Preface

Spectral analysis focuses on the relation between an object and its spectral characteristics, sometimes referred to as the duality relation. The object dealt with in this book is the graph (both finite and infinite), which is a very common concept in mathematics that can be traced back to Euler's paper on seven bridges of Königsberg in 1736. A graph consists of vertices with adjacency relations forming edges, and the spectral characteristics we discuss are eigenvalues of a certain matrix associated with a graph, more generally, its spectral distribution. The research in this line, bearing a discrete analogue of spectral geometry, has been developed under the name of *spectral graph theory*. (See Biggs [17], Brouwer and Haemers [28], Chung [35], Cvetković *et al*. [39–41], Godsil and Royle [53], and references cited therein.)

We are particularly interested in the spectral analysis of large graphs or of growing graphs, being motivated by the recent trend of complex network theory. Since the epoch-making papers by Barabási and Albert [14] and Watts and Strogatz [142], an enormous number of papers have been published covering a wide range of science. Accordingly, mathematical interest is steadily increasing, as is seen in the books by Blanchard and Volchenkov [18], Chung and Lu [36], Durrett [47], Lovász [95], and others.

The main purpose of this book is to outline the quantum probabilistic techniques in the spectral analysis of graphs and to provide some new results after the book by Hora and Obata [77]. The quantum (or non-commutative) probability theory, traced back to the famous work by von Neumann [140], is a non-commutative extension of the traditional probability theory and has developed along with quantum statistics as well as from purely mathematical interests. (See, e.g., Accardi *et al*. [5], Gudder [58], Meyer [101], Nica and Speicher [107], Parthasarathy [117], Speicher [131], and Voiculescu *et al*. [139].) As classical probabilistic concepts and techniques such as the law of large numbers, central limit theorem, large deviation principle, and so forth have been widely applied to asymptotic problems, it is highly anticipated that quantum probability theory will play an essential role in exploring statistical properties of non-commutative systems.

The first main topic of this book is the *method of quantum decomposition*, which makes it possible for us to study a classical random variable or a probability distribution within the framework of quantum probability. This method was first introduced explicitly by Hashimoto [65] in the study of central limit theorems for discrete groups and was applied to asymptotic spectral analysis of Hamming graphs by Hashimoto *et al*. [66]. In fact, the essence of quantum decomposition is based on the crucial observation of Accardi and Bożejko [2] that the three-term recurrence relation satisfied by orthogonal polynomials is nothing more than the interacting Fock space structure. In Chap. 6, we outline the basic idea and show concrete examples.

The second main topic of this book is *product structures of graphs*. In quantum probability, there are quite a few concepts of independence arising from non-commutativity: the free independence by Voiculescu [139], the monotone independence by Lu [96] and Muraki [102], the Boolean independence by Speicher and Woroudi [132], and many other variants. In connection with spectral analysis of graphs, Accardi *et al*. [6] first observed that the adjacency matrix of a comb product graph is decomposed into a sum of monotone-independent random variables. Similarly, it was found by Obata [108] that Boolean independence appears in a star product graph. In Chap. 7, we discuss product graphs from the point of view of quantum probability and some applications—for example, counting walks in restricted lattices by means of the Kronecker product. Free independence emerges naturally from free product graphs; however, due to page limitations we deal only with product structures that can be realized in the Cartesian product graph.

Another topic that we could not include in this book is the distance matrix and its entry-wise exponential called the Q-matrix. Conditional negativity of the distance matrix and positivity of the Q-matrix are interesting from several aspects. Furthermore, the Q-matrix gives rise to q-deformation of the vacuum spectral distribution. It is desirable to extend our methods to digraphs (directed graphs) and random (growing) graphs. Quantum probability seems to be useful also for operator calculus on graphs, as is mentioned in Schott and Staples [122]. Some relevant results along these directions are found in the references listed at the end of this book.

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