

Chapter 2

Signal Model for Synthetic Aperture Radar Images

Abstract This chapter introduces the SAR fundamental ideas and analysis methods employed in the subsequent chapters of the book. First, the geometrical configuration of the sensor relative to the imaged scene is described, followed by a brief overview of the time–frequency interpretation of the acquired signals. Afterwards, the principle of SAR image formation and the SAR tomography framework are introduced as the main classical processing tools.

2.1 SAR Acquisition Geometry

For the developments from this chapter, the considered raw signal model for synthetic aperture processing is described in the following. The geometry under consideration is presented in Fig. 2.1. The unit vector $\mathbf{u}$ describes the azimuth direction of the sensor. The position of the sensor's antenna phase center (APC) at a certain azimuth (slow) time t will be written as

$$\mathbf{r}_a(t) = \mathbf{r}_{a,0} + v_0 t \mathbf{u}, \quad (2.1)$$

where $\mathbf{r}_{a,0}$ is the APC position vector at $t = 0$ and v_0 the sensor's speed in zero-Doppler geometry. For a given target i having the position vector $\mathbf{r}_i$, the closest approach distance to the synthetic aperture is given by

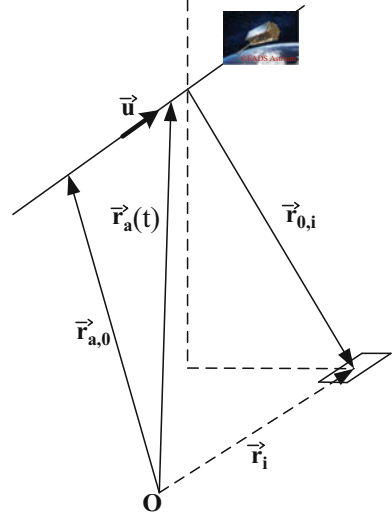
$$r_{0,i} = \|(\mathbf{r}_{a,0} - \mathbf{r}_i) - [(\mathbf{r}_{a,0} - \mathbf{r}_i) \cdot \mathbf{u}] \mathbf{u}\| \quad (2.2)$$

and the azimuth time at which this distance is attained can be expressed as

$$t_{r_i} = \frac{(\mathbf{r}_i - \mathbf{r}_{a,0}) \cdot \mathbf{u}}{v_0}. \quad (2.3)$$

The distance APC–target i as a function of azimuth time can be written in terms of the previously defined variables as follows:

Fig. 2.1 SAR acquisition geometry for a point having the position vector $\mathbf{r}_i$



$$\begin{aligned} \Delta r_i(t) &= \|\mathbf{r}_a(t) - \mathbf{r}_i\| \\ &= \sqrt{r_{0,i}^2 + [v_0(t - t_i)]^2}. \end{aligned} \quad (2.4)$$

After demodulation, the response from a set of N point scatterers located at $\mathbf{r}_i$ is a function of two variables (slow time and fast time):

$$s(t, \tau) = \sum_{i=1}^N A_i p_0 \left(\tau - \frac{2\Delta r_i(t)}{c} \right) \exp \left\{ -j \frac{4\pi f_c}{c} \Delta r_i(t) \right\} \text{rect} \left[\frac{t - t_i}{T_{ap}} \right], \quad (2.5)$$

where A_i is the complex amplitude of scatterer i , f_c is the central frequency, c is the speed of light, T_{ap} is the synthetic aperture duration, and $p_0(\tau)$ is the complex envelope of the transmitted signal as a function of the fast time τ . In (2.5), the function $\text{rect}(t/T_{ap})$ is a gate with length T_{ap} centered in the origin.

2.2 Azimuth Time–Frequency Representation

The azimuth phase history of an imaged target is actually a nonstationary signal whose location in time domain is given by the interval when the antenna beam illuminates the target. For this type of signals, the Fourier transform cannot provide information about the frequency content at certain time instants, but only an overview of the spectral composition on the whole analyzed period. This happens because the base on which a signal is decomposed in a Fourier transform has a theoretically infinite support which is not compatible with time-localized signals. A straightforward

solution to this problem is to perform a spectral analysis only on short intervals of the signal, method which is called the Short-Time Fourier Transform (STFT).

For a signal $x(t)$ the mathematic expression of the STFT is [1, 2]:

$$F_x(t, f) = \int_{-\infty}^{\infty} x(\tau) h^*(\tau - t) \exp(-j2\pi f \tau) d\tau, \quad (2.6)$$

where $h^*(\tau - t)$ is a windowing function centered in t . So in order to obtain the spectral representation around t the window $h^*(t)$ is time shifted with t and a Fourier transform is performed on the resulting windowed signal. The function $F_x(t, f)$ can also be viewed as the coefficient of the decomposition of signal $x(t)$ on a base signal defined as

$$h_{t,f}(\tau) = h(\tau - t) \exp(j2\pi f \tau), \quad (2.7)$$

where $h(t)$ is interpreted as a mother base function which is translated in time and frequency to obtain all possible base functions.

If the window $h(t)$ has unitary energy (the admissibility condition), the signal $x(t)$ is expressed as a linear combination of elementary atoms $h_{t,f}(\tau)$ multiplied by the coefficient $F_x(t, f)$:

$$\begin{aligned} x(\tau) &= \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} F_x(t, f) h_{t,f}(\tau) df \\ &= \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} F_x(t, f) h(\tau - t) \exp(j2\pi f \tau) df. \end{aligned} \quad (2.8)$$

The simplest pick for the window $h(t)$ is the rectangular window on a given analysis duration, but a wiser choice is a window which assures a good localization in time–frequency (e.g., Hamming, Hanning, or Gaussian windows) as well as a proper smoothness (rapid decrease, multiple times differentiable).

The squared magnitude of the Short-Time Fourier Transform is associated with the energy distribution of the signal in the time–frequency plane and is usually called a Spectrogram:

$$S_x(t, f) = |F_x(t, f)|^2 = \left| \int_{-\infty}^{\infty} x(\tau) h^*(\tau - t) \exp(-j2\pi f \tau) d\tau \right|^2. \quad (2.9)$$

The spectrogram can also be expressed using Parseval's theorem (or unitarity property of the Fourier transform):

$$\int_{-\infty}^{\infty} x_1(t) x_2^*(t) dt = \int_{-\infty}^{\infty} X_1(f) X_2^*(f) df, \quad (2.10)$$

which yields

$$S_x(t, f) = \left| \int_{-\infty}^{\infty} X(v) H^*(v - f) \exp(j2\pi vt) dv \right|^2. \quad (2.11)$$

In this case, the Fourier transform $H(v)$ of window h is employed as a sliding window which spans the entire spectrum. This standpoint resembles to a processing based on uniform continuous bank filter analysis with constant bandwidth.

The STFT has certain limitations especially when the analyzed signal is perfectly localized in time or in frequency. For instance, in case of a signal which is perfectly localized in time $x(t) = \delta(t - t_0)$, the spectrogram is actually the squared magnitude of the window placed around the moment t_0 , i.e., $S_x(t, f) = |h(t_0 - t)|^2$ and the representation is more adapted to the signal when the window is as short as possible. Contrariwise, for a signal perfectly localized in frequency $x(t) = \exp(j2\pi f_0 t)$ the spectrogram $S_x(t, f) = |H(f_0 - f)|^2$ is the squared magnitude of the window's Fourier transform around frequency f_0 . In this situation, the representation is more adapted to the signal when the window's spectrum is as narrow as possible. Therefore, the temporal and spectral lengths of the window condition the sharpness of the representation (good resolution in time or good resolution in frequency). Due to Heisenberg's uncertainty principle (the resolutions product $\Delta t \Delta f$ is lower bounded) is impossible to simultaneously obtain a very good resolution in both domains.

So, in the STFT case, the time–frequency analysis is performed by decomposing the signal on atoms which pave the time–frequency plane with constant time resolution δt and constant frequency resolution δf . The window which provides the best compromise between the two resolutions is the Gaussian window.

2.3 Time-Domain SAR Image Formation

A straightforward way to focus a SAR image is by convolving the raw data (in slow time-fast time domain) with a shift-varying filter [3, 4]. A first approach of this method is to perform a correlation in both slow and fast time domains of the raw data with the expected SAR signature for each imaged point. Assuming that we want to compute the response of a point k situated on a certain grid at the position $\mathbf{r}_k$, its SAR signature will be

$$p_k(t, \tau) = p_0 \left(\tau - \frac{2\|\mathbf{r}_a(t) - \mathbf{r}_k\|}{c} \right) \exp \left(-j \frac{4\pi f_c}{c} \|\mathbf{r}_a(t) - \mathbf{r}_k\| \right) \text{rect} \left[\frac{t - t_k}{T_{ap}} \right], \quad (2.12)$$

while the imaging equation can be expressed as a double integral:

$$g_{MF}(\mathbf{r}_k) = \int_t \int_\tau s(t, \tau) p_k^*(t, \tau) dt d\tau. \quad (2.13)$$

From the implementation standpoint, the two-dimensional integral is actually a double sum over the discrete values of (t, τ) on a uniform grid determined by the

pulse repetition frequency (PRF) in azimuth and the range sampling frequency (F_s). If the grid comprises M pulses in azimuth and for each pulse there are N_s samples in range, the number of required operations to compute the response is proportional to the product $M \times N_s$. In general, in order to generate a SAR image (practically a matrix of points for which the response has to be computed) with the matched filtering algorithm, the computational complexity is proportional to the number of points in the matrix times $M \times N_s$, which makes the method impractical in most cases.

A more efficient time domain focusing method is the back projection technique. This method enhances the computation time by computing separately and in different manners the matched filtered response in slow time and fast time domains, which leads to a separation of the double integral in (2.13). The first step is to perform a convolution in the fast time domain between the raw data and a reversed conjugated version of the transmitted pulse (a matched filtering in range):

$$s_p(t, \tau) = s(t, \tau) * p_0^*(-\tau), \quad (2.14)$$

which leads to a function consisting in a range profile (or range compressed response) for each azimuth time t :

$$s_{rp}(t, r) = s_p(t, 2r/c). \quad (2.15)$$

The imaging equation will be expressed as a single integral representing the coherent integration of the range compressed responses of a grid point placed at $\mathbf{r}_k$ from the entire azimuth aperture:

$$g_{BP}(\mathbf{r}_k) = \int_t s_p \left(t, \frac{2\Delta r_i(t)}{c} \right) \exp \left[j \frac{4\pi f_c}{c} \Delta r_i(t) \right] dt. \quad (2.16)$$

Practically, the implementation of (2.16) consists in a discrete-time convolution to obtain $s_p(t, \tau)$ and a summation over all the pulses. The discrete raw signal can be written as $s[m, n] = s(m\delta t, n2\delta r/c)$, where $\delta t = \frac{1}{PRF}$ is the slow time spacing and δr the range spacing. Similarly, the range focused signal is $s_{rp}[m, n] = s_{rp}(m\delta t, n\delta r)$. With this notations, the azimuth focusing of the range compressed signal is implemented as

$$g_{BP}(\mathbf{r}_k) = \sum_m s_{rp}(m\delta t, \|\mathbf{r}_a(m\delta t) - \mathbf{r}_k\|) \exp \left[j \frac{4\pi f_c}{c} \|\mathbf{r}_a(m\delta t) - \mathbf{r}_k\| \right]. \quad (2.17)$$

The name back projection comes from the fact that for each synthetic aperture location (slow time) the range compressed response of a target is traced back in the fast time domain in order to isolate the azimuth response of the envisaged target from other reflections. The value $s_{rp}(m\delta t, \|\mathbf{r}_a(m\delta t) - \mathbf{r}_k\|)$ is not implicitly available because $s_{rp}(t, \tau)$ is stored on the uniform grid $s_{rp}[m, n]$ which obviously will not

contain exactly the point $(m\delta t, \|\mathbf{r}_a(m\delta t) - \mathbf{r}_k\|)$. So in order to obtain the required value, a 1D interpolation has to be performed along the fast time axis. No interpolation along the slow time axis is necessary because each discrete value of t is actually the slow time of a pulse.

2.4 Differential (4D) SAR Tomography Framework

This subsection makes a review of the four-dimensional SAR imaging model (differential tomography) [5] adapted to SAR images focused on a certain grid using the presented time domain methods. The envisaged geometry is shown in Fig. 2.2. We consider N satellite tracks corresponding to the time instants t_n relative to the first track. The orthogonal baseline between a current track n and the first one is denoted $b_{\perp,n}$. The SAR response of a grid point k for track n is noted $g_n(\mathbf{r}_k)$. The responses for each track $g_n(\mathbf{r}_k)$ are grouped in a vector with N components denoted $g(\mathbf{r}_k)$. Considering the case of the presented time domain methods, the so-called deramping [6] is implicitly performed (geometric phase compensation), and each element of $g(\mathbf{r}_k)$ can be decomposed as [5, 7]:

$$g_n(\mathbf{r}_k) = \int_{\Delta s} \gamma_0(s) \exp \left[-j \frac{4\pi f_c}{c} (r_{n,s} - r_{n,k}) \right] \exp \left[j \frac{4\pi f_c}{c} d(s, t_n) \right] ds, \quad (2.18)$$

where $\gamma_0(s)$ is the reflectivity profile along elevation having the support Δs and $d(s, t_n)$ is the deformation in the line of sight of a target placed at the elevation s . Considering the baselines much smaller than the distances orbit-target, after some calculations and approximations Eq. (2.18) becomes:

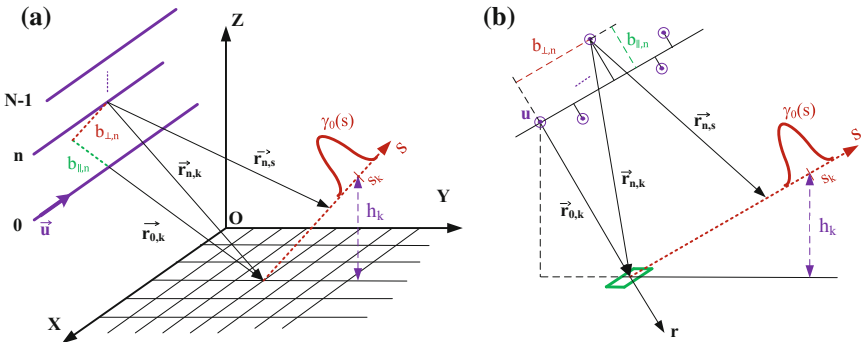


Fig. 2.2 Four-dimensional tomography geometry: **a** 3D view and **b** 2D section parallel to yOz plane

$$\begin{aligned}
g_n(\mathbf{r}_k) &= \int_{\Delta s} \gamma_0(s) \exp \left\{ j \frac{4\pi f_c}{c} \left[-\frac{s^2}{2r_{0,k}} + s \frac{b_{\perp,n}}{r_{0,k}} + d(s, t_n) \right] \right\} ds \\
&= \int_{\Delta s} \gamma(s) \exp \left\{ j \frac{4\pi f_c}{c} \left[s \frac{b_{\perp,n}}{r_{0,k}} + d(s, t_n) \right] \right\} ds,
\end{aligned} \tag{2.19}$$

where $\gamma(s) = \gamma_0(s) \exp \left(-j \frac{4\pi f_c}{c} \frac{s^2}{2r_{0,k}} \right)$ is a modified reflectivity profile which incorporates the s^2 -dependent phase term associated to the curvature of the incident wavefront (which corrupts the phase of the final image if it is not mitigated).

Onwards, a Fourier expansion is applied to each displacement term $d(s, t_n)$ and the vector of received and focused data has the following expression:

$$g(\mathbf{r}_k) = \int_{\Delta s} \int_{\Delta v} p_\gamma(s, v) \mathbf{a}(s, v) ds dv, \tag{2.20}$$

where $p_\gamma(s, v) = \gamma(s)p(s, v)$ and $p(s, v)$ is the elevation-velocity (EV) spectral distribution of the displacement terms, Δv is the velocity support of $p_\gamma(s, v)$ and $\mathbf{a}(s, v)$ is the steering vector whose elements are defined as

$$a_n(s, v) = \exp \left[j \frac{4\pi f_c}{c} \left(s \frac{b_{\perp,n}}{r_{0,k}} + vt_n \right) \right], \tag{2.21}$$

where v is the deformation/displacement velocity.

Because the $(b_{\perp,n}, t_n)$ pairs are sparse and nonuniform, a typical method used for reconstructing the function $p_\gamma(s, v)$ is the Capon filter [8, 9]:

$$\hat{p}_\gamma(s, v) = \frac{\mathbf{a}^H(s, v) \hat{\mathbf{R}}^{-1} g(\mathbf{r}_k)}{\mathbf{a}^H(s, v) \hat{\mathbf{R}}^{-1} \mathbf{a}(s, v)}, \tag{2.22}$$

where $\hat{\mathbf{R}}$ is a multi-look estimate of the data vector $g(\mathbf{r}_k)$ covariance matrix. The power spectral density (PSD) is then obtained as the power of $\hat{p}_\gamma(s, v)$. The elevation and mean displacement velocities of the targets can be extracted as the peaks in the elevation-velocity PSD.

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