

Preface

The study of convergence of infinite series is a very old art. In ancient times, people were interested in orthodox examination for convergence of infinite series. Divergent series, i.e., infinite series which do not converge, was of no interest to them until the advent of L. Euler (1707–1783), who took up a serious study of divergent series. He was later followed by a galaxy of very great mathematicians.

Study of divergent series is the foundation of summability theory. Summability theory has many utilities in analysis and applied mathematics. An engineer or physicist, who works on Fourier series, Fourier transforms or analytic continuation, can find summability theory very useful for his/her research.

In the present book, some of the contributions of the author to classical summability theory are highlighted, thereby supplementing, the material already available in standard texts on summability theory.

There are six chapters in all. The salient features of each chapter are listed below. In Chap. 1, after a very brief introduction, we recall well-known definitions and concepts. We state and prove Silverman–Toeplitz theorem, Schur’s theorem and then deduce Steinhaus theorem. We introduce a sequence space Λ_r , $r \geq 1$ being a fixed integer and make a detailed study of the space Λ_r , especially from the point of view of sequences of zeros and ones. We prove a Steinhaus type result involving the space Λ_r , which improves Steinhaus theorem. We prove some more Steinhaus type theorems too.

Chapter 2 deals with the core of a sequence. We present an improvement of Sherbakhoff’s result, which leads to a short and very elegant proof of Knopp’s core theorem. We also present some nice properties of the class (ℓ, ℓ) of infinite matrices.

Chapter 3 is devoted to a detailed study of some special methods of summability, viz., the Abel method, the Weighted mean method, the Euler method and the (M, λ_n) or Natarajan method. We bring out the connection between the Abel method and the Natarajan method. Some product theorems involving certain summability methods are also proved.

In Chap. 4, some nicer properties of the (M, λ_n) method are established. Further, we prove a few results on the Cauchy multiplication of certain summable series.

In Chap. 5, a new definition of convergence of a double sequence and a double series is introduced. In the context of this new definition, Silverman–Toeplitz theorem for 4-dimensional infinite matrices is proved. We also prove Schur’s and Steinhaus theorems for 4-dimensional infinite matrices.

Finally in Chap. 6, we introduce the Nörlund, the Weighted mean and the $(M, \lambda_{m,n})$ or Natarajan methods for double sequences and double series and study some of their properties.

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