Preface

It is a well-known story that in August 1834, John Scott Russel observed a large solitary wave in a shallow water channel in Scotland. He notes in his first paper [109] on the subject that

> I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed…

This solitary wave is now known as an example of a *soliton* and is described by a solution of the Korteweg-de Vries (KdV) equation [137]. The KdV equation describes one-dimensional wave propagation, such as beach waves parallel to the coastline or waves in narrow canal, and is obtained in the leading order approximation of an asymptotic perturbation theory under the assumptions of weak nonlinearity (small amplitude) and weak dispersion (long waves). The KdV equation has rich mathematical structure, including the existence of N-soliton solutions and the Lax pair for the inverse scattering method, and it is a prototype equation of the 1 + 1 dimensional integrable systems (see, e.g., [1, 2, 22, 48, 78, 89, 97, 98]).

In 1973, Kadomtsev and Petviashvili [61] proposed a 2 + 1 dimensional dispersive wave equation to study the stability of one-soliton solution of the KdV equation under the influence of weak transversal perturbations in the y-direction. This equation is now referred to as the KP equation, and it is given by

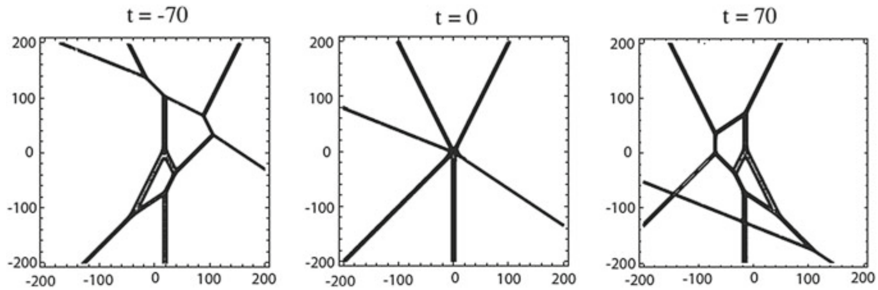
$$\frac{\partial}{\partial x}\left(-4\frac{\partial u}{\partial t}+6u\frac{\partial u}{\partial x}+\frac{\partial^3 u}{\partial x^3}\right)+3\frac{\partial^2 u}{\partial y^2}=0,$$

where $u=u(x,y,t)$ presents the wave amplitude at the point $(x,\ y)$ in the xy-plane for fixed time t. The KP equation can be used to describe shallow water waves (see, e.g., [1, 70, 79]), and in particular, the equation provides an excellent model for the *resonant* interaction of those waves [74, 87, 88]. It turns out that the KP equation has a much richer structure than the KdV equation, and is considered to be the most fundamental integrable system in the sense that many known integrable systems can be derived as special reductions of the KP hierarchy, which is defined as the set of the KP equation together with its infinitely many symmetries (see, e.g., [2, 37, 63, 75, 98]). The KP equation has been recognized to be related to various areas of mathematics and physics, such as algebraic and enumerative geometry, representation theory, random matrix theory, and quantum field theory (there are numerous papers related to these topics; see, e.g., [5, 6, 8, 12, 23, 46, 54, 60, 64, 67, 76, 95, 96, 117, 129, 134] and the references therein).

One of the main breakthroughs in the KP theory was given by Sato in [111–114], who realized that solutions of the KP equation could be written in terms of points of an infinite-dimensional Grassmannian. This book deals with a real, finite-dimensional version of the Sato theory. In particular, we are interested in solutions that are *regular* in the entire xy-plane, where they are localized along certain rays. Such solutions are called *line-soliton solutions* or *KP solitons* in this book, and they can be constructed from points of the real finite-dimensional Grassmannian.

Because of the nonlinearity in the KP equation, the solutions form very complex web-like patterns in the xy-plane which are generated by resonant interactions among several obliquely propagating line solitons. The set of figures below illustrates an example of such patterns. Each figure shows the contour plot of the solution at a fixed time t in the xy-plane with x in the horizontal and y in the vertical directions.

The main aim of this book is to give a geometric and combinatorial classification of the patterns generated by the line-soliton solutions as in the figures. There are mainly three parts: In Chaps. 1–3, I provide a brief introduction of the Sato theory of the KP equation and the KP solitons, which is the main subject of this book. Chapters 4 and 5 give an invitation to the *totally nonnegative* Grassmannians and present their parameterizations, which provides the mathematical foundation of the KP solitons. Then, Chaps. 6–8 present a classification theorem of the KP solitons and describe the structure of the spatial patterns, referred to as *soliton graphs*, generated by the KP solitons.



John Scott Russel continues on in his book [110] to say that

> This is a most beautiful and extraordinary phenomenon: the first day I saw it was the happiest day of my life. Nobody has ever had the good fortune to see it before or, at all events, to know what it meant. It is now known as the solitary wave of translation. No one before had fancied a solitary wave as a possible thing.

I hope the present book is successful in convincing you (the readers) that "*this (two-dimensional wave pattern generated by the KP equation) is a most beautiful and extraordinary phenomena*" of two-dimensional nonlinear wave dynamics, and have no doubt that observing it, for example at a beach or even in a numerical simulation of shallow water waves, will be one of the *happiest* moments of your life.

For more than 10 years, I have been working on the subject related to this book with several people. I am most grateful to my research collaborators Sarbarish Chakravarty, Lauren Williams, Ken-Ichi Maruno, Harry Yeh, Chiu-Yen Kao, Masayuki Oikawa, Hidekazu Tsuji, Gino Biondini, and my former Ph.D. students, Yuhan Jia, and Jihui Huang.

Most of the materials in this book are based on several series of lectures. The first two series were given at the Chinese Academy of Science in Beijing, June of 2008 and July of 2009. I would like to thank Qing-Ping Liu, Xing-Biao Hu, and Ke Wu for the invitation, and Tian Kai for taking lecture notes.

In the winter quarter of 2010 at the Ohio State University, I delivered a graduate class titled "*Nonlinear Waves*," which gives an overview of the background information for Chaps. 1 and 2 of this book. I would like to thank the students in the class for their comments and discussions. In particular, I am grateful to Chuanzhong Li, a visiting student from China, for his many useful suggestions.

During the second half of the year of 2012, I visited several institutes and Universities to give a series of lectures on the subjects related to this book. First, in June, I was invited to give a series of lectures titled "*Mathematical foundation of integrable systems and their applications*" at the University of Roma Tre, and would like to thank Decio Levi for the invitation and discussions. Then, in July, I was at the Institute of Mathematics, Academia Sinica in Taipei, to present "*Real Grassmannian and KP solitons*." I would like to thank Jyh-Hao Lee and Derchyi Wu for the invitation and their kind hospitality during my stay in Taipei. In September, I delivered a series of lectures titled "*Real Grassmann varieties and their applications to integrable systems*" to the graduate students of Kyushu University in Japan, and would like to thank Hiroaki Hiraoka for the invitation and his kind hospitality. Then in October, I gave a similar lecture series at Nagoya University and would like to thank Masashi Hamanaka for his kind hospitality during my stay in Nagoya.

I was then selected to be a principal speaker at the NSF-CBMS conference at UTPA in May, 2013, and delivered 10 lectures on "*Solitons in two-dimensional*

water waves and applications to tsunami." I would very much like to thank Ken-Ichi Maruno and Virgil Pierce for organizing such an excellent conference.

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