

Contents

1 Preliminaries	1
1.1 Vector Spaces	1
1.2 Metric Spaces	5
1.3 Lebesgue Integration	11
1.4 Zorn's Lemma	18
1.5 Absolute Continuity	18
2 Inner Product Spaces	21
2.1 Definition and Examples	21
2.2 Norm of a Vector	26
2.3 Inner Product Spaces as Metric Spaces	34
2.4 The Space $L^2(X, \mathfrak{M}, \mu)$	40
2.5 A Subspace of $L^2(X, \mathfrak{M}, \mu)$	46
2.6 The Hilbert Space $A(\Omega)$	48
2.7 Direct Sum of Hilbert Spaces	53
2.8 Orthogonal Complements	59
2.9 Complete Orthonormal Sets	78
2.10 Orthogonal Decomposition and Riesz Representation	102
2.11 Approximation in Hilbert Spaces	123
2.12 Weak Convergence	127
2.13 Applications	137
3 Linear Operators	153
3.1 Basic Definitions	153
3.2 Bounded and Continuous Linear Operators	156
3.3 The Algebra of Operators	167
3.4 Sesquilinear Forms	175
3.5 The Adjoint Operator	182
3.6 Some Special Classes of Operators	192
3.7 Normal, Unitary and Isometric Operators	205

3.8	Orthogonal Projections	216
3.9	Polar Decomposition.	222
3.10	An Application	229
4	Spectral Theory and Special Classes of Operators	233
4.1	Spectral Notions	233
4.2	Resolvent Equation and Spectral Radius.	238
4.3	Spectral Mapping Theorem for Polynomials.	242
4.4	Spectrum of Various Classes of Operators	248
4.5	Compact Linear Operators	263
4.6	Hilbert–Schmidt Operators	279
4.7	The Trace Class	285
4.8	Spectral Decomposition for Compact Normal Operators.	294
4.9	Spectral Measure and Integral.	305
4.10	Spectral Theorem for Self-adjoint Operators.	317
4.11	Spectral Mapping Theorem For Bounded Normal Operators	331
4.12	Spectral Theorem for Bounded Normal Operators	337
4.13	Invariant Subspaces	343
4.14	Unbounded Operators.	351
5	Banach Spaces	373
5.1	Definition and Examples.	373
5.2	Finite-Dimensional Spaces and Riesz Lemma.	384
5.3	Linear Functionals and Hahn–Banach Theorem	393
5.4	Baire Category Theorem and Uniform Boundedness Principle	401
5.5	Open Mapping and Closed Graph Theorems	409
6	Hints and Solutions	417
6.1	Problem Set 2.1	417
6.2	Problem Set 2.2	418
6.3	Problem Set 2.3	422
6.4	Problem Set 2.4	427
6.5	Problem Set 2.5	428
6.6	Problem Set 2.6	428
6.7	Problem Set 2.8	429
6.8	Problem Set 2.9	437
6.9	Problem Set 2.10	442
6.10	Problem Set 2.11	448
6.11	Problem Set 2.12	450
6.12	Problem Set 3.2	454
6.13	Problem Set 3.3	462
6.14	Problem Set 3.4	465
6.15	Problem Set 3.5	466
6.16	Problem Set 3.6	466
6.17	Problem Set 3.7	470

6.18	Problem Set 3.8	475
6.19	Problem Set 3.9	478
6.20	Problem Set 4.1	479
6.21	Problem Set 4.2	485
6.22	Problem Set 4.4	487
6.23	Problem Set 4.5	487
6.24	Problem Set 4.6	504
6.25	Problem Set 4.7	507
6.26	Problem Set 4.8	511
6.27	Problem Set 4.9	512
References		515
Index		517

Elements of Hilbert Spaces and Operator Theory

Vasudeva, H.L.

2017, XIII, 522 p. 5 illus., Hardcover

ISBN: 978-981-10-3019-2