

Preface

The theory of measures of noncompactness constitutes a very important branch of nonlinear functional analysis. It finds a lot of applications in operator theory. First of all, it allows us to select very significant class of operators being generalizations of compact operators. Those operators are known as operators satisfying the Darbo condition or contractions with respect to a measure of noncompactness as well as condensing operators.

Measures of noncompactness are widely applied in fixed point theory and are especially useful in investigations connected with the theories of differential equations, integral equations, functional integral equations and in optimization theory.

The present book contains a comprehensive treatment of the theory of measures of noncompactness and its applications. The book is addressed to the large audience of mathematical community applying the methods and tools of nonlinear analysis and investigating a lot of topics connected with that large branch of mathematics.

This book can also serve as a source of examples, references and new approaches associated with the theory of measures of noncompactness and its numerous valuable suggestions.

Apart from Preface, the present book consists of 13 chapters treating various topics of analysis in which the theory of measures of noncompactness is applicable. Each chapter is self-contained and contributed by specialists using in their research the methods and techniques associated with the theory of measures of noncompactness. All chapters contain several examples and complete references for the topics considered in them.

In Chap. 1 “Measures of Noncompactness in the Space of Continuous and Bounded Functions Defined on the Real Half-Axis” the authors present a review of results concerning measures of noncompactness in the space of real functions defined, continuous and bounded on the real half-axis and equipped with the classical supremum norm. Moreover, measures of noncompactness in a more general space of functions defined, continuous on real half-axis and tempered by a given function, are investigated. A number of applications to the solvability of

nonlinear integral equations of measures of noncompactness in question are included.

Chapter 2 entitled “Measures of Noncompactness and their Applications” provides a survey of some results concerning classical measures of noncompactness in miscellaneous spaces. In particular, some fixed point theorems are established for different classes of operators. Moreover, some applications of measures of noncompactness to functional equations, including nonlinear integral equations of fractional orders, implicit fractional integral equations and the so-called q -integral equations of fractional orders, are presented.

“On Some Results Using Measures of Noncompactness” creates Chap. 3 of our book. It contains a review of some results on measures of noncompactness including the classical Kuratowski, Hausdorff, inner Hausdorff and separation measures of noncompactness as well as measures defined in an axiomatic way. Apart from this, the role of the mentioned measures of noncompactness in fixed point theory and operator theory is discussed. Further, the authors presented methods of characterization of compact matrix operators between some BK spaces, matrix domains of triangles in certain sequence spaces and spaces of strongly summable and bounded sequences.

Next, Chap. 4 “Space of Functions with Growths Tempered by a Modulus of Continuity” discusses the space of functions defined on a compact metric space and having growths (increments) tempered by a given modulus of continuity. The authors present a measure of noncompactness defined in the space in question and show its applicability in establishing some existence results for nonlinear quadratic integral equations. A few examples illustrating the discussed results are included.

“Measure of Noncompactness in Functional Fractional Calculus” constitutes the next Chap. 5 devoted to the application of measures of noncompactness in the study of the Cauchy problem for fractional differential equations. This chapter presents the extension of a few classical results from the theory of ordinary differential equations in Banach spaces to the theory of fractional differential equations. A review of such results is given.

The subsequent Chap. 6 entitled “Measures of Weak Noncompactness and Fixed Points” is dedicated to exhibiting the importance of the use of measures of weak noncompactness in topological fixed point theory. This chapter shows simultaneously the applicability of the theory of measures of weak noncompactness in the theory of integral and partial differential equations. A few results concerning the existence of fixed points of operators acting in Banach algebras are also included.

Chapter 7 “The Class of F -Contraction Mappings with a Measure of Noncompactness” deals with the discussion of the existence of fixed points for some classes of operators. The results contained in the chapter are obtained with the help of measures of noncompactness. The usefulness of the mentioned existence results in the theory of functional and integral equations is demonstrated.

Next, Chap. 8 “On the Measure of Noncompactness in Banach Spaces and Application to the Theory of Differential and Integral Equations” presents a review of some important generalizations of Darbo’s fixed point theorem. The results provided in the chapter are applied to the study of the existence and qualitative

behaviour of solutions of some integral and differential equations with feedback control.

In Chap. 9 “Partial Hadamard-Stieltjes Fractional Integral Equations in Banach Spaces”, a few existence results concerning some classes of functional partial integral equations with the use of Hadamard fractional operators are given. The main tool used in considerations is the combination of the technique of measures of noncompactness with fixed point theorems of Darbo and Mönch type.

The next Chap. 10 entitled “On the Aronszajn Property for Differential Equations of Fractional Order in Banach Spaces” presents a survey of results concerning some topological properties of the set of solutions of differential and integro-differential equations of fractional order in Banach spaces. The so-called Aronszajn property creates the considerations of this chapter. The Kuratowski measure of noncompactness is the main tool exploited in the chapter in question.

“On the Qualitative Behaviours of Nonlinear Functional Differential Systems of Third Order” is Chap. 11 that discusses new sufficient conditions for the boundedness and global asymptotic stability of solutions of some nonlinear delay differential systems of third order. The technique of Lyapunov–Krasovskii functionals in combination with LaSalle’s invariant principle is basic tools used for reasonings in this chapter.

In Chap. 12 “On the Approximation of Solutions to a Fixed Point Problem with Inequality Constraints in a Banach Space Partially Ordered by a Cone” a few results are given on the approximation of solutions to a fixed point problem concerning mappings defined on a metric space furnished with two partial orders, under constraint inequalities.

The last Chap. 13 “A Short Survey on Dislocated Metric Spaces via Fixed Point Theory” presents a short review of results on dislocated metric and b -metric spaces via the fixed point theory for operators acting on the mentioned dislocated metric spaces.

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