}] \sin \beta_{b1} = 0 \end{array} \right. \quad (2.11)$$

$$\left\{ \begin{array}{l} (I_{s1}/r_{bs1}^2)\ddot{u}_{s1} + C_{os1}\dot{u}_{s1} + k_{osc}(u_{s1} - \frac{r_{bs1}}{r_{c2}}u_{c2}) \\ - \sum_{i=1}^4 [k_{sp1i}(t)f(\delta_{sp1i}, b_{s1}) + C_{sp1i}\dot{\delta}_{sp1i}] \cos \beta_{b1} = 0 \\ m_{s1}(\ddot{x}_{s1} - 2\omega_{c1}\dot{y}_{s1} - \omega_{c1}^2 x_{s1}) + C_{s1}(\dot{x}_{s1} - \omega_{c1}y_{s1}) + k_{s1}x_{s1} \\ + \sum_{i=1}^4 [k_{sp1i}(t)f(\delta_{sp1i}, b_{s1}) + C_{sp1i}\dot{\delta}_{sp1i}] \sin \theta_{s1i} \cos \beta_{b1} = 0 \\ m_{s1}(\ddot{y}_{s1} + 2\omega_{c1}\dot{x}_{s1} - \omega_{c1}^2 y_{s1}) + C_{s1}(\dot{y}_{s1} + \omega_{c1}x_{s1}) + k_{s1}y_{s1} \\ + \sum_{i=1}^4 [k_{sp1i}(t)f(\delta_{sp1i}, b_{s1}) + C_{sp1i}\dot{\delta}_{sp1i}] \cos \theta_{s1i} \cos \beta_{b1} = 0 \\ m_{s1}\ddot{z}_{s1} + C_{s1}\dot{z}_{s1} + k_{s1}z_{s1} + \sum_{i=1}^4 (k_{sp1i}(t)f(\delta_{sp1i}, b_{s1}) + C_{sp1i}\dot{\delta}_{sp1i}) \sin \beta_{b1} = 0 \end{array} \right. \quad (2.12)$$

Dynamic equations in the 2nd stage are represented in Eqs. (2.13)–(2.16).

$$\left\{ \begin{array}{l} (I_{c2}/r_{c2}^2)\ddot{u}_{c2} + C_{oc2}\dot{u}_{c2} + k_{osc}(u_{c2} - \frac{r_{c2}}{r_{bs2}}u_{s1}) + \sum_{i=1}^3 (k_{p2}\delta_{c\eta2i} + C_{p2}\dot{\delta}_{c\eta2i}) = 0 \\ m_{c2}(\ddot{x}_{c2} - 2\omega_{c2}\dot{y}_{c2} - \omega_{c2}^2 x_{c2}) + C_{c2}(\dot{x}_{c2} - \omega_{c2}y_{c2}) + k_{c2}x_{c2} \\ - k_{p2} \sum_{i=1}^3 (\delta_{c\xi2i} \cos \psi_{2i} + \delta_{c\eta2i} \sin \psi_{2i}) \\ - C_{p2} \sum_{i=1}^3 (\dot{\delta}_{c\xi2i} \cos \psi_{2i} + \dot{\delta}_{c\eta2i} \sin \psi_{2i}) = 0 \\ m_{c2}(\ddot{y}_{c2} + 2\omega_{c2}\dot{x}_{c2} - \omega_{c2}^2 y_{c2}) + C_{c2}(\dot{y}_{c2} + \omega_{c2}x_{c2}) + k_{c2}y_{c2} \\ - k_{p2} \sum_{i=1}^3 (\delta_{c\xi2i} \sin \psi_{2i} + \delta_{c\eta2i} \cos \psi_{2i}) \\ - C_{p2} \sum_{i=1}^3 (\dot{\delta}_{c\xi2i} \sin \psi_{2i} + \dot{\delta}_{c\eta2i} \cos \psi_{2i}) = 0 \\ m_{c2}\ddot{z}_{c2} + C_{c2}\dot{z}_{c2} + k_{c2}z_{c2} - k_{p2} \sum_{i=1}^3 \delta_{c\xi2i} - C_{p2} \sum_{i=1}^3 \dot{\delta}_{c\xi2i} = 0 \end{array} \right. \quad (2.13)$$

$$\begin{cases}
(I_{p2}/r_{bp2}^2)\ddot{u}_{p2i} - [k_{rp2i}(t)f(\delta_{rp2i}, b_{r2}) + C_{rp2i}\dot{\delta}_{rp2i}] \cos \beta_{b2} \\
+ [k_{sp2i}(t)f(\delta_{sp2i}, b_{s2}) + C_{sp2i}\dot{\delta}_{sp2i}] \cos \beta_{b2} = 0 \\
m_{p2}(\ddot{\xi}_{p2i} - 2\omega_{c2}\dot{\eta}_{p2i} - \omega_{c2}^2\xi_{p2i}) + C_{p2}(\dot{\delta}_{c\xi2i} - \omega_{c2}\delta_{c\xi2i}) + k_{p2}\delta_{c\xi2i} \\
+ [k_{rp2i}(t)f(\delta_{rp2i}, b_{r2}) + C_{rp2i}\dot{\delta}_{rp2i}] \sin \alpha_{t2} \cos \beta_{b2} - [k_{sp2i}(t)f(\delta_{sp2i}, b_{s2}) \\
+ C_{sp2i}\dot{\delta}_{sp2i}] \sin \alpha_{t2} \cos \beta_{b2} = 0 \\
m_{p2}(\ddot{\eta}_{p2i} + 2\omega_{c2}\dot{\xi}_{p2i} - \omega_{c2}^2\eta_{p2i}) + C_{p2}(\dot{\delta}_{c\eta2i} + \omega_{c2}\delta_{c\eta2i}) + k_{p2}\delta_{c\eta2i} \\
- [k_{rp2i}(t)f(\delta_{rp2i}, b_{r2}) + C_{rp2i}\dot{\delta}_{rp2i}] \cos \alpha_{t2} \cos \beta_{b2} - [k_{sp2i}(t)f(\delta_{sp2i}, b_{s2}) \\
+ C_{sp2i}\dot{\delta}_{sp2i}] \cos \alpha_{t2} \cos \beta_{b2} = 0 \\
m_{p2}\ddot{\xi}_{p2i} + C_{p2}\dot{\delta}_{c\xi2i} + k_{p2}\delta_{c\xi2i} + [k_{rp2i}(t)f(\delta_{rp2i}, b_{r2}) + C_{rp2i}\dot{\delta}_{rp2i}] \sin \beta_{b2} \\
- [k_{sp2i}(t)f(\delta_{sp2i}, b_{s2}) + C_{sp2i}\dot{\delta}_{sp2i}] \sin \beta_{b2} = 0
\end{cases} \quad (2.14)$$

$$\begin{cases}
(I_{r2}/r_{br2}^2)\ddot{u}_{r2} + C_{or2}\dot{u}_{r2} + k_{or2}u_{r2} \\
+ \sum_{i=1}^3 [k_{rp2i}(t)f(\delta_{rp2i}, b_{r2}) + C_{rp2i}\dot{\delta}_{rp2i}] \cos \beta_{b2} = 0 \\
m_{r2}(\ddot{x}_{r2} - 2\omega_{c2}\dot{y}_{r2} - \omega_{c2}^2x_{r2}) + C_{r2}(\dot{x}_{r2} - \omega_{c2}y_{r2}) + k_{r2}x_{r2} \\
- \sum_{i=1}^3 [k_{rp2i}(t)f(\delta_{rp2i}, b_{r2}) + C_{rp2i}\dot{\delta}_{rp2i}] \sin \theta_{r2i} \cos \beta_{b2} = 0 \\
m_{r2}(\ddot{y}_{r2} + 2\omega_{c2}\dot{x}_{r2} - \omega_{c2}^2y_{r2}) + C_{r2}(\dot{y}_{r2} + \omega_{c2}x_{r2}) + k_{r2}y_{r2} \\
+ \sum_{i=1}^3 [k_{rp2i}(t)f(\delta_{rp2i}, b_{r2}) + C_{rp2i}\dot{\delta}_{rp2i}] \cos \theta_{r2i} \cos \beta_{b2} = 0 \\
m_{r2}\ddot{z}_{r2} + C_{r2}\dot{z}_{r2} + k_{r2}z_{r2} \\
- \sum_{i=1}^3 [k_{rp2i}(t)f(\delta_{rp2i}, b_{r2}) + C_{rp2i}\dot{\delta}_{rp2i}] \sin \beta_{b2} = 0
\end{cases} \quad (2.15)$$

$$\begin{cases}
(I_{s2}/r_{bs2}^2)\ddot{u}_{s2} - \sum_{i=1}^3 [k_{sp2i}(t)f(\delta_{sp2i}, b_{s2}) + C_{sp2i}\dot{\delta}_{sp2i}] \cos \beta_{b2} \\
+ C_{7s2}\left(\dot{\theta}_{s2} - \frac{r_{s2}}{r_7}\dot{\theta}_7\right) + K_{7s2}\left(\theta_{s2} - \frac{r_{s2}}{r_7}\theta_7\right) = 0 \\
m_{s2}(\ddot{x}_{s2} - 2\omega_{c2}\dot{y}_{s2} - \omega_{c2}^2x_{s2}) + C_{s2}(\dot{x}_{s2} - \omega_{c2}y_{s2}) + k_{s2}x_{s2} \\
+ \sum_{i=1}^3 [k_{sp2i}(t)f(\delta_{sp2i}, b_{s2}) + C_{sp2i}\dot{\delta}_{sp2i}] \sin \theta_{s2i} \cos \beta_{b2} = 0 \\
m_{s2}(\ddot{y}_{s2} + 2\omega_{c2}\dot{x}_{s2} - \omega_{c2}^2y_{s2}) + C_{s2}(\dot{y}_{s2} + \omega_{c2}x_{s2}) + k_{s2}y_{s2} \\
+ \sum_{i=1}^3 [k_{sp2i}(t)f(\delta_{sp2i}, b_{s2}) + C_{sp2i}\dot{\delta}_{sp2i}] \cos \theta_{s2i} \cos \beta_{b2} = 0 \\
m_{s2}\ddot{z}_{s2} + C_{s2}\dot{z}_{s2} + k_{s2}z_{s2} + \sum_{i=1}^3 (k_{sp2i}(t)f(\delta_{sp2i}, b_{s2}) + C_{sp2i}\dot{\delta}_{sp2i}) \sin \beta_{b2} = 0
\end{cases} \quad (2.16)$$

Dynamics equations of the high-speed stage are shown as follows.

$$\begin{cases} (I_7/r_7^2)\ddot{\theta}_7 + (C_{78}\dot{x}_{78} + k_{78}x_{78})r_{b7} \cos \beta_{b3} + C_{7s_2}(\dot{\theta}_7 - \frac{r_7}{r_{s2}}\dot{\theta}_{s2}) + k_{7s_2}(\theta_7 - \frac{r_7}{r_{s2}}\theta_{s2}) = 0 \\ m_7\ddot{x}_7 + (C_{78}\dot{x}_{78} + k_{78}x_{78}) \cos \beta_{b3} \sin \alpha_{t78} + C_{7x}\dot{x}_7 + k_{7x}x_7 = 0 \\ m_7\ddot{y}_7 + (C_{78}\dot{x}_{78} + k_{78}x_{78}) \cos \beta_{b3} \cos \alpha_{t78} + C_{7y}\dot{y}_7 + k_{7y}y_7 = 0 \\ m_7\ddot{z}_7 + (C_{78}\dot{x}_{78} + k_{78}x_{78}) \sin \beta_{b3} + C_{7x}\dot{z}_7 + k_{7z}z_7 = 0 \end{cases} \quad (2.17)$$

$$\begin{cases} (I_8/r_8^2)\ddot{\theta}_8 - (C_{78}\dot{x}_{78} + k_{78}x_{78})r_{b8} \cos \beta_{b78} = -T_{out}/r_8 \\ m_8\ddot{x}_8 - (C_{78}\dot{x}_{78} + k_{78}x_{78}) \cos \beta_{b78} \sin \alpha_{t78} + c_{8x}\dot{x}_8 + k_{8x}x_8 = 0 \\ m_8\ddot{y}_8 - (C_{78}\dot{x}_{78} + k_{78}x_{78}) \cos \beta_{b78} \cos \alpha_{t78} + c_{8y}\dot{y}_8 + k_{8y}y_8 = 0 \\ m_8\ddot{z}_8 - (C_{78}\dot{x}_{78} + k_{78}x_{78}) \sin \beta_{b78} + c_{8z}\dot{z}_8 + k_{8z}z_8 = 0 \end{cases} \quad (2.18)$$

In Eqs. (2.9)–(2.18), $\delta_{c\xi ij}$, $\delta_{c\eta ij}$, $\delta_{c\zeta ij}$ are relative displacements between carrier and planet gear along ξ, η, ζ axis, respectively, in rotational coordinate system which is presented in Eq. (2.19).

$$\begin{cases} \delta_{c\xi ij} = \xi_{pij} - x_{ci} \cos \psi_{ij} - y_{ci} \sin \psi_{ij} \\ \delta_{c\eta ij} = \eta_{pij} + x_{ci} \sin \psi_{ij} - y_{ci} \cos \psi_{ij} + u_{ci} \\ \delta_{c\zeta ij} = \xi_{pij} - z_{ci} \end{cases} \quad (2.19)$$

where δ_{rpij} is relative displacement along the meshing line between ring gear and planet which is presented in Eq. (2.20).

$$\delta_{rpij} = (u_{ri} - u_{pij} - x_{ri} \sin \theta_{rij} + y_{ri} \cos \theta_{rij} + \xi_{pij} \sin \alpha_{ti} - \eta_{pij} \cos \alpha_{ti}) \cos \beta_{bi} + (\xi_{pij} - z_{ri}) \sin \beta_{bi} + e_{rpij}(t) \quad (2.20)$$

where δ_{spij} is relative displacement along the meshing line between sun gear and planet which is presented in Eq. (2.21).

$$\delta_{spij} = (-u_{si} + u_{pij} + x_{si} \sin \theta_{sij} + y_{si} \cos \theta_{sij} - \xi_{pij} \sin \alpha_{ti} - \eta_{pij} \cos \alpha_{ti}) \cos \beta_{bi} + (z_{si} - \xi_{pij}) \sin \beta_{bi} + e_{spij}(t) \quad (2.21)$$

The subscript i is the stage number and j is the planet number. Their relationship is presented in Eq. (2.22)

$$i = \begin{cases} 1, & j = 1, \dots, 4 \\ 2, & j = 1, \dots, 3 \end{cases} \quad (2.22)$$

The total relative displacement along the meshing line of high-speed stage is

$$x_{78} = [(r_{b7}\theta_7 - r_{b8}\theta_8) + (y_7 - y_8) \cos \alpha_{t78} + (x_7 - x_8) \sin \alpha_{t78}] \cos \beta_{b78} - e_{78} \quad (2.23)$$

where r_{b7} , r_{b8} , α_{t78} , β_{b78} , and e_{78} , respectively, represent the base circle radius of wheel, the base circle radius of pinion, the working transverse pressure angle, the base helix angle, and the dynamic transmission error of high-speed stage.

The other terms are described as follows.

I_{c1} , I_{r1} , I_{p1} , I_{s1} are rotational inertia of carrier, ring gear, planets, and sun gear in the 1st stage.

I_{c2} , I_{r2} , I_{p2} , I_{s2} are rotational inertia of carrier, ring gear, planets, and sun gear in the 2nd stage.

m_{c1} , m_{r1} , m_{p1} , m_{s1} are mass of carrier, ring gear, planets, and sun gear in the 1st stage.

m_{c2} , m_{r2} , m_{p2} , m_{s2} are mass of carrier, ring gear, planets, and sun gear in the 2nd stage.

k_{oc1} , k_{or1} are torsional stiffness of carrier and ring gear in the 1st stage.

k_{oc2} , k_{or2} are torsional stiffness of carrier and ring gear in the 2nd stage.

k_{osc} is coupled torsional stiffness of the shaft connecting two stages in series.

k_{c1} , k_{r1} , k_{p1} , k_{s1} are supporting stiffness of carrier, ring gear, planets, and sun gear in the 1st stage.

k_{c2} , k_{r2} , k_{p2} , k_{s2} are supporting stiffness of carrier, ring gear, planets, and sun gear in the 2nd stage.

k_{rp1i} is mesh stiffness between ring gear and the i th planet gear in the 1st stage.

k_{sp1i} is mesh stiffness between sun gear and the i th planet gear in the 1st stage.

k_{rp2i} is mesh stiffness between ring gear and the i th planet gear in the 2nd stage.

k_{sp2i} is mesh stiffness between sun gear and the i th planet gear in the 2nd stage.

C_{rp1i} is mesh damping between ring gear and the i th planet gear in the 1st stage.

C_{sp1i} is mesh damping between sun gear and the i th planet gear in the 1st stage.

C_{rp2i} is mesh damping between ring gear and the i th planet gear in the 2nd stage.

C_{sp2i} is mesh damping between sun gear and the i th planet gear in the 2nd stage.

C_{c1} , C_{r1} , C_{p1} , C_{s1} are supporting damping of carrier, ring gear, planets, and sun gear in the 1st stage.

C_{c2} , C_{r2} , C_{p2} , C_{s2} are supporting damping of carrier, ring gear, planets, and sun gear in the 2nd stage.

β_{b1} , β_{b2} are base helix angles of two stages.

α_{t1} , α_{t2} are the working transverse pressure angle of two stages.

θ_{s1i} , θ_{r1i} are, respectively, the external and inner mesh phase angles in the 1st stage.

θ_{s2i} , θ_{r2i} are, respectively, the external and inner mesh phase angles in the 2nd stage.

k_{7s2} is the torsional stiffness between the 2nd-stage sun gear and high-speed stage wheel.

C_{7s2} is the torsional damping between the 2nd-stage sun gear and high-speed stage wheel.

k_{7x} , k_{7y} , k_{7z} are the supporting stiffness of wheel in x , y , z directions, respectively.

k_{8x} , k_{8y} , k_{8z} are the supporting stiffness of pinion in x , y , z directions, respectively.

C_{7x} , C_{7y} , C_{7z} are the supporting damping of wheel in x , y , z directions, respectively.

C_{8x} , C_{8y} , C_{8z} are the supporting damping of pinion in x , y , z directions, respectively.

Assembling the equations in matrix form, the global equation of motion of the planetary gearing system is written as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F} \quad (2.24)$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, and $\mathbf{F}$, respectively, represent mass matrix, damping matrix, stiffness matrix, and exciting force vector. $\mathbf{x}$, $\dot{\mathbf{x}}$, $\ddot{\mathbf{x}}$ are the vectors of vibration displacement.

2.2 Torsional Dynamics of the Gearbox

The parameters of the above planetary system in this book are listed in Table 2.2.

According to Eqs. (2.9)–(2.18), the obtained nature frequencies and mode shapes of the system are shown in Table 2.3.

The modal energy distribution in each node of the system is obtained as shown in Table 2.4. Nodes 1–7 denote the planet carrier, 4 planet gears, annular gear, and sun gear in the first stage separately. Nodes 8–13 denote the planet carrier, 3 planet gears, annular gear, and sun gear in the second stage separately. Nodes 14 and 15 are output parallel gears.

2.3 Load Factors of Gear-Rotor System in Gearbox

2.3.1 Analysis of System Excitation and Damping

The equation of the two-parameter Weibull distribution is used to describe the wind speed [8, 9]:

$$f_v(v) = \left(\frac{k}{c}\right) \left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^k}, \quad 0 < v < \infty \quad (2.25)$$

Table 2.2 The parameters of the planetary gear system

	z	$m_n(\text{mm})$	β (°)	α (°)	B (mm)	m (kg)
Sun gear	29 (1st)	18	20	7.17	470(1st)	713 (1st)
	28 (2nd)	(1st)	(1st)	(1st)	260(2nd)	139 (2nd)
Ring gear	99 (1st)	12	20	8.67		555 (1st)
	122 (2nd)	(2nd)	(2nd)	(2nd)		265 (2nd)
Planet gear	35 (1st)					4419 (1st)
	47 (2nd)					1537 (2nd)

Table 2.3 Nature frequencies and mode shapes of the planetary system

Order	Frequency (Hz)	Mode shape descriptions
1	27.8	Vibration of the output parallel gear
2	67.3	Vibration of the planet gear in the 1st stage
3	114.1	Vibration of the planet gear in the 2nd stage
4	134.2	Vibration of the annular gear in the 1st stage
5	144.7	Vibrations of the sun gear in the 2nd stage and the gear in the high-speed stage
6	189.0	Vibration of the planet carrier in the 1st stage
7	194.0	Vibration of the sun gear in the 1st stage and the planet carrier in the 2nd stage
8	194.3	Vibration of the annular gear in the 2nd sage
9	235.2	Vibration of the planet carrier in the 1st and 2nd stages
10	253.9	Vibration of the planet carrier in the 2nd stage
11	309.1	Vibration of the planet carrier in the 1st stage
12	332.4	Vibration of the planet carrier in the 1st stage

where v , k , and c represent the wind speed, the shape parameter, and the scale parameter separately, and the unit is m/s.

The mathematic expectation and variance of the wind speed can be determined based on Eq. (2.25). The expectation of the wind speed can be expressed as

$$E(v) = \bar{v} = \int_0^{+\infty} v \cdot f_v(v) dv = c \cdot \Gamma\left(1 + \frac{1}{k}\right) \quad (2.26)$$

The expression of the variance of wind speed is

$$D(v) = \int_0^{+\infty} [v - E(v)]^2 f_v(v) dv = c^2 \cdot \left\{ \Gamma\left(1 + \frac{2}{k}\right) - \left[\Gamma\left(1 + \frac{1}{k}\right) \right]^2 \right\} \quad (2.27)$$

where $\Gamma(x)$ is gamma function in Eqs. (2.26) and (2.27).

The parameters of wind turbine and wind speed are listed in Table 2.5.

According to Eqs. (2.26) and (2.27), the two parameters are determined, k is 5.5 and c is 11.9089. The time-domain model of the random wind based on the two-parameter Weibull distribution function is shown in Fig. 2.4 in 100 s.

Kinetic energy generated by the blade which is driven by natural wind per unit time is numerically equal to the output power of the blade, namely the input power of wind turbine gearbox. The relation between the input torque and the input

Table 2.4 The modal energy distributions of the planetary system of the first 12 orders

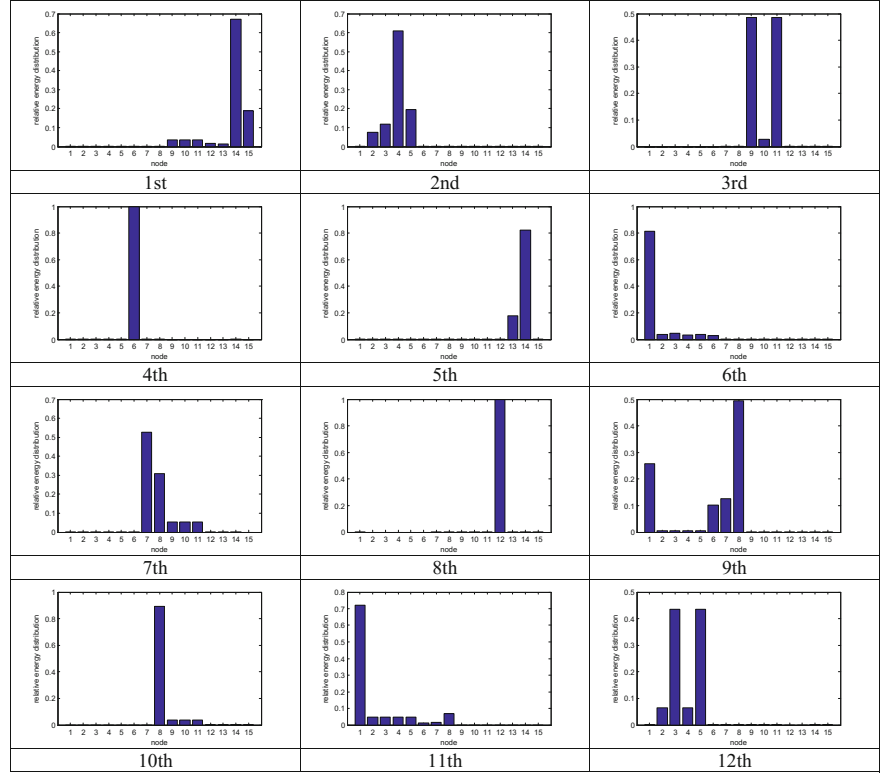
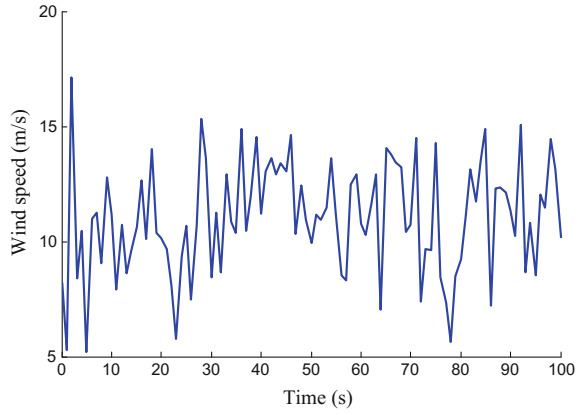


Table 2.5 Parameter values of the wind speeds

Parameters	Value
Rated design power P (kW)	3300
Rated working speed n_e (r/min)	13
Rated torque T_e (kNm)	2424.052
Rated wind speed V_e (m/s)	13.5
Radius of blade R (m)	52
Density of wind ρ (kg/m ³)	1.225
Average wind speed v_m (m/s)	11
Standard deviation σ_m (m/s)	2.27
Cut in speed $V_{\text{cut in}}$ (m/s)	3.5
Cut off speed $V_{\text{cut off}}$ (m/s)	25
Blade top speed ratio λ	9
Power coefficient C_p (λ)	0.485

Fig. 2.4 Simulation of the random wind speeds



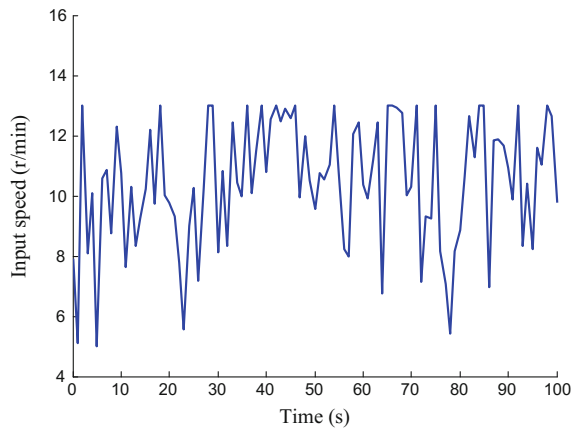
working speed of variable pitch can be obtained by the wind energy and blade top speed ratio. They are

$$T_{in} = \begin{cases} 0 & v < V_{cut\ in} \\ \frac{T_c}{V_e^2} \cdot v^2 & V_{cut\ in} \leq v \leq V_e \\ \frac{T_c}{V_e} & V_e \leq v \leq V_{cut\ off} \\ 0 & v > V_{cut\ off} \end{cases} \quad (2.28)$$

$$n_{in} = \begin{cases} 0 & v < V_{cut\ in} \\ \frac{n_e}{V_e} \cdot v & V_{cut\ in} \leq v \leq V_e \\ n_e & V_e \leq v \leq V_{cut\ off} \\ 0 & v > V_{cut\ off} \end{cases} \quad (2.29)$$

According to Eqs. (2.28) and (2.29), the simulated input working speeds are shown in Fig. 2.5.

Fig. 2.5 Simulation of the input working speeds



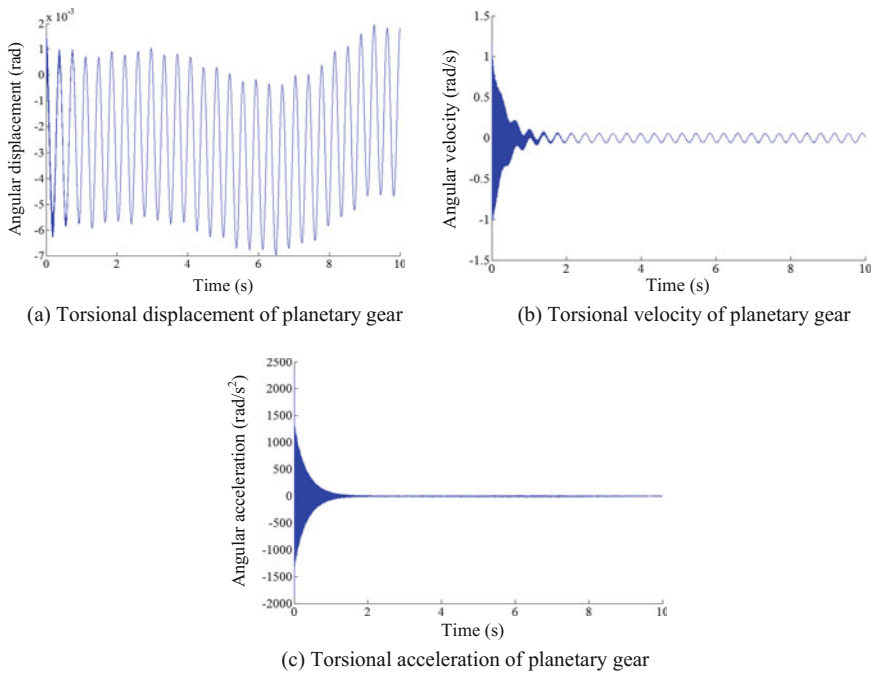


Fig. 2.6 Time-domain results of the planetary gear in the first-stage components. **a** Torsional displacement of planetary gear. **b** Torsional velocity of planetary gear. **c** Torsional acceleration of planetary gear

For the time-varying input load, the torsional displacement, velocity and acceleration of the sun gear and the planetary gear in the first-stage components are shown in Figs. 2.6 and 2.7, respectively. And the frequency spectrum results of torsional velocity and acceleration of planetary gear are shown in Figs. 2.8 and 2.9, respectively.

As shown in Figs. 2.6 and 2.7, the vibration displacement, velocity, and acceleration of the sun gear and planetary gear in the first-stage components along the torsional direction have same changing trends with the external load excitation. It mainly affects low-frequency vibration by the external load excitation while it affects the high-frequency vibration by the internal load excitation. If the input load is not stable, it will lead to relatively complex changing of the transverse vibration, and it has a very little influence on the vibration amplitude affected by the variable external load (Figs. 2.8 and 2.9).

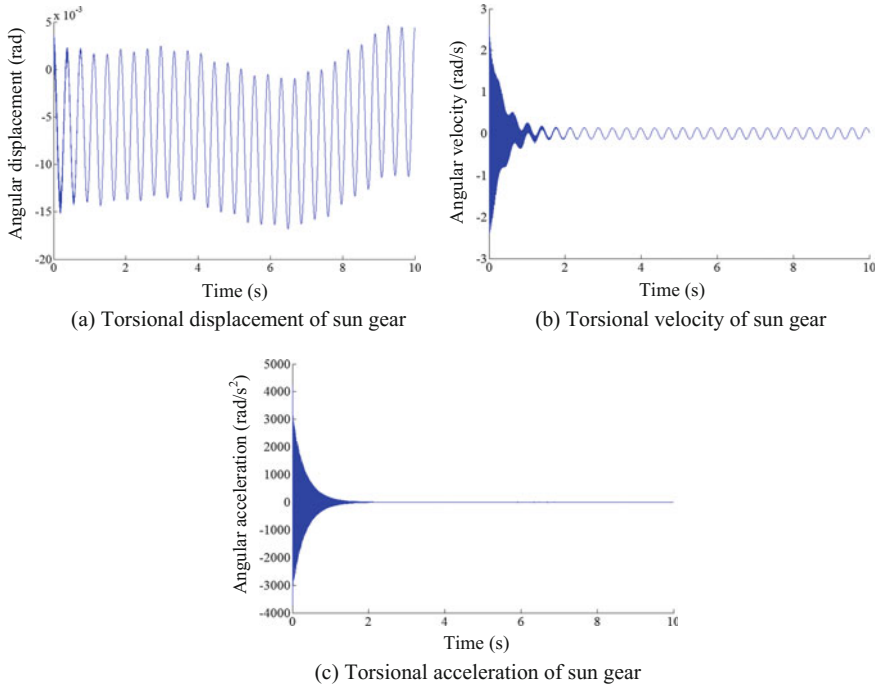


Fig. 2.7 Time-domain results of the sun gear in the first-stage components. **a** Torsional displacement of sun gear. **b** Torsional velocity of sun gear. **c** Torsional acceleration of sun gear

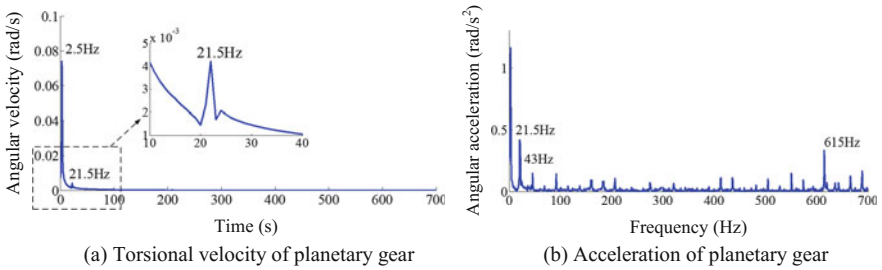


Fig. 2.8 Frequency spectrum results of torsional velocity and acceleration of planetary gear. **a** Torsional velocity of planetary gear. **b** Acceleration of planetary gear

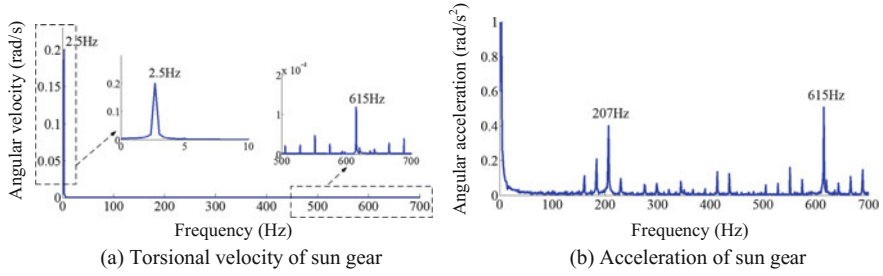


Fig. 2.9 Frequency spectrum results of torsional velocity and acceleration of sun gear. **a** Torsional velocity of sun gear. **b** Acceleration of sun gear

2.3.2 Load Factors of Gear-Rotor System

The dynamics Eq. (2.24) can be expressed as

$$M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + [K_m + \Delta K(t)] \times [\mathbf{x} + \mathbf{x}_s - \mathbf{e}(t)] = \mathbf{F}_s + \Delta \mathbf{F}(t) \quad (2.30)$$

where $\mathbf{x}_s$, $\mathbf{x}$, $\mathbf{F}_s$, K_m , $\Delta K(t)$, and $\Delta \mathbf{F}(t)$ represent the static relative displacement vector, the dynamic relative displacement vector, the static load, the average stiffness, the variable stiffness, and the variable part of external excitation separately.

Ignoring the smaller values, the expanded expression of Eq. (2.30) is rewritten as

$$M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + K_m\mathbf{x} = \Delta K(t)\mathbf{e}(t) + \Delta \mathbf{F}(t) \quad (2.31)$$

where $\Delta K(t)\mathbf{e}(t)$ is the additional dynamic load caused by internal excitation and $\Delta \mathbf{F}(t)$ is the additional dynamic load caused by external excitation. The ratio of the sum of the additional internal (external) dynamic load is the tangential load application factor (dynamic load factor).

(a) The application factor

K_A is defined as the application factor of the external factors of the gear meshing to the additional dynamic load. For the wind turbine transmission system, the additional external dynamic load is mainly caused by the variation of the wind load. The common method to obtain the factor relies on the look-up table method; dynamic analysis results are used to obtain more accurate result here. K_A is expressed as

$$K_A = (\mathbf{F}_\tau + \mathbf{F}_{ex}) / \mathbf{F}_\tau \quad (2.32)$$

where F_τ and $F_{ex}(F_{ex} = k_m x_i)$ represent the tangential load of the gear set and the additional external dynamic load, respectively.

(b) The dynamic load factor

K_V is defined as the dynamic factor considering the influence of the manufacture accuracy and the working speed to the additional dynamic load. It is the important measurement of the dynamic characteristics of the system, which shows the condition of maximum load in the moving process. In traditional design, dynamic load factor is determined by existing chart according to accuracy of gear and working speed. The expression of dynamic load factor which is obtained from comprehensive dynamic analysis is

$$K_V = (F_\tau + F_{in}) / F_\tau \quad (2.33)$$

In Eq. (2.33), F_{in} is additional internal dynamic load.

According to ISO 81400-4-2005: Wind turbines-Part 4 Design and Specification of Gearboxes, the life expectancy is 20 years under the circumstance of meeting design reliability requirement of 0.99. The application factor and the dynamic load factor are shown in Figs. 2.10, 2.11, 2.12, 2.13, 2.14, 2.15, 2.16, 2.17, 2.18, and 2.19 under the circumstance when system reliability requirement is 0.99.

The variation ranges of K_A and K_V of each gear pair, as shown in Figs. 2.18 and 2.19, have no obvious difference between the 1st and 2nd stages. K_A of the 1st-stage sun-planet and ring-planet gear pair is 1.10. K_A of the 2nd-stage sun-planet gear pair is 1.10. K_A of the 2nd-stage ring-planet gear pair is 1.12. K_A of the high-speed stage is 1.27, which is significantly increased. K_V of the 1st-stage sun-planet and

Fig. 2.10 K_A of the 1st-stage sun-planet gear pair

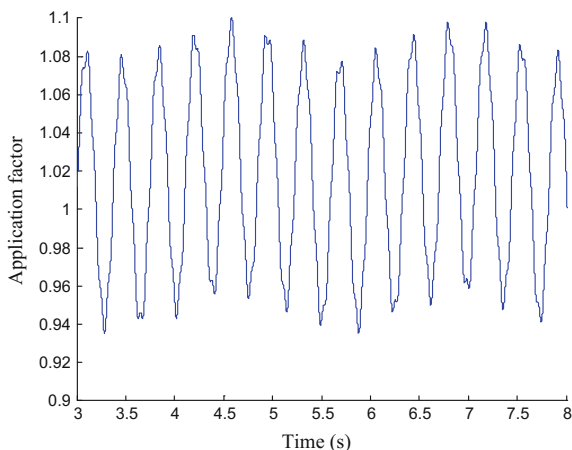


Fig. 2.11 K_V of the 1st-stage sun–planet gear pair

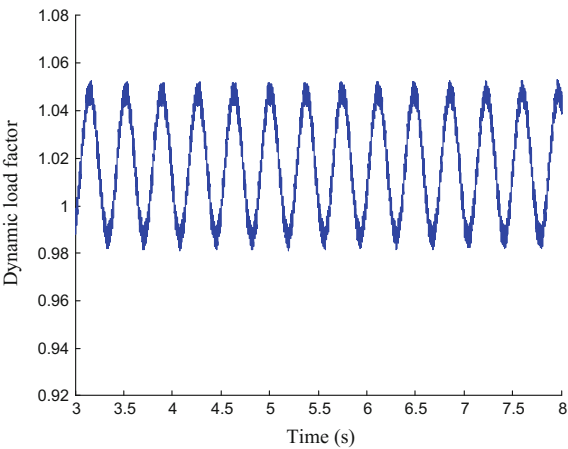


Fig. 2.12 K_A of the 1st-stage ring–planet gear pair

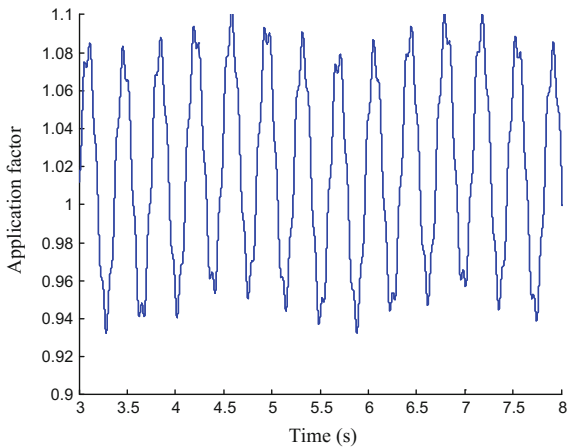


Fig. 2.13 K_V of the 1st-stage ring–planet gear pair

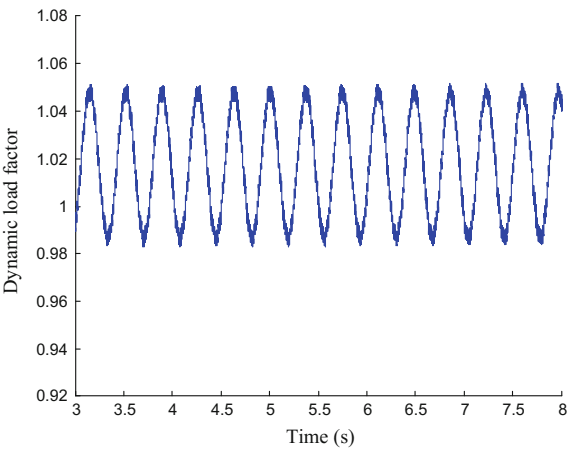


Fig. 2.14 K_A of the 2nd-stage sun-planet gear pair

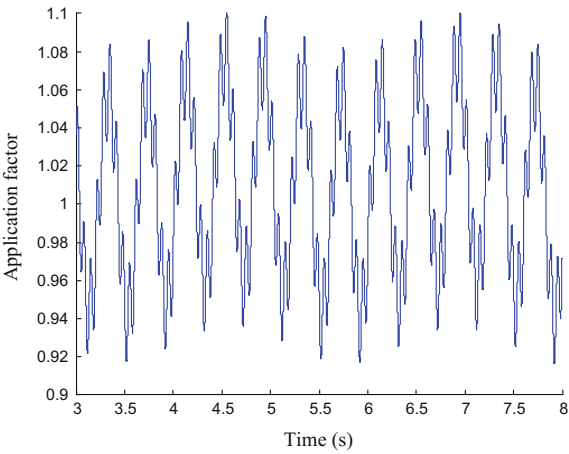


Fig. 2.15 K_V of the 2nd-stage sun-planet gear pair

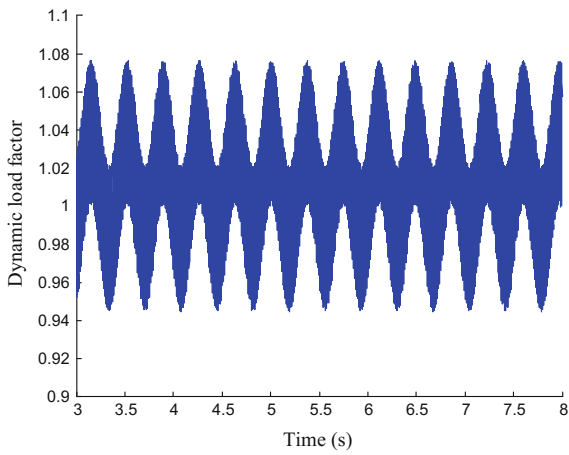


Fig. 2.16 K_A of the 2nd-stage ring-planet gear pair

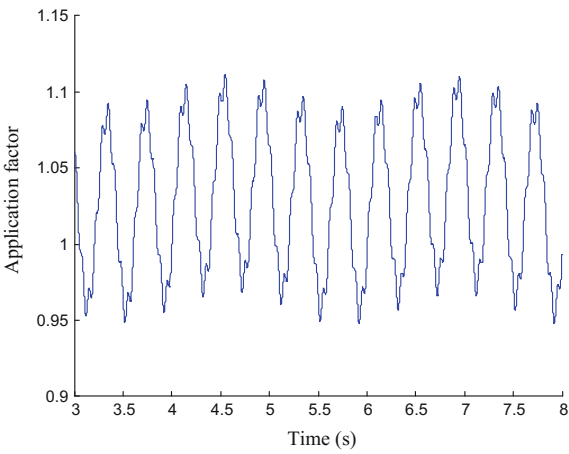


Fig. 2.17 K_V of the 2nd-stage ring-planet gear pair

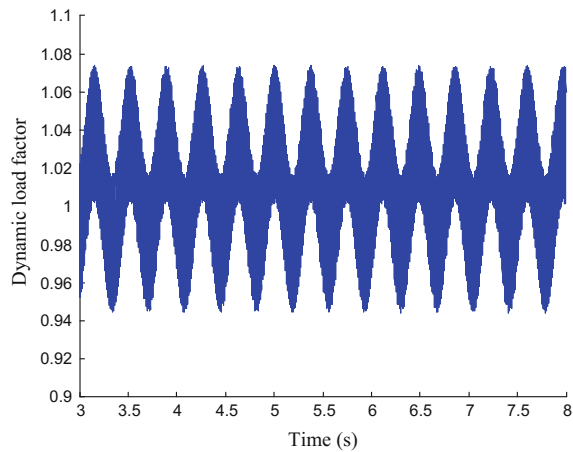


Fig. 2.18 K_A of high-speed stage

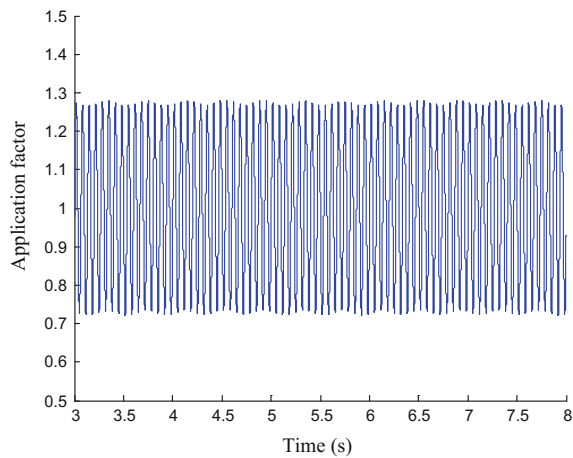
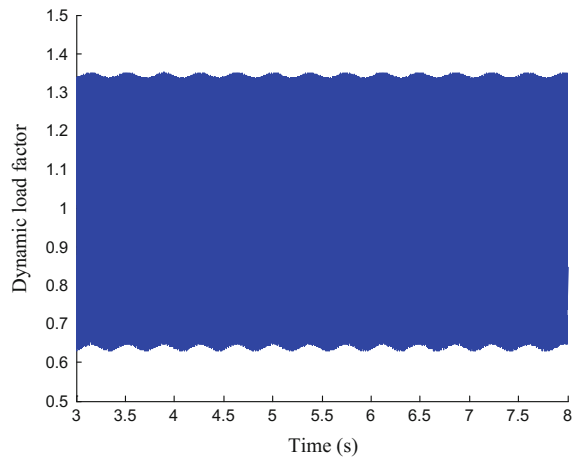


Fig. 2.19 K_V of high-speed stage



ring–planet gear pair is 1.05. K_V of the 2nd-stage sun–planet and ring–planet gear pair is 1.08 and 1.07, respectively. K_V of high-speed stage is 1.36, which is significantly increased similarly.

The computation program of K_A is attached in the Appendix A. The computation program of dynamic load factor K_V is similar to that of K_A , and it is ignored here.

2.4 Load-Sharing Coefficients Based on Torsional Dynamics

Load balance of a planetary gear train mainly refers to the engaging forces between each planet gear in planetary gear train. The basic floating components, such as the sun gear, the annular gear, or the planet carrier, are frequently used to realize better load balance among the planet gears. The floating component can move in the radial direction without radial support, which can make sure each planet gear shares the load equably under the unbalanced force. Load balance structure can decrease the load-sharing coefficient of the planetary gear transmission, reduce the noise, and improve the stability and the reliability of the system.

The supporting stiffness, mesh stiffness, and torsional stiffness of each gear in the system are calculated according to ANSI/AGMA ISO 1328-1 standard. The values are shown in Table 2.6.

The inner mesh force F_{rpij} and the external mesh force F_{spij} can be calculated by classical Runge–Kutta method according to Eqs. (2.9)–(2.18). Subsequently, planet load-sharing coefficient (PLSC) is defined as

$$b_{rpij} = NF_{rpij} / \sum_{j=1}^N (F_{rpij}), b_{spij} = NF_{spij} / \sum_{j=1}^N (F_{spij}) \quad (2.34)$$

The maximum of each PLSC in the same stage is defined as the system load-sharing coefficient (SLSC) as

$$B_{ri} = \frac{N \times \max(F_{rij})}{\sum_{j=1}^N F_{rij}}, B_{si} = \frac{N \times \max(F_{sij})}{\sum_{j=1}^N F_{sij}} \quad (2.35)$$

where b_{rpij} is the inner meshing PLSC of the j th planet in the i th stage, b_{spij} is the external meshing PLSC of the j th planet in the i th stage, B_{ri} is the inner meshing

Table 2.6 Stiffness values in the torsional model ($\times 10^9 \text{ N/m}$)

Stage	k_s	k_r	k_p	k_{oc}	k_{or}	$\bar{k}_{sp}$	$\bar{k}_{rp}$
1st	1.02	1.88	18.6	0.8	438.7	9.30	8.93
2nd	0.60	1.08	10.30	1.5	514.2	5.51	5.12

SLSC in the i th stage, B_{si} is the external meshing SLSC in the i th stage. N is the number of the planets in each stage.

2.4.1 Calculation of Load-Sharing Coefficients

The numerical solution for dynamical equations is obtained by the fourth-order Runge–Kutta method. Subsequently, the PLSC curve of each planet with rotation angle of the carrier in the 1th (θ_{c1}) is obtained according to Eq. (2.23) shown in Figs. 2.20 and 2.21. The SLSC curve of each stage is obtained according to Eq. (2.24) shown in Fig. 2.22. Finally, the maximum of SLSC for each planet gear is obtained, $\max(S_{r1}) = 1.066$, $\max(S_{s1}) = 1.062$, $\max(S_{r2}) = 1.130$, $\max(S_{s2}) = 1.107$, and is defined as the load-sharing coefficient.

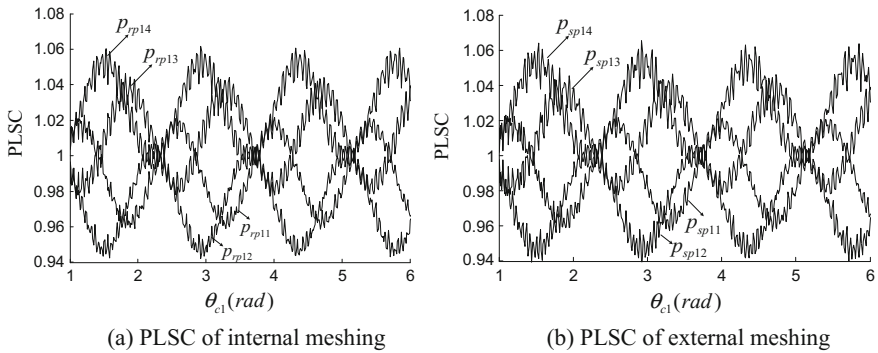


Fig. 2.20 PLSC in the 1st-stage planetary transmission. **a** PLSC of internal meshing. **b** PLSC of external meshing

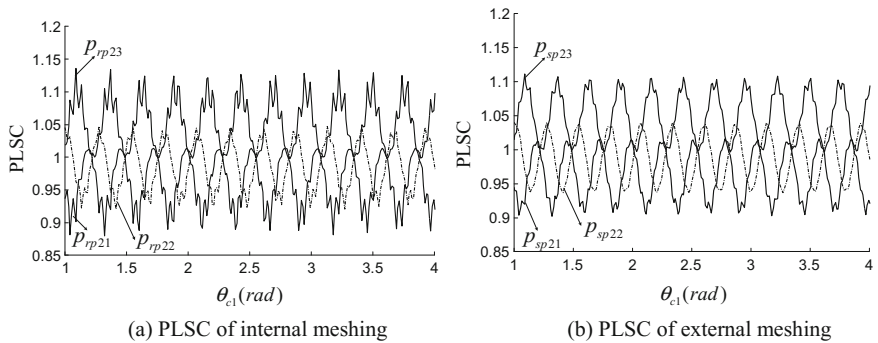


Fig. 2.21 PLSC in the 2nd-stage planetary transmission. **a** PLSC of internal meshing. **b** PLSC of external meshing

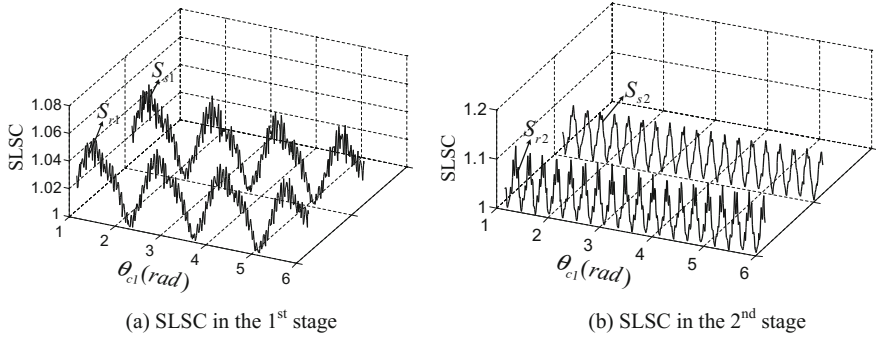


Fig. 2.22 SLSC of planetary transmission. **a** SLSC in the 1st stage. **b** SLSC in the 2nd stage

2.4.2 Load-Sharing Structure Design

Load-sharing performance of the planetary transmission can be improved by adopting appropriate load-sharing structure in practice. Two types of load-sharing structures are discussed according to the characteristics of the planetary model in this chapter.

(1) The elastic support of the planet gear

Elastic support of the planet gear is an effective load-sharing method and is becoming the future development direction of the load-sharing mechanism for the planetary transmission of the high-power wind turbine gearbox. The planets and the carrier in the same stage are connected by flexible pin to make the planets elastic float. SLSC is obtained by decreasing the planetary supporting stiffness to simulate the effect of the flexible pin. Since the SLSC variation between the external and internal meshing is mainly consistent shown in Fig. 2.22, the following calculation takes the external meshing for instance. S_{sj} is reduced from 1.07 to 1.02 in the 1st stage and from 1.15 to 1.01 in the 2nd stage with the elastic supporting planets in Fig. 2.23. It indicates that the elastic supporting planets can improve load-sharing performance especially in the 2nd stage.

Each planet floating displacement range is represented by the shadow in Fig. 2.24. The value is $a = 17 \mu\text{m}$, $b = 7.5 \mu\text{m}$, $c = 7 \mu\text{m}$, $d = 3 \mu\text{m}$, $e = 10 \mu\text{m}$, and $f = 57 \mu\text{m}$. All displacements are within the limit of maximum deflection according to HG/T 4082-2009, and the dynamic stability of the system is not disturbed.

(2) The center part floating with clearance

Floating center part (sun gear, ring gear or carrier) of the planetary system can also make the load of the planets more uniform by allowing radial and torsional deflection of float parts. Double (single)-tooth gear coupling is used as the floating component to achieve clearance float. The supporting forces of center part P_x and P_y

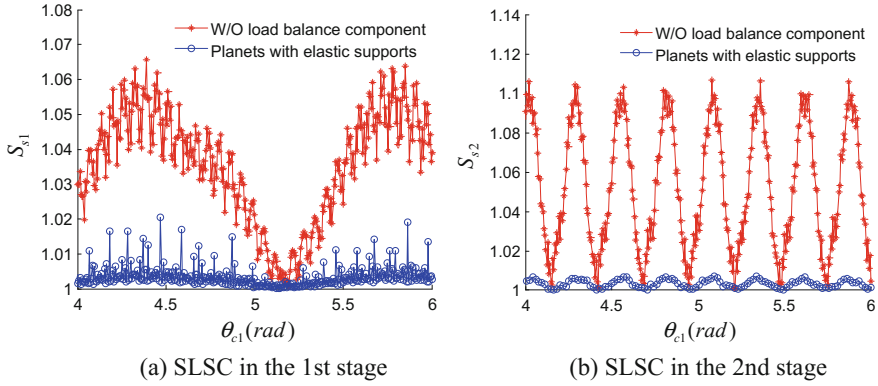


Fig. 2.23 SLSC of planets elastic support structure. **a** SLSC in the 1st stage. **b** SLSC in the 2nd stage

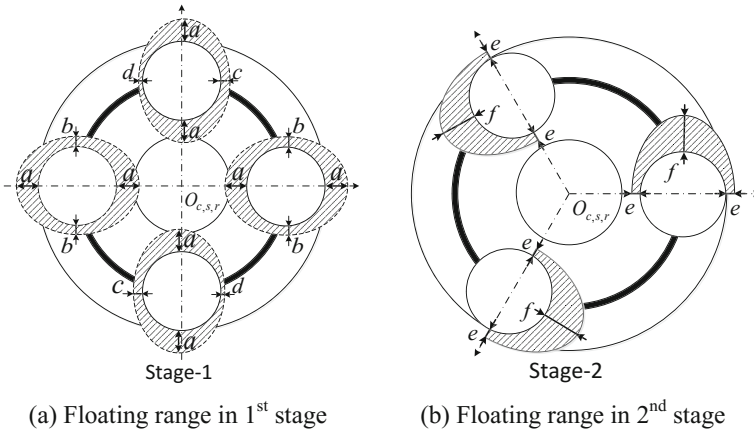


Fig. 2.24 Floating range of planets. **a** Floating range in 1st stage. **b** Floating range in 2nd stage

are presented in Eqs. (2.36)–(2.37) after considering supporting clearance nonlinearity. P_x and P_y are taken into dynamical equations and S_{si} is obtained.

$$P_x = \begin{cases} k_{si,ri,ci}(x_{si,ri,ci} - r_{si,ri,ci}), & |x_{si,ri,ci}| \geq r_{si,ri,ci} \\ 0, & |x_{si,ri,ci}| < r_{si,ri,ci} \end{cases} \quad (2.36)$$

$$P_y = \begin{cases} k_{si,ri,ci}(y_{si,ri,ci} - r_{si,ri,ci}), & |y_{si,ri,ci}| \geq r_{si,ri,ci} \\ 0, & |y_{si,ri,ci}| < r_{si,ri,ci} \end{cases} \quad (2.37)$$

where P_x and P_y are the supporting forces of center part in X_c - and Y_c -directions, respectively. r_{si} , r_{ri} , and r_{ci} are maximum permissible offset of center part presented in Eq. (2.38)–(2.39) [10].

$$\begin{cases} r_{s1,r1} = E_{s1} + 4\sqrt{2}E_{p1} \cos \alpha_{t1} + E_{r1} + 4E_{c1} \cos \alpha_{t1} + 2E_{\tau1} \cos \alpha_{t1} \\ r_{c1} = \frac{E_{s1} + E_{r1}}{2 \cos \alpha_{t1}} + 2E_{p1} + E_{c1} + E_{\tau1} \\ (N = 4) \end{cases} \quad (2.38)$$

$$\begin{cases} r_{s2,r2} = E_{s2} + \frac{8}{3}E_{p2} \cos \alpha_{t2} + E_{r2} + 2E_{c2} \cos \alpha_{t2} + \frac{4}{3}E_{\tau2} \cos \alpha_{t2} \\ r_{c2} = \frac{E_{s2} + E_{r2}}{2 \cos \alpha_{t2}} + \frac{4}{3}E_{p2} + E_{c2} + E_{\tau2} \\ (N = 3) \end{cases} \quad (2.39)$$

S_{si} of different floating parts are compared in Fig. 2.25 and is reduced obviously by floating any center part for both two stages. In particular, the S_{s1} is reduced from 1.068 to 1.002 and S_{s2} from 1.107 to 1.003 under sun gear floating. SLSC of the sun gear floating is minimum due to its little inertia and flexible movement. Floating center part of one stage has no influence on SLSC of the other stage.

SLSC can be reduced more in coupled floating mode, such as the sun gear of the 1st stage and the carrier of the 2nd stage floating together, compared with only one center part floating, as shown in Fig. 2.26. The value of SLSC with different load-sharing structure is listed in Table 2.7, where W/O represents system without load-sharing structure. It can be found that coupled floating mode, with a long spindle connecting sun gear of the 1st stage and carrier of the 2nd stage, can achieving a better load-sharing performance and is more suitable than any other structure for the two-stage planetary transmission.

Each center floating part displacement range is represented by the shadow in Fig. 2.27. The floating value is $g = 20 \mu\text{m}$, $h = 70 \mu\text{m}$, $i = 6 \mu\text{m}$, $j = 3 \mu\text{m}$, $k = 1 \mu\text{m}$, $l = 4 \mu\text{m}$, $m = 8 \mu\text{m}$, $n = 73 \mu\text{m}$, $o = 7 \mu\text{m}$, $p = 3 \mu\text{m}$, and $q = 2 \mu\text{m}$. All displacements are within the limit of maximum deflection according to HG/T 4082-2009, and dynamic stability of the system is not disturbed.

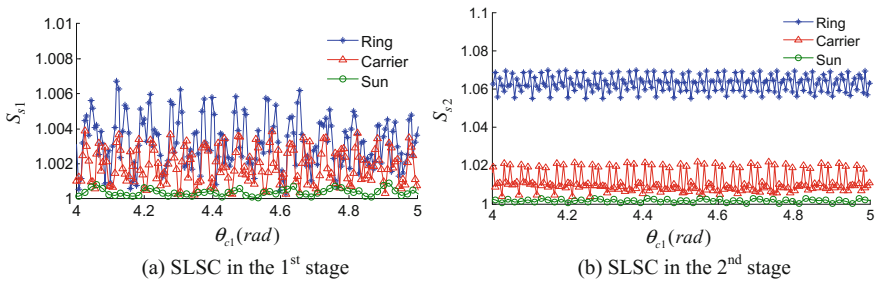


Fig. 2.25 SLSC of center part floating structure. **a** SLSC in the 1st stage. **b** SLSC in the 2nd stage

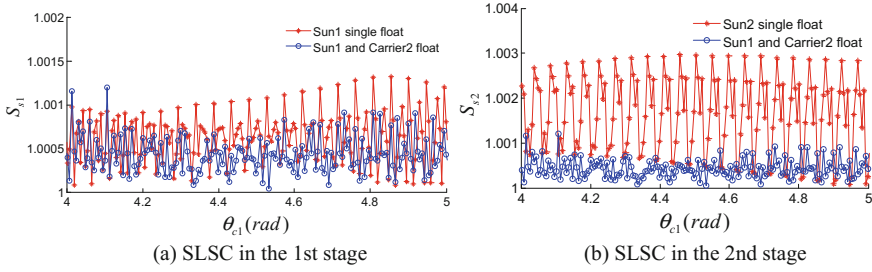


Fig. 2.26 SLSC of coupled float structure. **a** SLSC in the 1st stage. **b** SLSC in the 2nd stage

Table 2.7 SLSC of different load-sharing structure

		S_{s1}	S_{s2}
W/O		1.0681	1.1071
Planets elastic support		1.0231	1.0133
Center part floating	Ring	1.0073	1.0720
	Carrier	1.0043	1.0221
	Sun	1.0015	1.0030
	Coupled float	1.0012	1.0018

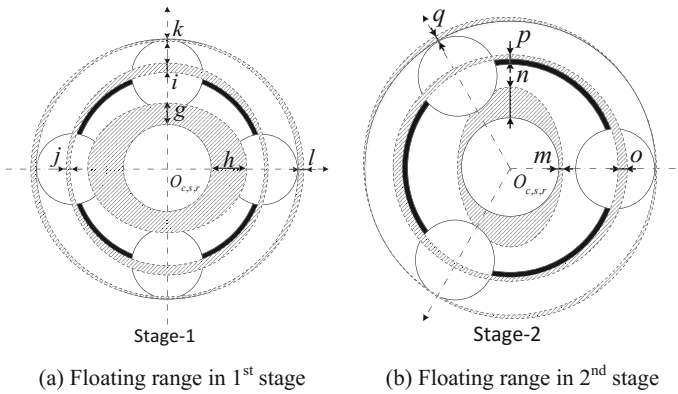
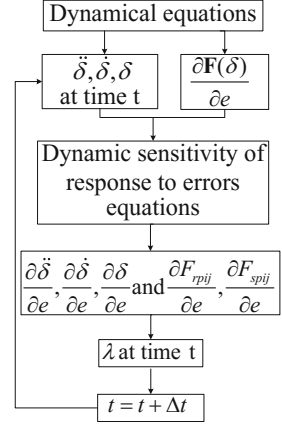


Fig. 2.27 Floating range of center floating parts. **a** Floating range in 1st stage. **b** Floating range in 2nd stage

2.4.3 Load-Sharing Performance Effected by the Errors

Since the component errors are major factors to make the planetary load distribution uneven, it is important to decrease the effect of the component errors on the load-sharing performance. The errors considered in this chapter are shown in

Fig. 2.28 Calculating flowchart of λ



nomenclature. λ is the dynamical sensitivity of PLSC to the errors, bigger λ means larger influence of the error on load-sharing performance. Direct derivative method is applied to obtain λ in this chapter. The calculating flowchart of λ is shown in Fig. 2.28. Equations (2.9)–(2.16) are organized into a matrix form with ignoring its tooth backlash nonlinearity and represented as follows

$$\mathbf{M}\ddot{\delta} + \mathbf{C}\dot{\delta} + \mathbf{K}\delta = \mathbf{F}(\delta) \quad (2.40)$$

Taking partial derivative of both sides with the errors, Eq. (2.28) is rewritten as

$$\mathbf{M}\frac{\partial\ddot{\delta}}{\partial e} + \mathbf{C}\frac{\partial\dot{\delta}}{\partial e} + \mathbf{K}\frac{\partial\delta}{\partial e} = \frac{\partial\mathbf{F}(\delta)}{\partial e} - \left(\frac{\partial\mathbf{M}}{\partial e}\ddot{\delta} + \frac{\partial\mathbf{C}}{\partial e}\dot{\delta} + \frac{\partial\mathbf{K}}{\partial e}\delta \right) \quad (2.41)$$

Since $\frac{\partial\mathbf{M}}{\partial e}\ddot{\delta} + \frac{\partial\mathbf{C}}{\partial e}\dot{\delta} + \frac{\partial\mathbf{K}}{\partial e}\delta = 0$, Eq. (2.29) can be simplified as follows:

$$\mathbf{M}\left(\frac{\partial\delta}{\partial e}\right)'' + \mathbf{C}\left(\frac{\partial\delta}{\partial e}\right)' + \mathbf{K}\left(\frac{\partial\delta}{\partial e}\right) = \tilde{\mathbf{F}}\left(\frac{\partial\delta}{\partial e}\right) \quad (2.42)$$

Partial derivative of dynamic response on the error is obtained by classical Runge–Kutta method, and the sensitivity of SLSC to the errors can be obtained in Eq. (2.43).

$$\lambda = \frac{\partial p_{rpij,spjij}}{\partial e} = N \frac{F'_{rpij,spjij} \left(\sum_{i=1}^N F_{rpij,spjij} \right) + F_{rpij,spjij} \left(\sum_{i=1}^N F'_{rpij,spjij} \right)}{\left(\sum_{i=1}^N F_{rpij,spjij} \right)^2} \quad (2.43)$$

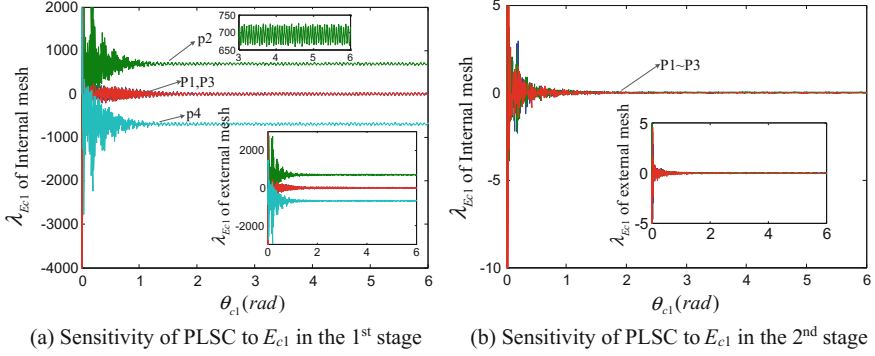


Fig. 2.29 Influence of E_{c1} on PLSC. **a** Sensitivity of PLSC to E_{c1} in the 1st stage. **b** Sensitivity of PLSC to E_{c1} in the 2nd stage

In Eq. (2.43), $F'_{rpj,spj}$ is partial derivative of the internal or external meshing force on error with its expression is

$$F'_{rpj,spj} = \frac{\partial F_{rpj,spj}}{\partial e} = F \left(\frac{\partial \delta}{\partial e}, \frac{\partial \dot{\delta}}{\partial e} \right)_{rpj,spj} \quad (2.44)$$

Taking E_{c1} for instance, dynamic sensitivity of PLSC to E_{c1} (λ_{Ec1}) is obtained without load-sharing structure in Fig. 2.29. λ_{Ec1} fluctuates within a certain range after the system maintains a stable state. Figure 2.29a shows that λ_{Ec1} of P_{12} is the largest of the four planets and λ_{Ec1} of P_{14} is opposite to P_{12} due to the symmetrical location of planets. λ_{Ec1} in the 2nd stage is close to 0 in Fig. 2.29b. It indicates that E_{c1} only affects PLSC in the same stage and has no influence on the other, with the same regularity as the other type of errors. The following research takes external meshing of P_{12} in the 1st stage for instance.

Dynamic sensitivity of PLSC to each type of error without load-sharing structure is given in Fig. 2.30. It shows that $E_{\tau 12}$ and E_{p12} have tremendous influence on PLSC compared with other type of errors in Fig. 2.30, which adequate attention should be paid on planetary transmission design. Sensitivity to $\varepsilon_{r1,s1,p12}$ is almost zero shown in Fig. 2.30e which can be ignored in planetary load-sharing research.

Figure 2.31 lists the root mean square (RMS) of sensitivities by each type of load-sharing structure this paper discussed above. It shows that when a component of planetary transmission is used as the load-sharing structure, the sensitivity to the relevant error is reduced. So it is concluded that adopting load-sharing structure not only can improve load-sharing performance but also can reduce the requirement of component precision for planetary transmission.

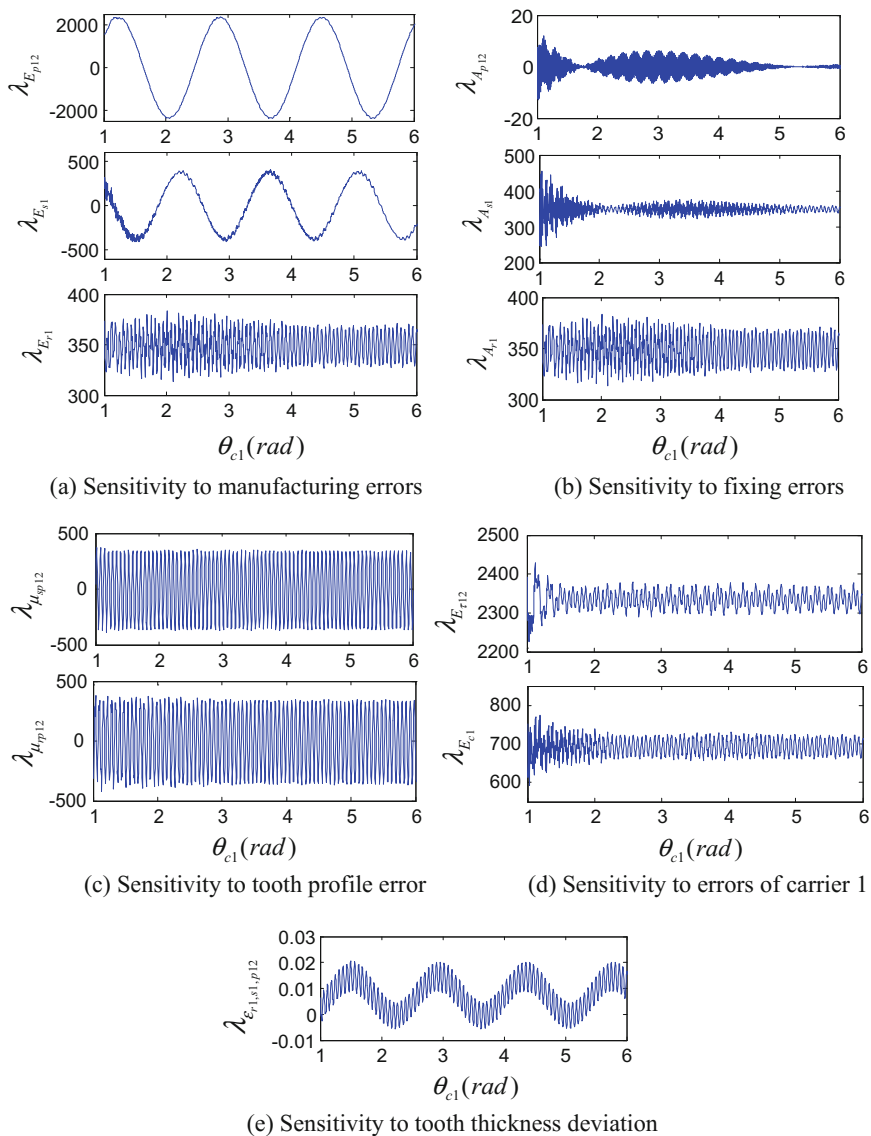


Fig. 2.30 Dynamic sensitivity of PLSC to errors. **a** Sensitivity to manufacturing errors. **b** Sensitivity to fixing errors. **c** Sensitivity to tooth profile error. **d** Sensitivity to errors of carrier 1. **e** Sensitivity to tooth thickness deviation

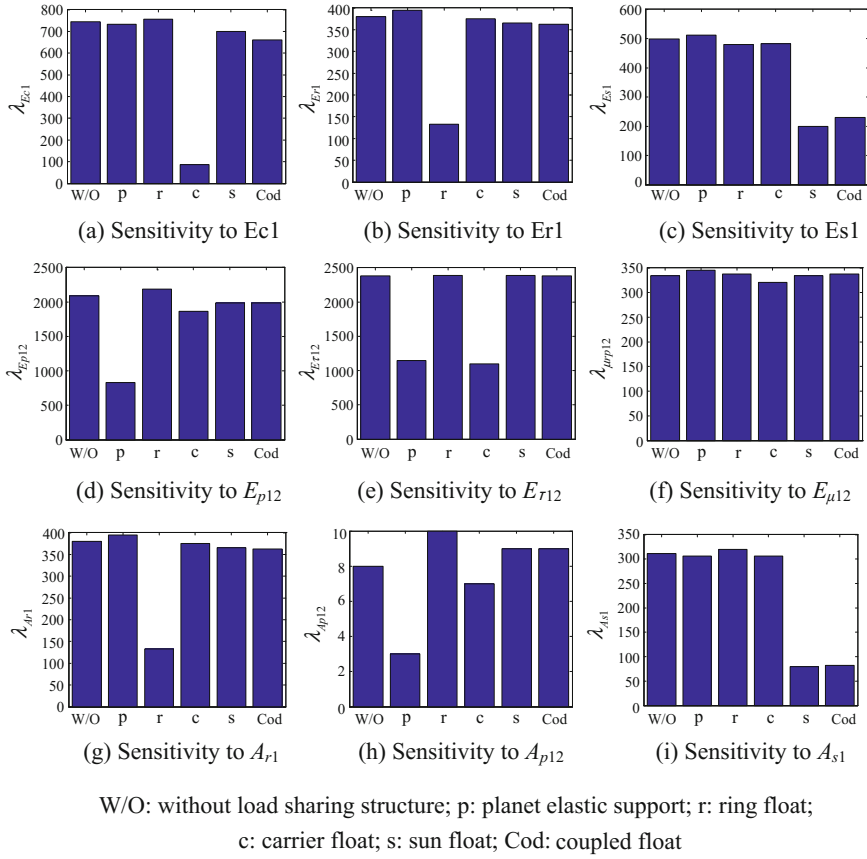


Fig. 2.31 RMS of sensitivity to errors with different load-sharing structure. **a** Sensitivity to E_{c1} . **b** Sensitivity to E_{r1} . **c** Sensitivity to E_{s1} . **d** Sensitivity to E_{p12} . **e** Sensitivity to E_{r12} . **f** Sensitivity to $E_{\mu12}$. **g** Sensitivity to A_{r1} . **h** Sensitivity to A_{p12} . **i** Sensitivity to A_{s1} . W/O without load sharing structure; p planet elastic support; r ring float; c carrier float; s sun float; Cod coupled float

2.5 Experimental Test of Load Balances in Gearbox

Taking a planetary gearbox with single stage as an example, load-sharing experiment is carried out by using two load-sharing structures, W/O load balance structure, and floating sun gear with single-tooth coupling. Test bench layout is shown in Fig. 2.32. Electronic measure method was adopted in the experiment to acquire planetary gear load, testing points were arranged in the ring gear, as shown in Fig. 2.33.

The gearbox was loaded by 0, 50, and 100 % of the rated load with the motor speed 13 r/min, and the planetary strains were measured, as shown in Figs. 2.34 and 2.35.

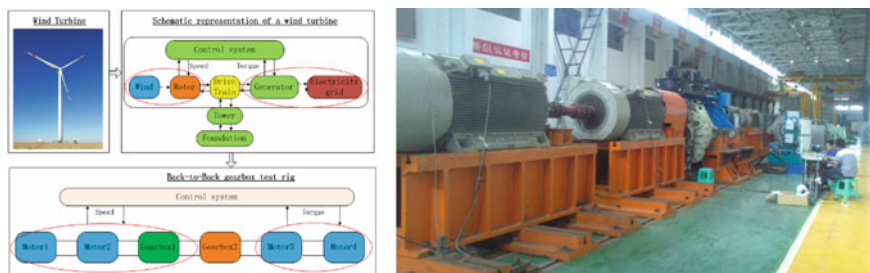


Fig. 2.32 Test bench of the load-sharing experiment

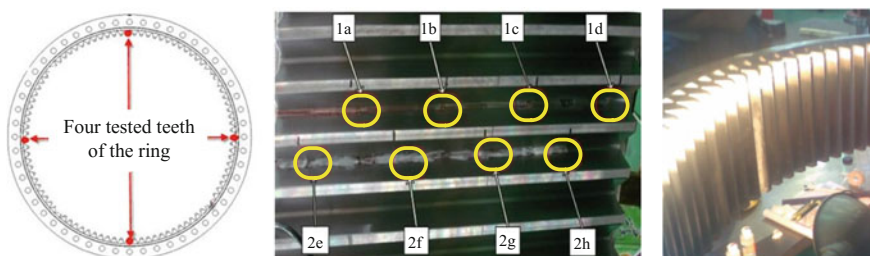


Fig. 2.33 Testing points arrangement

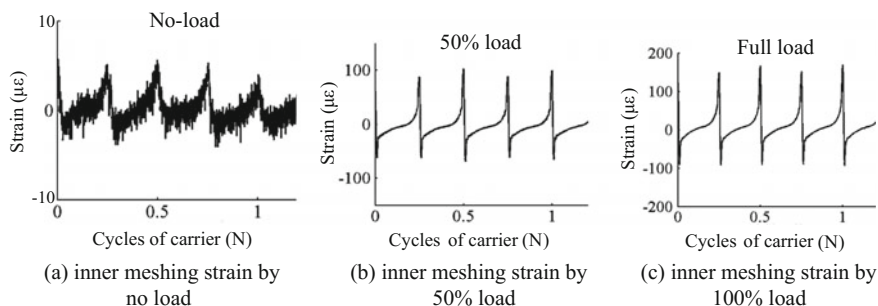


Fig. 2.34 Planetary strain W/O balance mechanism. **a** inner meshing strain by no load. **b** Inner meshing strain by 50 % load. **c** Inner meshing strain by 100 % load

The inner meshing strain is increased with the increasing of the load. The results of the experiment indicate that load fluctuation of the gear teeth is significantly reduced by floating sun gear and PLSC decreased correspondingly. It is concluded that the calculation results are agreed well with experiment. Due to the influence of the actual working conditions on the sharing load performance, such as manufacturing and installation errors, there is a little difference between the experimental and theoretical value, but same regularity they presented is consistent.

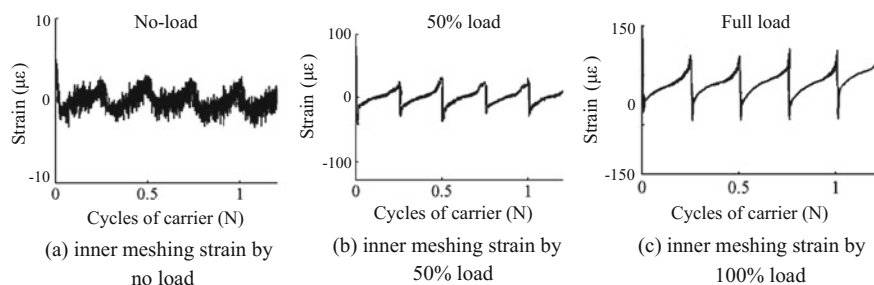


Fig. 2.35 Planetary strain with sun gear floating. **a** Inner meshing strain by no load. **b** Inner meshing strain by 50 % load. **c** Inner meshing strain by 100 % load

Table 2.8 SLSC of experiment results vs numerical calculations

		Exp. value	Cal. value	Error %
W/O load balance mechanism	50 % load	1.174	1.042	12.7
	100 % load	1.141	1.065	7.1
Sun gear float	50 % load	1.1061	1.001	10.5
	100 % load	1.087	1.002	8.5

Table 2.8 lists the comparative results of SLSC by experiment vs numerical calculation in the case of 50 and 100 % load, respectively.

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Dynamics and Vibration Analyses of Gearbox in Wind
Turbine

Han, Q.; Wei, J.; Han, Q.; Zhang, H.

2017, XII, 164 p. 88 illus., 68 illus. in color., Hardcover

ISBN: 978-981-10-2746-8