

Contents

Part I Modeling

1	Introduction	3
1.1	Vibration Tests	5
1.1.1	Free Vibration Test	6
1.1.2	Forced Vibration Test	7
1.1.3	Ambient Vibration Test	8
1.2	Uncertainties	9
1.2.1	Variability and Identification Uncertainty	9
1.2.2	Sources of Identification Uncertainty	10
1.3	OMA Methods	10
1.4	Non-Bayesian Approach	11
1.4.1	Eliminating Random Response	12
1.4.2	Exploiting Statistics	12
1.4.3	Identification Uncertainty	14
1.5	Bayesian Approach	15
1.5.1	Philosophy	15
1.5.2	Posterior Distribution and Statistics	16
1.5.3	Computing Posterior Statistics	17
1.5.4	Formulations and Algorithms	17
1.5.5	Maximum Likelihood Estimation	18
1.5.6	Drawbacks and Limitations	18
1.6	Overview of This Book	19
1.6.1	Modeling	20
1.6.2	Inference	20
1.6.3	Algorithms	21
1.6.4	Uncertainty Laws	21
1.7	How to Use This Book	22
1.7.1	Student	22
1.7.2	Researcher	24

1.7.3	Practitioner	25
1.7.4	Supporting Resources	25
	References.	26
2	Spectral Analysis of Deterministic Process	29
2.1	Periodic Process (Fourier Series)	30
2.1.1	Complex Exponential Form.	32
2.1.2	Parseval Equality.	34
2.2	Non-periodic Process (Fourier Transform)	35
2.2.1	From Fourier Series to Fourier Transform.	35
2.2.2	Properties of Fourier Transform.	37
2.2.3	Dirac Delta Function	38
2.2.4	Parseval Equality.	38
2.3	Discrete-Time Approximation with FFT.	39
2.3.1	Fast Fourier Transform	40
2.3.2	Approximating Fourier Transform and Fourier Series	42
2.3.3	Parseval Equality.	43
2.4	Distortions in Fourier Series	43
2.4.1	Nyquist Frequency	44
2.4.2	Aliasing.	44
2.4.3	Leakage.	46
2.5	Distortions in Fourier Transform	49
2.6	Summary of FFT Approximations	50
2.7	Summary of Fourier Formulas, Units and Conventions	50
2.7.1	Multiplier in Fourier Transform.	50
2.8	Connecting Theory with Matlab	53
2.9	FFT Algorithm	54
2.9.1	Basic Idea	55
2.9.2	Computational Effort.	56
	References.	57
3	Structural Dynamics and Modal Testing	59
3.1	SDOF Dynamics.	60
3.1.1	Natural Frequency.	61
3.1.2	Damping Ratio	63
3.1.3	Damped Free Vibration	63
3.1.4	Logarithmic Decrement Method	67
3.1.5	Harmonic Excitation	68
3.1.6	Simplifying Algebra with Complex Number	71
3.1.7	Dynamic Amplification	72
3.1.8	Half-Power Bandwidth Method.	74
3.1.9	Principle of Superposition	77
3.1.10	Periodic Excitation	78

3.1.11	Ideal Impulse Excitation	80
3.1.12	Arbitrary Excitation	81
3.1.13	Summary of SDOF Response	83
3.2	MDOF Dynamics	83
3.2.1	Natural Frequencies and Mode Shapes	86
3.2.2	Eigenvalue Problem	88
3.2.3	Modal Superposition and Classical Damping	92
3.2.4	Rayleigh Quotient	94
3.3	Remarks on Damping	98
3.4	Harmonic Load Test	102
3.4.1	Collocated Setup	102
3.4.2	Least Squares Approach	103
3.5	Impact Hammer Test	105
3.5.1	Frequency Response	105
3.5.2	Least Squares Approach	107
3.5.3	Covering DOFs in Multiple Setups	108
3.6	State-Space Approach	110
3.6.1	Matrix Exponential	112
3.6.2	Eigenvalue Properties of System Matrix	113
3.7	Time Integration Scheme	117
3.7.1	Numerical Stability and Accuracy	117
3.7.2	Discrete-Time State-Space Analysis	120
3.8	Newmark Scheme	122
3.8.1	SDOF Linear Acceleration	123
3.8.2	SDOF General Scheme	124
3.8.3	General MDOF Scheme	125
3.8.4	Parameters and Numerical Stability	126
3.8.5	Derivation of Stability Criterion	127
	References	130
4	Spectral Analysis of Stationary Stochastic Process	133
4.1	Correlation Function	134
4.2	Power Spectral Density	136
4.3	Fourier Series, Fourier Transform and PSD	137
4.4	Continuous-Time Sample Process	140
4.4.1	Sample Correlation Function	140
4.4.2	Sample Power Spectral Density	142
4.4.3	Wiener-Khinchin Formula	142
4.4.4	Parseval Equality	143
4.4.5	White Noise	144
4.5	Discrete-Time Sample Process	147
4.5.1	Sample Correlation Function	147
4.5.2	Sample Power Spectral Density	149

4.5.3	Wiener-Khinchin Formula	150
4.5.4	Parseval Equality	152
4.6	Averaging Sample PSD	153
4.7	Distortions in Sample Estimators	156
4.7.1	Sample Correlation Function	156
4.7.2	Sample PSD	157
4.8	Second Order Statistics of Scaled DTFT	163
4.8.1	Complex Covariance and Pseudo-covariance Matrix	164
4.8.2	Convolution Formula.	164
4.8.3	Long-Data Asymptotics of Scaled FFT	167
4.8.4	How Long Is Long?	169
4.9	Asymptotic Distribution of Scaled FFT	171
4.10	Asymptotic Distribution of Sample PSD.	172
4.10.1	Scalar Process	173
4.11	Summary of Fourier Formulas, Units and Conventions	174
4.11.1	Multiplier in Wiener-Khinchin Formula	174
4.11.2	One-Sided Versus Two-Sided Spectrum	174
	References.	177
5	Stochastic Structural Dynamics	179
5.1	Stationary SDOF Response.	180
5.1.1	Scaled Fourier Transform	181
5.1.2	Power Spectral Density	182
5.1.3	Response Variance	183
5.1.4	Response to White Noise	184
5.2	Stationary MDOF Response	185
5.2.1	Scaled Fourier Transform	186
5.2.2	Power Spectral Density	187
5.2.3	Response Variance	188
5.2.4	Mode Shape Scaling	188
5.3	Transient Response Variance	191
5.3.1	Governing Equation.	192
5.3.2	Solution Procedure	193
5.3.3	Limiting Stationary Value	194
5.3.4	Response to White Noise	195
5.4	Transient Response Correlation	197
5.4.1	Governing Equation.	198
5.4.2	Limiting Stationary Value	199
5.4.3	Response to White Noise	201
5.5	Summary of Theories and Connections	203
	References.	204

6	Measurement Basics	205
6.1	Data Acquisition Process	205
6.2	Channel Noise	206
6.3	Sensor/Hardware Noise	207
6.4	Sensor Principle	209
6.5	Aliasing	212
6.6	Quantization Error	213
6.6.1	Statistical Properties	215
6.6.2	Power Spectral Density	215
6.7	Synchronization	216
6.8	Channel Noise Calibration	218
6.8.1	Base Isolation	219
6.8.2	Huddle Test	220
6.8.3	Three Channel Analysis	222
	References	224
7	Ambient Data Modeling and Analysis	225
7.1	Resonance Band Characteristics	226
7.1.1	Single Mode	227
7.1.2	Multi-mode	228
7.2	PSD Spectrum	228
7.2.1	Procedure	229
7.3	Singular Value Spectrum	231
7.3.1	Single Mode	232
7.3.2	Multi-mode	234
7.4	Illustration with Field Data	237
7.4.1	Time Histories	238
7.4.2	Sample PSD (No Averaging)	238
7.4.3	Sample PSD (Averaged)	239
7.4.4	Singular Value Spectrum	240
7.5	Asynchronous Data	241
7.5.1	Two Measurement Groups	242
7.5.2	Multiple Measurement Groups	247
7.6	Microtremor Data	249
7.6.1	Background Seismic Noise	249
7.6.2	Site Amplification and H/V Spectrum	252
7.7	Simulation of Ambient Data	255
7.7.1	Gaussian Scalar Process	255
7.7.2	Gaussian Vector Process	258
7.7.3	Quantifying Noise Level	260
	References	261

Part II Inference

8	Bayesian Inference	265
8.1	Bayes' Theorem	266
8.2	Updating Knowledge Using Data	267
8.3	System Identification Framework	268
8.4	Identifiability	268
8.5	Globally Identifiable Problems	274
8.5.1	Quality of Gaussian Approximation	275
8.6	Locally Identifiable Problems	283
8.7	Unidentifiable Problems	284
8.8	Model Class Selection	285
8.8.1	Comparing Model Classes with Evidence	285
8.8.2	Model Trade-off	286
	References.	288
9	Classical Statistical Inference	291
9.1	Statistical Estimators	293
9.1.1	Quality Statistics	293
9.1.2	Bias and Convergence	294
9.1.3	Empirical Statistics	294
9.2	Maximum Likelihood Estimator	295
9.3	Cramér-Rao Bound	300
9.3.1	Easier but Looser Bounds	307
9.3.2	General Form	310
9.3.3	Derivation	310
9.4	Fisher Information Matrix for Gaussian Data	312
9.4.1	Real Gaussian	312
9.4.2	Complex Gaussian	315
9.5	Asymptotic Properties of ML Estimator	316
9.6	Comparison with Bayesian Inference	319
9.6.1	Philosophical Perspectives	319
9.6.2	Maximum Likelihood Estimator	320
9.6.3	Cramér-Rao Bound and Uncertainty Law	321
	References.	324
10	Bayesian OMA Formulation	325
10.1	Single Setup Data	325
10.1.1	Likelihood Function	326
10.1.2	Single Mode	329
10.2	Remarks to Formulation	329
10.2.1	Complex Gaussian FFT	330
10.2.2	Selected Frequency Band	330
10.2.3	Prediction Error Model	331
10.2.4	Measurement Type	332

10.2.5	Mode Shape Scaling	333
10.2.6	Leakage	335
10.3	Multi-setup Data	336
10.3.1	Global and Local Mode Shape	337
10.3.2	Reference DOFs	337
10.3.3	Parameters in Different Setups	338
10.3.4	Likelihood Function	339
10.3.5	Single Mode	340
10.4	Asynchronous Data	341
10.4.1	PSD Matrix	341
10.4.2	Single Mode	343
11	Bayesian OMA Computation	345
11.1	Posterior Most Probable Value	346
11.2	Posterior Covariance Matrix	348
11.2.1	Mapping with Free Parameters	348
11.2.2	Transformation of Covariance Matrix	349
11.2.3	Hessian of Composite Function	349
11.2.4	Transformation Invariance	351
11.2.5	Constraint Singularity	352
11.2.6	Pseudo-inverse	353
11.2.7	Singular Vector Formula	355
11.2.8	Dimensionless Hessian	356
11.3	Mode Shape Uncertainty	359
11.3.1	Norm Constraint Singularity	360
11.3.2	Stochastic Representation	360
11.3.3	Expected MAC and Mode Shape c.o.v	361
	Reference	362
 Part III Algorithms		
12	Single Mode Problem	365
12.1	Alternative Form of NLLF	366
12.2	Algorithm for MPV	368
12.3	High s/n Asymptotics of MPV	368
12.3.1	Initial Guess of MPV	370
12.4	Posterior Covariance Matrix	370
12.4.1	General Expressions	371
12.4.2	Condensed Expressions	372
12.5	Synthetic Data Examples	374
12.6	Laboratory/Field Data Examples	381
	References	390

13	Multi-mode Problem	391
13.1	Mode Shape Subspace	393
13.1.1	Orthonormal Basis Representation	393
13.2	Alternative Form of NLLF	394
13.3	Most Probable Mode Shape Basis	396
13.3.1	Hyper Angle Representation	396
13.3.2	Rotation Matrix	397
13.3.3	Newton Iteration	398
13.4	Most Probable Spectral Parameters	402
13.4.1	Parameterizing Structured Matrices	402
13.5	Algorithm for MPV	403
13.6	High s/n Asymptotics of MPV	404
13.6.1	Initial Guess of MPV	405
13.7	Posterior Covariance Matrix	405
13.7.1	General Expressions	407
13.7.2	Condensed Expressions	408
13.8	Illustrative Examples	413
	References	418
14	Multi-setup Problem	419
14.1	Local Least Squares	420
14.2	Global Least Squares	422
14.2.1	Partial Solutions	423
14.2.2	Limiting Behavior of Solution	424
14.2.3	Iterative Algorithm	425
14.2.4	Reference Condensation	426
14.3	Bayesian Method	427
14.3.1	Alternative Form of NLLF	428
14.3.2	Partial MPV of Global Mode Shape	430
14.3.3	Algorithm for MPV	431
14.3.4	High s/n Asymptotic MPV	431
14.3.5	Initial Guess	433
14.3.6	Asymptotic Weight for Global Least Squares	433
14.3.7	Posterior Covariance Matrix	434
14.4	Representative Statistics	437
14.5	Field Applications	438
	References	450

Part IV Uncertainty Laws

15	Managing Identification Uncertainties	455
15.1	Context and Key Formulas	456
15.2	Understanding Uncertainty Laws	460
15.2.1	Data Length and Usable Bandwidth	461

15.2.2	Signal-to-Noise Ratio	462
15.2.3	First Order Effect of Modal s/n Ratio	463
15.2.4	Governing Uncertainty	463
15.3	Demonstrative Examples.	464
15.4	Planning Ambient Vibration Tests	467
15.4.1	Simple Rule of Thumb	469
15.4.2	Accounting for Channel Noise and Measured DOFs	469
15.5	Common Sense.	471
16	Theory of Uncertainty Laws.	473
16.1	Long Data Asymptotics	474
16.1.1	Fisher Information Matrix	475
16.2	Small Damping Asymptotics	479
16.3	Asymptotic Decoupling	479
16.3.1	Scalar Parameter	479
16.3.2	Vector-Valued Parameter.	481
16.4	Leading Order Uncertainty	482
16.5	First Order Effect of Signal-to-Noise Ratio.	485
16.6	Other Data Types	488
	Appendix A Asymptotics of $\Sigma_k D_k^a (\beta_k - 1)^b \beta_k^c$	489
	Appendix B Derivation of Small Damping Asymptotics (Zeroth Order).	493
	Appendix C First Order of NLLF Derivatives w.r.t. f, ζ, S	496
	Reference	498
	Appendix A. Complex Gaussian and Wishart Distribution	499
A.1	Complex Gaussian Vectors	499
A.2	Covariance and Pseudo-covariance Matrix.	501
A.3	Complex Gaussian PDF.	502
A.4	Circular Symmetry.	505
A.5	Complex Wishart Distribution	507
A.5.1	Scalar Variable.	509
	References	510
	Appendix B. Hessian Under Constraints.	511
B.1	Direct Formula.	514
B.1.1	Derivation.	517

B.2 Lagrange Multiplier Formula	519
B.2.1 Derivation.	520
B.2.2 Transformation Invariance	522
Appendix C. Mathematical Tools.	525
C.1 Asymptotics	525
C.2 Linear Algebra	526
C.2.1 Linear Independence, Span, Basis and Dimension	526
C.2.2 Linear Transformation, Rank and Nullity	527
C.2.3 Euclidean Norm, Inner Product and Orthogonality	527
C.2.4 Eigenvalue Problem	528
C.2.5 Diagonalizable Matrices and Their Functions.	529
C.2.6 Real Symmetric and Hermitian Matrices	530
C.3 Lagrange Multiplier Method.	532
C.4 Minimizing Quadratic Forms	533
C.5 Identities and Inequalities.	535
C.5.1 Matrix Inverse Lemma and Determinant Theorem	536
C.5.2 Block Matrix Determinant and Inverse.	536
C.5.3 Derivatives of Log-Determinant and Inverse	536
C.5.4 Gradient and Hessian of Rayleigh Quotient	537
C.5.5 Gradient and Hessian of Unit Vector	537
C.5.6 Cauchy-Schwartz Inequality	538
C.5.7 Inequality for Partition of Inverse.	539
C.5.8 Inequality for Covariance of Two Vectors	539
References	540
Index	541

Operational Modal Analysis

Modeling, Bayesian Inference, Uncertainty Laws

Au, S.-K.

2017, XXIII, 542 p. 158 illus., 28 illus. in color.,

Hardcover

ISBN: 978-981-10-4117-4