

# Preface

## Aim of This book

The aim of this book is to (1) to explore the relationship between principal component analysis (PCA), neural network, and learning algorithms and provide an introduction to adaptive PCA methods and (2) to present many novel PCA algorithms, their extension/generalizations, and their performance analysis.

In data analysis, one very important linear technique to extract information from data is principal component analysis (PCA). Here, the principal components (PCs) are the directions in which the data have the largest variances and capture most of the information content of data. They correspond to the eigenvectors associated with the largest eigenvalues of the autocorrelation matrix of the data vectors. On the contrary, the eigenvectors that correspond to the smallest eigenvalues of the autocorrelation matrix of the data vectors are defined as the minor components (MCs) and are the directions in which the data have the smallest variances (they represent the noise in the data). Expressing data vectors in terms of the minor components is called minor component analysis (MCA). Through PCA, many variables can be represented by few components, so PCA can be considered as either a feature extraction or a data compression technology. Now, PCA has been successfully applied to many data processing problems, such as high-resolution spectral estimation, system identification, image compression, and pattern recognition. MCA is mainly used to solve total least squares problem, which is a technology widely used to compensate for data errors in parameter estimation or system identification. However, how can we obtain the principal components or minor components from a stochastic data stream?

This book aims to provide a relatively complete view of neural network-based principal component analysis or principal subspace tracking algorithms and present many novel PCA algorithms, their performance analysis, and their extension/generalizations.

## Novel Algorithms and Extensions

It is well known that many methods exist for the computation of principal components, such as the power method, eigenvalue decomposition (ED), singular value decomposition (SVD), and neural network algorithms. Neural network approaches on PCA pursue an effective “online” approach to update the eigen direction after each presentation of a data point, which possess many obvious advantages. Many neural network learning algorithms have been proposed to extract PC, and this has been an active field for around two decades up to now.

This book is not oriented toward all neural network algorithms for PCA, but to some novel neural algorithms and extensions of PCA, which can be summarized as follows.

- (1) Compared with most neural principal component learning algorithms, the number of neural networks for minor component analysis is somewhat smaller. A norm divergence problem exists in some existing MCA algorithms. To guarantee the convergence, it is necessary to use self-stabilizing algorithms. In these self-stabilizing algorithms, the weight vector length converges to a fixed value independent of the presented input vector. In this book, the self-stabilizing algorithms are discussed in detail and some novel self-stabilizing MCA learning algorithms are introduced.
- (2) Most neural PCA algorithms only focus on eigenvector extraction using uncoupled rules, and a serious speed-stability problem exists in most uncoupled rules. To overcome this problem, several coupled PCA algorithms are introduced and their performances are analyzed in this book.
- (3) Most neural algorithms only deal with either principal component extraction or minor component extraction. Are there such algorithms as dual-purpose subspace tracking algorithm, which are capable of both PC and MC extractions by simply switching the sign in the same learning rule? This book will develop a few dual algorithms for such purposes.
- (4) The convergence of PCA neural learning algorithms is a difficult topic for direct study and analysis. Traditionally, the convergence of these algorithms is indirectly analyzed via certain deterministic continuous-time (DCT) systems. The DCT method is based on a fundamental theorem of stochastic approximation theory, and some crucial conditions must be satisfied, which are not reasonable requirements to be imposed in many practical applications. Recently, deterministic discrete-time (DDT) systems have been proposed instead to indirectly interpret the dynamics of neural network learning algorithms described by stochastic discrete-time system. This book will discuss the DDT method in detail.
- (5) It is well known that generalized eigen decomposition (GED) plays very important roles in various signal processing applications, and PCA can be seen as a special case of GED problem. The GED neural algorithms will be also discussed in detail.

- (6) An important aspect of the generalization of classic PCA is the cross-correlation problem, which studies the maximization of the cross-correlation between two stochastic signals. The neural learning algorithm to extract cross-correlation feature between two high-dimensional data streams will be studied in this book as well.

## Prerequisites

The mathematical background required for reader is that of college calculus and probability theory. Readers should be familiar with basic linear algebra and numerical analysis as well as the fundamentals of statistics, such as the basics of least squares, and preferably, but not necessarily, stochastic algorithms. Although the book focuses on neural networks, they are presented only by their learning law, which is simply an iterative algorithm. Therefore, no a priori knowledge of neural networks is required. Basic background in mathematics is provided in the review chapter for convenience.

Some of the materials presented in this book have been published in the archival literature over the last several years by the authors, and they are included in this book after necessary modifications or updates to ensure accuracy, relevance, completeness, and coherence. This book also puts effort into presenting as many contributions by other researchers in this field as possible. This is a fast-growing area, so it is impossible to make sure that all works published to date are included. However, we still have made special efforts to filter through major contributions and to provide an extensive bibliography for further reference. Nevertheless, we realize that there may be oversights on critical contributions on this subject. For these, we would like to offer our apology. More importantly, our sincere thanks go to the many researchers whose contributions have established a solid foundation for the topics treated in this book.

## Outline of the Book

Chapter 2 reviews some important concepts and theorems of matrix analysis and optimization theory. We discuss some basic concepts, properties, and theorems related to matrix analysis, with the emphasis on singular value decomposition and eigenvalue decomposition. We also introduce some methods of gradient analysis and optimization theory, which are all important tools which will be instrumental for our theoretical analysis in the subsequent chapters.

In Chap. 3, we discuss the principal component analysis networks and algorithms. The first half of this chapter analyzes the problem, basic theorems, and SVD-based methods of principal component analysis. The second half of this chapter studies the principal component analysis networks in detail, which falls

into the following classes, such as Hebbian rule-based, LMS error-based, optimization-based, anti-Hebbian rule-based, nonlinear, constrained, and localized PCA, providing a theoretical analysis of major networks.

Chapter 4 studies the minor component analysis networks and algorithms. First, we analyze the problem of the minor component analysis and its classical application in total least squares estimation. Second, we present some classical anti-Hebbian rule-based MCA algorithms, especially analyzing the divergence (sudden, dynamical, and numerical) property and self-stabilizing property of some MCA algorithms. This chapter concludes with a self-stabilizing MCA algorithm and a novel neural algorithm for total least squares filtering of ours, with the simulations and application presented as aid to the understanding of our algorithm.

Chapter 5 addresses the theoretical issue of the dual-purpose principal and minor component analyses. We analyze the merit of dual-purpose algorithms in application and theory analysis and introduce existing dual-purpose methods, such as Chen's, Hasan's, and Peng's algorithms. Two important dual-purpose algorithms of ours are presented. Also, the information criterion, its landscape and gradient flow, global convergence analysis, and numerical consideration are analyzed. This is one of the most important chapters in this book.

Chapter 6 deals with the stability and convergence analysis of PCA or MCA neural network algorithms. The performance analysis methods are classified into three classes, namely the deterministic continuous-time (DCT) system, the stochastic discrete-time (SDT) system, and the deterministic discrete-time (DDT) system, which are discussed in detail. We briefly review the DDT system of Oja's PCA algorithm and give a detailed analysis of the DDT systems of a new self-stabilizing MCA algorithm and Chen's unified PCA/MCA algorithm.

Chapter 7 studies the generalized feature extraction method. First, we review the generalized Hermitian eigenvalue problem. Second, a few existing adaptive algorithms to extract generalized eigen pairs are discussed. Third, a minor generalized eigenvector extraction algorithm and its convergence analysis via the DDT method are presented. Finally, we analyze a novel adaptive algorithm for generalized coupled eigen pairs of ours in detail, and a few simulation and application experiments are provided.

Chapter 8 analyzes the demerits of the existing uncoupled feature extraction algorithm, introduces Moller's coupled principal component analysis neural algorithm, and concludes with our two algorithms, the one of which is a unified and coupled self-stabilizing algorithm for minor and principal eigen pair extraction algorithms and the other an adaptive coupled generalized eigen pair extraction algorithms.

Chapter 9 presents the generalization of feature extraction from autocorrelation matrix to cross-association matrix. We briefly review the cross-correlation asymmetric network and analyze Feng's neural networks for extracting cross-correlation features. Then, an effective neural algorithm for extracting cross-correlation feature

between two high-dimensional data streams is proposed and analyzed. Finally, a novel coupled neural network-based algorithm to extract the principal singular triplet of a cross-correlation matrix between two high-dimensional data streams is presented and analyzed in detail.

## Suggested Sequence of Reading

This book aims to provide a relatively complete and coherent view of neural network-based principal component analysis or principal subspace tracking algorithms. This book can be divided into four parts, namely preliminary knowledge, neural network-based principal component learning algorithm, performance analysis of algorithms, and generalizations and extensions of PCA algorithms. For readers who are interested in general principal component analysis and future research directions, a complete reading of this book is recommended. For readers who are just interested in some specific subjects, selected chapters and reading sequences are recommended as follows.

- (1) Numerical calculation of principal components

Chapter 2 → Chapter 3 → Chapter 4

- (2) Performance analysis of neural network-based PCA algorithms

Chapter 3 → Chapter 4 → Chapter 6

- (3) Neural network-based PCA algorithms and their extensions

Chapter 3 → Chapter 4 → Chapter 7 → Chapter 8 → Chapter 9

Principal Component Analysis Networks and Algorithms

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2017, XXII, 323 p. 86 illus., 41 illus. in color., Hardcover

ISBN: 978-981-10-2913-4