

Probabilistic Analysis of Performance Measures of Redundant Systems Under Weibull Failure and Repair Laws

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Abstract The aim of the present paper is to analyze probabilistically various performance measures of two redundant systems under Weibull failure and repair activities using semi-Markov processes and regenerative point technique. Two stochastic models comprise of one original and one duplicate units are developed with the provision of a single repair facility and priority. All repairs and preventive maintenance, after a pre-specific time, are perfect. Recurrence relations of availability, mean time to system failure and profit function for both the models are derived. To highlight the importance of the system, numerical and graphical results for the difference of both models are obtained with respect to failure rate of original unit.

Keywords Weibull failure and repair laws • Redundant system • Preventive maintenance • Priority • Availability

1 Introduction

In the current age of science and technology, reliability, availability, and expected profit of a system play a very dominant role in the market of IT, communication, computers electrical, and manufacturing systems. During the last few decades, many reliability improvement techniques have been developed by researchers. Redundancy, provision of spare unit, is one technique that is used to enhance the reliability of the system. Standby redundant systems have been studied by many authors such as Cao and Wu [1], Goel and Sharma [3], Chandrasekhar et al. [2], Mahmoud and Moshref [9], Moghaddass et al. [10], Wu and Wu [11], and Kumar

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and Saini [6] discussed two-unit cold standby systems under different set of assumptions such as repair, replacement, random shocks, priority, preventive maintenance, inspection, and constant repair and failure distributions. The Weibull distribution has been used in many different fields with many applications. The hazard function of the Weibull distribution can be increasing, decreasing, or constant. It is a very attractive property of Weibull distribution. For many years, researchers Gupta et al. [4] suggest a reliability model for a cold standby system using Weibull distribution for repair and failure rates. Kishan and Jain [8] analyzed a two-unit parallel system by classical and Bayesian approach with Weibull failure and repair laws. Kumar and Malik [5] developed a reliability model for single unit systems in which all random variables are arbitrary distributed like Weibull distribution. Recently, Kumar et al. [7] analyzed a non-identical unit's redundant system using the concept of preventive maintenance, priority, and Weibull distribution for all-time variables.

The main objective of the present paper is to analyze two reliability models of two non-identical units by using Weibull distribution for all-time variables with different scale and equal shape parameters. For, concrete study, difference graphs are derived for various reliability characteristics of both models. The following reliability characteristics of interest to system developers by using regenerative point technique and semi-Markov technique are evaluated.

1. Steady state transition probabilities of different states for both reliability models
2. Mean sojourn times at different states for both reliability models,
3. Reliability and MTSF for both models,
4. Availability of both models,
5. Expected busy period of server due to repair and preventive maintenance for both models, and
6. Net profit of both the models.

2 System Model Description, Notations and States of the Model

The system consists of two units, one original and other duplicate, in cold standby redundancy pattern. In both models, system starts working in S_0 state and upon failure of one unit system also remains operative and failed unit undergoes for repair or replacement. The system stops working upon failure of both units simultaneously in both models. In model-I all repair activities performed in FCFS manner while in model-II priority to preventive maintenance of original unit is given over repair of duplicate unit. All random variables associated with repair and failure activities are independent having Weibull density function with different scale parameters and common shape parameter as follows: failure times of the

original and duplicate unit are denoted by $f(t) = \beta\eta t^{\eta-1} \exp(-\beta t^\eta)$, $f_2(t) = h\eta t^{\eta-1} \exp(-ht^\eta)$, $g(t) = \alpha\eta t^{\eta-1} \exp(-\alpha t^\eta)$, $g_1(t) = \gamma\eta t^{\eta-1} \exp(-\gamma t^\eta)$, $f_1(t) = k\eta t^{\eta-1} \exp(-kt^\eta)$ and $f_3(t) = l\eta t^{\eta-1} \exp(-lt^\eta)$ with $t \geq 0$ and $\theta, \eta, \alpha, \beta, h, k, l > 0$.

3 Notations

O	Original operative unit
DCs	Duplicate unit in standby
Do	Operative duplicative unit
Fur/FUR	Original unit failed and under repair/continuously under repair
DFur/DFUR	Duplicate unit failed and under repair/continuously under repair
DPm/DWPm	Duplicate unit under/waiting for preventive maintenance
Pm/WPm	Original unit under/waiting for preventive maintenance
PM/WPM	Original unit continuously under/waiting for preventive Maintenance
DPM/DWPM	Duplicate unit continuously under/waiting for preventive maintenance
Fwr/DFwr	Original unit after failure under/waiting for repair
FWR/DFWR	Duplicate unit after failure continuously under/waiting for repair

4 Various Transition States of the System Models

In view of the above notations and assumptions, the system may be in one of the following states:

Common in Model-I and II

$S_0 = (O, DCs)$, $S_1 = (Pm, Do)$, $S_2 = (Fur, Do)$, $S_3 = (O, DFur)$, $S_4 = (O, DPm)$, $S_5 = (DPM, Fwr)$, $S_6 = (FUR, DFwr)$, $S_7 = (FUR, DWPM)$, $S_8 = (DPM, WPm)$, $S_9 = (PM, DWPM)$, $S_{10} = (PM, DFwr)$, and $S_{11} = (Fwr, DFUR)$.

Distinct states in Model-I and II

$S_{12} = (DFUR, WPm)$, and $S_{12}(Pm, DFwr)$.

5 Transition Probabilities

By considering simple probabilistic arguments and using below mentioned formula, we easily obtained transition probabilities for all possible states of both models for a particular value of the shape parameter $\eta = 1$. The probabilities of model-I are given below and for model-II are derived in similar way by using the following formula:

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)dt \quad (1)$$

For Model-I

$$\begin{aligned} p_{01} &= \frac{\alpha}{\alpha+\beta}, p_{02} = \frac{\beta}{\alpha+\beta}, p_{27} = \frac{\alpha}{\alpha+h+k} = p_{24,7}, p_{10} = \frac{\gamma}{\alpha+h+\gamma}, \\ p_{1,10} &= \frac{h}{\alpha+\gamma+h} = p_{13,10}, p_{19} = \frac{\alpha}{\alpha+\gamma+h} = p_{14,9}, p_{3,12} = \frac{\alpha}{\alpha+\beta+l} = p_{31,12}, \\ p_{20} &= \frac{k}{\alpha+k+h}, p_{26} = \frac{h}{\alpha+h+k} = p_{23,6}, p_{30} = \frac{l}{l+\alpha+\beta}, p_{3,11} = \frac{\beta}{\alpha+\beta+l} = p_{32,11}, \\ p_{40} &= \frac{\gamma}{\alpha+\beta+\gamma}, p_{45} = \frac{\beta}{\alpha+\beta+\gamma} = p_{42,5}, p_{48} = \frac{\alpha}{\alpha+\beta+\gamma} = p_{41,8}, \\ p_{52} &= p_{63} = p_{74} = p_{81} = p_{94} = p_{10,3} = p_{11,2} = p_{12,1} = 1 \end{aligned} \quad (2)$$

6 Mean Sojourn Times

The expected time to stay in any particular state by system prior visiting to any other position is called mean sojourn time. If T_i is sojourn time at state S_i , the mean sojourn time (ψ_i) is obtained by formula $\psi_i = \int P(T_i > t)dt$. Mean sojourn time of the model-I is given below for rest states we can obtained in similar way.

$$\psi_0 = \frac{\Gamma(1+1/\eta)}{(\alpha+\beta)^{1/\eta}}, \psi_3 = \frac{\Gamma(1+1/\eta)}{(\alpha+\beta+l)^{1/\eta}}, \psi_2 = \frac{\Gamma(1+1/\eta)}{(\alpha+k+h)^{1/\eta}}, \psi_4 = \frac{\Gamma(1+1/\eta)}{(\alpha+\beta+\gamma)^{1/\eta}}, \psi_1 = \frac{\Gamma(1+1/\eta)}{(\alpha+\gamma+h)^{1/\eta}}$$

7 Reliability Measures

7.1 Mean Time to System Failure

By using probability concepts, certain recursive relations for reliability analysis of a system model are obtained for cumulative density function between regenerative states where failed state is an absorbing state:

$$R_i(t) = \sum_j Q_{i,j}(t) \Theta R_j(t) + \sum_k Q_{i,k}(t), \quad \begin{cases} i=0, 1, 2, 3, 4 \text{ for Model-I} \\ i=0, 1, 2, 3, 4 \text{ for Model-II} \end{cases} \quad (3)$$

The mean time to system failure (MTSF) is obtained by taking LST of above relations and using the formula appended below

$$\lim_{s \rightarrow 0} \frac{1 - V_0^{**}(s)}{s}.$$

7.2 Availability Analysis

Let $Av_i(t)$ denotes the up-state probability at regenerative state S_i at $t = 0$. The recursive relations for $Av_i(t)$ are given as

$$Av_i(t) = \sum_j q_{i,j}(t) \Theta Av_j(t) + M_i(t), \quad \begin{cases} i=0, 1, 2, 3, 4 \text{ for Model-I} \\ i=0, 1, 2, 3, 4, 12 \text{ for Model-II} \end{cases} \quad (4)$$

The probability to stay operative in a particular regenerative state is denoted by $M_i(t)$ up to a time point without any transition. Taking LT from Eq. (4) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s \rightarrow 0} s A_0^*(s)$$

7.3 Some Other Reliability Characteristics

By using probabilistic arguments, recurrence relations for other reliability characteristics such as busy period of server due to preventive maintenance, repair, expected number of repairs, preventive maintenance, and expected number of visits by the server, respectively, denoted by $B_i^p(t)$, $B_i^R(t)$, $R_i^r(t)$, $R_i^p(t)$, and $N_i(t)$ are derived between regenerative states as follows:

$$\begin{aligned} B_i^p(t) &= \sum_j q_{i,j}(t) \Theta B_j^p(t) + W_i(t), & \begin{cases} i=0, 1, 2, 3, 4 \text{ for Model-I} \\ i=0, 1, 2, 3, 4, 12 \text{ for Model-II} \end{cases} \\ B_i^r(t) &= \sum_j q_{i,j}(t) \Theta B_j^r(t) + W_i(t), & \begin{cases} i=0, 1, 2, 3, 4 \text{ for Model-I} \\ i=0, 1, 2, 3, 4, 12 \text{ for Model-II} \end{cases} \end{aligned}$$

$$\begin{aligned}
R_i^{pm}(t) &= \sum_j Q_{i,j}^{(n)}(t) [\delta_j + R_j^{pm}(t)], & \begin{cases} i=0, 1, 2, 3, 4 \text{ for Model-I} \\ i=0, 1, 2, 3, 4, 12 \text{ for Model-II} \end{cases} \\
R_i^r(t) &= \sum_j Q_{i,j}^{(n)}(t) [\delta_j + R_j^r(t)], & \begin{cases} i=0, 1, 2, 3, 4 \text{ for Model-I} \\ i=0, 1, 2, 3, 4, 12 \text{ for Model-II} \end{cases} \\
N_i(t) &= \sum_j Q_{i,j}^{(n)}(t) [\delta_j + N_i(t)], & \begin{cases} i=0, 1, 2, 3, 4 \text{ for Model-I} \\ i=0, 1, 2, 3, 4, 12 \text{ for Model-II} \end{cases} \quad (5)
\end{aligned}$$

Taking LT of above relations and solving for $B_0^{*P}(s)$, $B_0^{*R}(s)$, $R_i^{*r}(s)$, $R_i^{*pm}(s)$, and $N_i^{**}(t)$. The time for which server is busy due to preventive maintenance, repair, expected number of preventive maintenance, repairs, and expected number of visits is given by

$$\begin{aligned}
B_0^P &= \lim_{s \rightarrow 0} s B_0^{*P}(s), \quad B_0^R = \lim_{s \rightarrow 0} s B_0^{*R}(s), \quad N_0(\infty) = \lim_{s \rightarrow 0} s \tilde{N}_0(s) \\
R_0^r(\infty) &= \lim_{s \rightarrow 0} s \tilde{R}_0^r(s), \quad R_0^p(\infty) = \lim_{s \rightarrow 0} s \tilde{R}_0^p(s)
\end{aligned}$$

8 Profit Analysis

The net expected profit incurred to the system model by defining various costs as K_i in steady state can be obtained for both models as follows:

$$P = K_0 A v_0 - K_1 B_0^R - K_2 B_0^P - K_3 R_0^R - K_4 R_0^P - K_5 N_0$$

8.1 Comparative Study

The availability and profit function for both models are obtained for a set of particular values $\alpha = 2$, $\eta = 0.5$, $\gamma = 5$, $k = 1.5$, $h = 0.009$, and $l = 1.4$. The model-II is more available and profitable than that of the model-I for all cases in which shape parameter $\eta = 0.5, 1, 2$. The availability and profit of both models are increased by increasing the value of $\gamma = 5$ to $\gamma = 7$. But, all reliability characteristics decrease with the increase in shape parameter. Hence, we can concluded on the basis of present study that the concept of priority to preventive maintenance of original unit is given over repair of duplicate unit is more profitable.

9 Graphical Results

See Figs. 1 and 2.

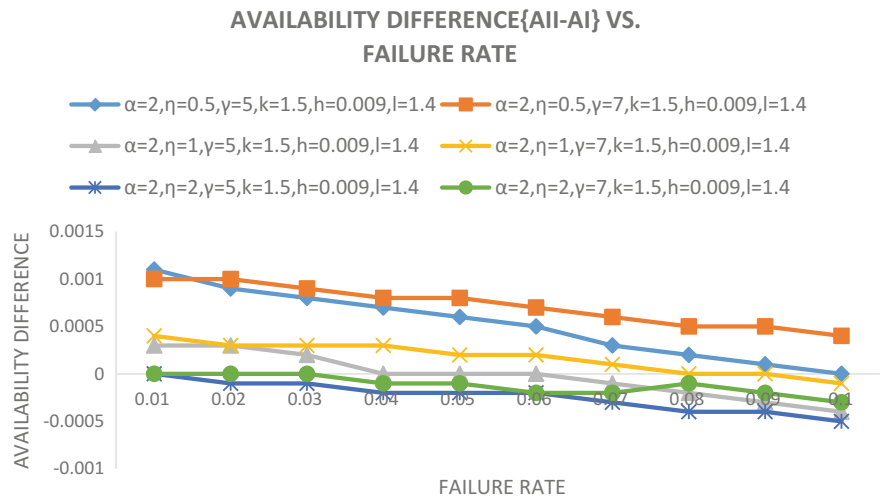


Fig. 1 Availability (AI-AII) versus failure rate (β) for shape parameter $\eta = 0.5, 1, 2$

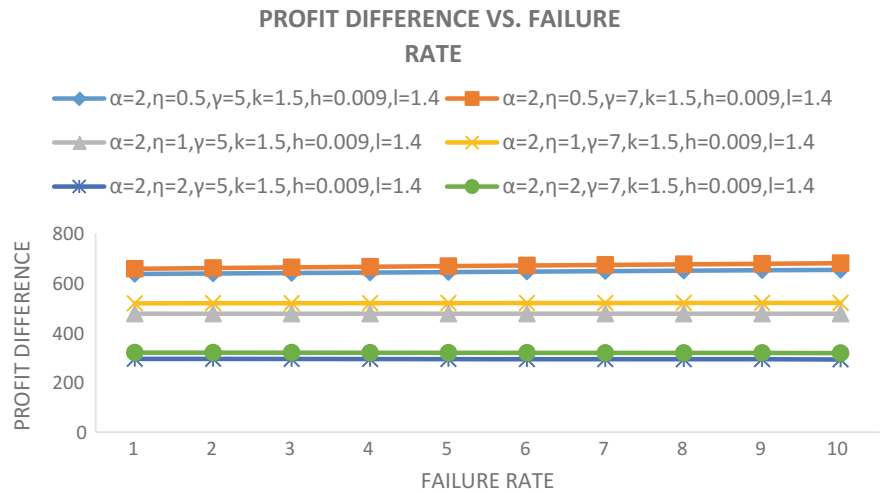


Fig. 2 Profit (PI-PII) versus failure rate (β) for shape parameter $\eta = 0.5, 1, 2$

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