The Compact Approximation Property for Weighted Spaces of Holomorphic Mappings

Manjul Gupta and Deepika Baweja

Abstract In this paper, we examine the compact approximation property for the weighted spaces of holomorphic functions. We show that a Banach space E has the compact approximation property if and only if the predual $\mathcal{G}_v(U)$ of the space $H_v(U)$ consisting of all holomorphic mappings $f : U \to \mathbb{C}$ (complex plane) with $\sup_{x \in U} v(x)\|f(x)\| < \infty$ has the compact approximation property, where v is a radial weight defined on a balanced open subset U of E such that $H_v(U)$ contains all the polynomials. We have also studied the compact approximation property for the weighted (LB)-space $VH(E)$ of holomorphic mappings and its predual $VG(E)$ for a countable decreasing family V of radial rapidly decreasing weights on E.

Keywords Weighted spaces of holomorphic mappings · Approximation property · Compact approximation property

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1 Introduction

The approximation property plays a vital role in the structural study of Banach spaces and appeared for the first time in the book by Banach [4]. A systematic study of this concept was taken up by Grothendieck [26] in the year 1955 who considered the approximation property, bounded approximation property, and the basis property. At present, we have several variants of this property such as metric approximation property, compact approximation property, strong approximation property,

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p-approximation property, and ideal approximation property, cf. [8, 20–22, 34–36, 45, 50], etc. As the identity operator on the space is approximated by linear operators having simpler representation in the study of the approximation property, there are three standard tools for studying approximation property for spaces of holomorphic mappings: ϵ-products, linearization, and $\mathcal{S}$-absolute decompositions. The notion of ϵ-products for locally convex spaces X and Y written as $X\epsilon Y$ introduced by L. Schwartz is defined as the space $\mathcal{L}_e(Y_c'; X)$ of all continuous linear operators from Y_c' to X, endowed with the topology of uniform convergence on equicontinuous subsets of Y, where Y_c' is the topological dual of Y equipped with the topology of uniform convergence on compact subsets of Y. Using the method of ϵ-products, the study of the approximation property for spaces of holomorphic mappings was initiated by Aron and Schottenloher in their pioneer work [2] and was further carried out in [14–19, 27, 29, 41, 49]. Through linearization results, one identifies a given class of holomorphic functions defined on an open subset U of a Banach space E with values in a Banach space F, with the space of continuous linear mappings from a certain Banach space G to F, i.e., a holomorphic mapping is being identified with a linear operator and so one can pursue the study of the approximation property for spaces of holomorphic mappings by this method. The first linearization theorem for such spaces was obtained by Mazet [38] in the year 1984. Almost six years later, J. Mujica obtained a linearization theorem for $\mathcal{H}^\infty(U; F)$, the space of bounded holomorphic mappings defined on an open subset U of a Banach space E with values in F; indeed, the space $\mathcal{H}^\infty(U; F)$ is being identified with $\mathcal{L}(\mathcal{G}^\infty(U); F)$ where $\mathcal{G}^\infty(U)$ is the predual of $\mathcal{H}^\infty(U)$. Using this linearization theorem, Mujica proved several results characterizing the approximation property for E in terms of the approximation property for $\mathcal{H}^\infty(U)$ and $\mathcal{G}^\infty(U)$. This study has further been continued by E. Caliskan in [15–19]. The study of the approximation property for a locally convex X having Schauder decomposition is characterized through the approximation property for the subspaces forming its Schauder decomposition; indeed, if a sequence $\{X_n\}_{n\geq 1}$ forms an $\mathcal{S}$-absolute decomposition for a locally convex space X, then X has the approximation property if and only if each X_n has the approximation property. As the sequence of spaces of m-homogenous polynomials forms an $\mathcal{S}$-absolute decomposition for their parent space, this method has been proved to be useful in such a study.

Weighted spaces of holomorphic functions defined on an open subset of a finite or infinite dimensional Banach space have been studied widely in the literature by several mathematicians. Whereas for the results in the finite dimensional case, we attribute to the contributions of K.D. Bierstedt, J. Bonet, A. Galbis, W.H. Summers, R.G. Meise, Rubel, and Shields [9–13, 46] etc., the infinite dimensional case was introduced by Garcia, Maestre, and Rueda in [25] and further investigated by Beltran [6, 7] Jorda [32], Rueda [47], etc. Though Mujica and Caliskan considered the approximation property for spaces of bounded holomorphic mappings, we initiated this study for weighted spaces in our work [27–29]. In the present article, we consider the compact approximation property for such spaces.

In Sect. 3, we show that a Banach space E has the compact approximation property if and only if the predual $\mathcal{G}_v(U)$ has the compact approximation property for a radial weight v defined on a balanced open subset U of E such that $H_v(U)$ contains all the polynomials. Also, it has been shown that E has the compact approximation property if and only if each weighted holomorphic mapping can be approximated by such a map with relatively compact range.

In Sect. 4, we introduce a locally convex topology $\tau'_{\mathcal{M}}$ and prove a characterization for the $\tau'_{\mathcal{M}}$-denseness of weighted spaces of holomorphic mappings with relatively compact range in $\mathcal{H}_v(U; F)$.

Finally, in the last section, we study the approximation properties for the weighted (LB)-spaces $VH(E)$ defined corresponding to a countable decreasing family V of radial rapidly decreasing weights and its predual $VG(E)$; indeed, it is proved that E has the approximation property if and only if $VG(E)$ has the approximation property. Also, this result holds for the compact approximation property for suitably restricted family V of weights.

2 Preliminaries

Throughout this paper, E and F denote complex Banach spaces with closed unit balls B_E and B_F, respectively. The symbols X' and X^*_b, respectively, stand for the algebraic and strong topological dual of a locally convex space X. The notation X^* is used for X^*_b in case of a normed space X. The symbols $\mathbb{N}$, $\mathbb{N}_0$, and $\mathbb{C}$ are, respectively, used for the set of natural numbers, $\mathbb{N} \cup \{0\}$, and the complex plane.

For each $m \in \mathbb{N}$, $\mathcal{L}(^m E; F)$ is the Banach space of all continuous m-linear mappings from E to F endowed with the sup norm. A mapping $P : E \to F$ is a *continuous m-homogeneous polynomial* if there exists a continuous m-linear map $A \in \mathcal{L}(^m E; F)$ such that $P(x) = A(x, \ldots, x), \quad x \in E$. The space of all m-homogeneous continuous polynomials from E to F is denoted by $\mathcal{P}(^m E; F)$ which is a Banach space endowed with the norm $\|P\| = \sup_{\|x\|\leq 1} \|P(x)\|$. For $F = \mathbb{C}$, $\mathcal{P}(^m E; \mathbb{C})$ is written as $\mathcal{P}(^m E)$. *A continuous polynomial* P is a mapping from E into F which can be represented as a sum $P = P_0 + P_1 + \cdots + P_k$ with $P_m \in \mathcal{P}(^m E; F)$ for $m = 0, 1, \ldots, k$. The vector space of all continuous polynomials from E into F is denoted by $\mathcal{P}(E; F)$. A polynomial $P \in \mathcal{P}(^m E, F)$ is said to be *compact* if it takes bounded subsets of E to relatively compact subsets of F or equivalently if $P(B_E)$ is relatively compact in F. The collection of all compact m-homogenous polynomials is denoted by $\mathcal{P}_k(^m E; F)$, and for $m = 1$, we get $\mathcal{K}(E; F)$, the class of all compact linear operators from E to F.

A mapping f from E to F is said to be *weakly uniformly continuous (weakly continuous)* on bounded sets if for each bounded subset B of E (for each $x \in B$) and $\epsilon > 0$ there exists $\phi_1, \ \phi_2, \ldots, \phi_n \in E^*$ such that

$$\|f(x) - f(y)\| < \epsilon$$

whenever $x, y \in B$ $(y \in B)$ with $|\phi_i(x-y)| < \delta$, for each $i = 1, 2, \ldots, n$. The space of all polynomials which are weakly uniformly continuous (weakly continuous) on bounded subsets of E is denoted by $\mathcal{P}_{wu}(E, F)(\mathcal{P}_w(E, F))$.

A mapping $f : U \to F$ is said to be *holomorphic*, if for each $\xi \in U$, there exists a ball $B(\xi, r)$ with center at ξ and radius $r > 0$, contained in U and a sequence $\{P^j f(\xi)\}_{j=0}^{\infty}$ of polynomials with $P^j f(\xi) \in \mathcal{P}(^j E; F)$, $j \in \mathbb{N}_0$ such that

$$f(x) = \sum_{j=0}^{\infty} P^j f(\xi)(x-\xi) \tag{1}$$

where the series converges uniformly for each $x \in B(\xi, r)$. The space of all holomorphic mappings from U to F is denoted by $\mathcal{H}(U; F)$. For $F = \mathbb{C}$, we write $\mathcal{H}(U)$ for $\mathcal{H}(U; \mathbb{C})$.

A weight v is a continuous and strictly positive function defined on an open subset U of a Banach space E. A weight v defined on (i) a balanced open set U is *radial* if $v(tx) = v(x)$ for all $x \in U$ and $t \in \mathbb{C}$ with $|t| = 1$ and (ii) E is said to be *rapidly decreasing* if $\sup_{x \in E} v(x)\|x\|^m < \infty$ for each $m \in \mathbb{N}_0$. Let us quote from [27] the following: The weighted space

$$\mathcal{H}_v(U; F) = \{f \in \mathcal{H}(U; F) : \|f\|_v = \sup_{x \in U} v(x)\|f(x)\| < \infty\}$$

of holomorphic functions is a Banach space endowed with the norm $\|\cdot\|_v$ with closed unit ball B_v. For $F = \mathbb{C}$, we write $\mathcal{H}_v(U) = \mathcal{H}_v(U; \mathbb{C})$.

Proposition 2.1 *Let v be a weight defined on an open subset U of a Banach space E. Then, for given $m \in \mathbb{N}$, following are equivalent:*

(a) $\mathcal{P}(^m E, F) \subset \mathcal{H}_v(U, F)$ *for each Banach space F.*
(b) $\mathcal{P}(^m E) \subset \mathcal{H}_v(U)$.

Proposition 2.2 *The topology $\tau_{\|\cdot\|_v}$ restricted to $\mathcal{P}(^m E)$ coincides with the sup norm topology.*

Since the closed unit ball B_v of $\mathcal{H}_v(U)$ is τ_0-compact, it follows by Ng's Theorem cf. [44], $\mathcal{H}_v(U)$ is a dual Banach space and its predual is defined as

$$G_v(U) = \{\phi \in \mathcal{H}_v(U)' : \phi | B_v \text{ is } \tau_0\text{-continuous }\}$$

which is endowed with the topology of uniform convergence on the set B_v.

Theorem 2.3 (Linearization Theorem) *For an open subset U of a Banach space E and a weight v on U, there exists a Banach space $\mathcal{G}_v(U)$ and a mapping $\Delta_v \in \mathcal{H}_v(U, \mathcal{G}_v(U))$ with the following property: For each Banach space F and each mapping $f \in \mathcal{H}_v(U, F)$, there is a unique operator $T_f \in \mathcal{L}(\mathcal{G}_v(U), F)$ such that $T_f \circ \Delta_v = f$. The correspondence Ψ between $\mathcal{H}_v(U, F)$ and $\mathcal{L}(\mathcal{G}_v(U), F)$ given by*

$$\Psi(f) = T_f$$

is an isometric isomorphism. The space $\mathcal{G}_v(U)$ is uniquely determined upto an isometric isomorphism by these properties.

A simple consequence of the above linearization theorem is

Proposition 2.4 *For a weight v defined on an open subset U of a Banach space E satisfying $\mathcal{P}(E) \subset \mathcal{H}_v(U)$, E is topologically isomorphic to a complemented subspace of $\mathcal{G}_v(U)$.*

Let us also recall the locally convex topology $\tau_{\mathcal{M}}$ on $\mathcal{H}_v(U, F)$ which is generated by the family $\{p_{\overline{\alpha},\overline{A}} : \overline{\alpha} = (\alpha_j) \in c_0^+,\ \overline{A} = (A_j),\ A_j$ being finite subset of U for each $j\}$ of semi-norms defined by

$$p_{\overline{\alpha},\overline{A}}(f) = \sup_{j\in\mathbb{N}}(\alpha_j \inf_{x\in A_j} v(x) \sup_{y\in A_j} \|f(y)\|).$$

It can be easily checked that

$$\tau_0 \leq \tau_{\mathcal{M}} \leq \tau_{\|.\|_v} \tag{2}$$

on $\mathcal{H}_v(U, F)$. For $v \equiv 1$, the space $\mathcal{H}_v(U, F) \equiv \mathcal{H}^\infty(U, F)$ and the topology $\tau_{\mathcal{M}} \equiv \tau_\gamma$ on $\mathcal{H}^\infty(U, F)$; cf. [41].

Proposition 2.5 *Let E and F be Banach spaces. For a weight v on an open subset U of E with $\mathcal{P}(E) \subset \mathcal{H}_v(U)$, $\tau_{\mathcal{M}}$ coincides with τ_0 on $\mathcal{P}(^m E; F)$ for each $m \in \mathbb{N}$.*

Proposition 2.6 *Let E and F be Banach spaces. For a radial weight v on a balanced open subset U of E with $\mathcal{P}(E) \subset \mathcal{H}_v(U)$, the space $\mathcal{P}(E; F)$ is $\tau_{\mathcal{M}}$-dense in $\mathcal{H}_v(U; F)$.*

Theorem 2.7 *Let E and F be Banach spaces, and v be a weight on an open subset U of E. Then, the mapping $\Psi : (\mathcal{H}_v(U; F), \tau_{\mathcal{M}}) \to (\mathcal{L}(\mathcal{G}_v(U); F), \tau_c)$ is a topological isomorphism.*

Let

$$\mathcal{H}_v(U) \otimes F = \{f \in \mathcal{H}_v(U, F) : f \text{ has finite dimensional range}\}$$

and

$$\mathcal{H}_v^c(U, F) = \{f \in \mathcal{H}_v(U, F) : vf \text{ has a relatively compact range}\}.$$

Then, we have

Proposition 2.8 *Let U be an open subset of a Banach space E and v be a weight on U. Then, for any Banach space F,*

(a) $f \in \mathcal{H}_v(U) \otimes F$ *if and only if* $T_f \in \mathcal{F}(\mathcal{G}_v(U); F)$*, and*
(b) $f \in \mathcal{H}_v^c(U; F)$ *if and only if* $T_f \in \mathcal{K}(\mathcal{G}_v(U); F)$*.*

A locally convex space X is said to have the *approximation property* if for every compact set K of X, a continuous semi-norm p on X and $\epsilon > 0$, there exists a finite rank operator $T \equiv T_{\epsilon,K}$ such that $\sup_{x\in K} p(T(x) - x) < \epsilon$ and the *compact approximation property* (CAP) if there is a compact linear operator T such that $\sup_{x\in K} p(T(x) - x) < \epsilon$.

The following is quoted from [27]

Theorem 2.9 *Let E be a Banach space and v be a radial weight on a balanced open subset U of E such that $H_v(U)$ contains all the polynomials. Then, E has the approximation property if and only if $\mathcal{G}_v(U)$ has the approximation property.*

Similar to the characterization of AP given by Grothedieck [26] and also given in [37], we have the following result from [16]

Theorem 2.10 *For a Banach space E, the following are equivalent:*

(i) *E has the compact approximation property.*
(ii) *For every Banach space F, $\overline{\mathcal{K}(E; E)}^{\tau_c} = \mathcal{L}(E; E)$.*
(iii) *For every Banach space F, $\overline{\mathcal{K}(F; E)}^{\tau_c} = \mathcal{L}(F; E)$.*
(iv) *For every Banach space F, $\overline{\mathcal{K}(E; F)}^{\tau_c} = \mathcal{K}(E; F)$.*

Using the definition of the CAP, one can easily prove

Proposition 2.11 *Let E be a Banach space with the compact approximation property. Then, each complemented subspace of E also has the compact approximation property.*

The space $\mathcal{Q}(^m E)$ defined as

$$\mathcal{Q}(^m E) = \{\phi \in \mathcal{P}(^m E)' : \phi|B_m \text{ is } \tau_0\text{-continuous}\}$$

is the predual of $\mathcal{P}(^m E)$, $m \in \mathbb{N}$, cf. [48]. It is a Banach space equipped with the topology of uniform convergence on B_m, the unit ball of $\mathcal{P}(^m E)$. Connecting the CAP for a Banach space E with the CAP for $\mathcal{Q}(^m E)$, E. Caliskan [16] proved.

Proposition 2.12 *Let E be a Banach space. Then, E the compact approximation property if and only if $\mathcal{Q}(^m E)$ has the compact approximation property for each $m \in \mathbb{N}$.*

Analogous to Proposition 2.2 in [42], we have.

Proposition 2.13 *Let E and F be Banach spaces such that E has the compact approximation property. Then, $\mathcal{P}_w(^m E; F)$ is τ_c-dense in $\mathcal{P}(^m E; F)$ for each $m \in \mathbb{N}$*

For the following, one may refer to [3], cf. also [1].

Proposition 2.14 *Let E and F be Banach spaces. Then, $\mathcal{P}_w(E; F) \subset \mathcal{P}_k(E; F)$.*

A sequence of subspaces $\{E_n\}_{n=1}^{\infty}$ of a Banach space E is called a *Schauder decomposition* of E if for each $x \in E$, there exists a unique sequence $\{x_n\}$ of vectors $x_n \in E_n$ for all n, such that

$$x = \sum_{n=1}^{\infty} x_n = \lim_{m\to\infty} u_m(x)$$

where the projection maps $\{u_m\}_{m=1}^{\infty}$ defined by $u_m(x) = \sum_{j=1}^{m} x_j$, $m \geq 1$ are continuous. Let $\mathcal{S} = \{(\alpha_n)\}_{n=1}^{\infty} : \alpha_n \in \mathbb{C},\ n \geq 1$ and $\limsup_{n\to\infty} |\alpha_n|^{\frac{1}{n}} \leq 1\}$. A Schauder decomposition $\{E_n\}_n$ is said to be $\mathcal{S}$-absolute if (i) for each $\beta = (\beta_j) \in \mathcal{S}$ and $x = \sum_{j=1}^{\infty} x_j \in E$, $\beta \cdot x = \sum_{j=1}^{\infty} \beta_j x_j \in E$ and (ii) if p is a continuous semi-norm on E and $\beta \in \mathcal{S}$, then $p_\beta(x) = \sum_{j=1}^{\infty} |\beta_j| p_\beta(x_j)$ defines a continuous semi-norm on E.

Following is proved in [15].

Proposition 2.15 *If $\{E_n\}_{n=0}^{\infty}$ is an $\mathcal{S}$-absolute decomposition of the locally convex space E, then E has the CAP if and only if each E_n has the CAP.*

For more background and details about the theory of infinite dimensional holomorphy, Schauder decompositions, and the approximation properties, we refer to [5, 23, 24, 26, 37, 40, 43] and the reference given therein.

3 The Compact Approximation Property for $\mathcal{G}_v(U)$

This section is devoted to the study of the compact approximation property for $H_v(U)$ and its predual $G_v(U)$.

Let us begin with

Lemma 3.1 *Let v be a weight on an open subset U of a Banach space E such that $\mathcal{P}(E) \subset H_v(U)$. Then,*

$$\sup v(x)\|x\|^m < \infty$$

for each $m \in \mathbb{N}$.

Proof Let $m \in \mathbb{N}$. For each $x \in U$, choose $\phi_x \in E^*$ such that $\|\phi_x\| = 1$ and $\phi_x(x) = \|x\|$. Write $B = \{\phi_x^m : x \in U\}$. Then, B is a $\|.\|$-bounded subset of $\mathcal{P}(^m E)$. Hence, by Proposition 2.2, B is $\|.\|_v$-bounded. Consequently,

$$\sup_{x\in U} v(x)\|x\|^m \le \sup_{x\in U}\sup_{y\in U} v(y)|\phi_x^m(y)| < \infty.$$

□

Theorem 3.2 *Let v be a radial weight on a balanced open subset U of a Banach space E such that $\mathcal{P}(E) \subset H_v(U)$. Then, the following assertions are equivalent:*

(i) *E has the compact approximation property.*
(ii) *$\overline{\mathcal{P}_v(E,F)}^{\tau_{\mathcal{M}}} = \mathcal{H}_v(U,F)$ for each Banach space F.*
(iii) *$\overline{\mathcal{P}_k(E,F)}^{\tau_{\mathcal{M}}} = \mathcal{H}_v(U,F)$ for each Banach space F.*
(iv) *$\overline{\mathcal{H}_v^c(U;F)}^{\tau_{\mathcal{M}}} = \mathcal{H}_v(U,F)$, for each Banach space F.*
(v) *$\mathcal{G}_v(U)$ has the compact approximation property.*

Proof (i)$\Rightarrow$ (ii): Let $f \in \mathcal{H}_v(U;F)$ and p be a $\tau_{\mathcal{M}}$-continuous semi-norm on $\mathcal{H}_v(U,F)$. Then, there exist $P \in \mathcal{P}(E;F)$ such that $p(f-P) < \frac{\epsilon}{2}$ by Proposition 2.6. Write $P = P_0 + P_1 + \cdots P_m$, $P_j \in \mathcal{P}(^jE,F)$, $0 \le j \le m$. Then, for each j, $0 \le j \le m$, there exist Q_j in $\mathcal{P}_w(^jE,F)$, such that

$$p(P_j - Q_j) < \frac{\epsilon}{2m}.$$

by using Propositions 2.5 and 2.13. Write $Q = Q_0 + Q_1 + \cdots + Q_k$. Clearly, $Q \in \mathcal{P}_w(E,F)$ and $p(f-Q) < \epsilon$.
(ii) $\Rightarrow$(iii) follows by Proposition 2.14.
(iii) $\Rightarrow$ (iv): It is enough to show that $\mathcal{P}_k(^jE;F) \subset \mathcal{H}_v^c(U;F)$ for each $j \in \mathbb{N}$. Consider $P \in \mathcal{P}_k(^jE;F)$. By Lemma 3.1, $\sup v(x)\|x\|^j = K_j < \infty$. Hence, $v(U)P(U) \subset K_jP(B_E)$. consequently, $v(U)P(U)$ is relatively compact in F.
(iv) $\Rightarrow$ (v): Take $F = \mathcal{G}_v(U)$ in (iv). Then, by Theorem 2.3 and the hypothesis, $\Delta_v \in \overline{\mathcal{H}_v^c(U;G_v(U))}^{\tau_{\mathcal{M}}}$. Now, $\overline{\mathcal{H}_v^c(U;\mathcal{G}_v(U))}^{\tau_{\mathcal{M}}}$ can be identified with $\overline{\mathcal{K}(\mathcal{G}_v(U),\mathcal{G}_v(U))}^{\tau_c}$ via the map Ψ in view of Theorem 2.7 and Proposition 2.8(b). Since $T_{\Delta_v} \circ \Delta_v = \Delta_v$, $\Psi(\Delta_v) = I$, the identity map on $\mathcal{G}_v(U)$. Thus, $I \in \overline{\mathcal{K}(\mathcal{G}_v(U);\mathcal{G}_v(U))}^{\tau_c}$.
(v) $\Rightarrow$(i) follows by Propositions 2.4 and 2.11. □

Proposition 3.3 *For a weight v defined on an open subset U of a Banach space E, $\overline{\mathcal{K}(\mathcal{G}_v(U),F)}^{\tau_c} = \mathcal{L}(\mathcal{G}_v(U);F)$ if and only if $\overline{\mathcal{H}_v^c(U;F)}^{\tau_{\mathcal{M}}} = \mathcal{H}_v(U;F)$ for each Banach space F.*

Proof Assume $\overline{\mathcal{K}(\mathcal{G}_v(U),F)}^{\tau_c} = \mathcal{L}(\mathcal{G}_v(U);F)$. Take $f \in \mathcal{H}_v(U;F)$. Then, by Theorem 2.3, $T_f \in \mathcal{L}(\mathcal{G}_v(U);F)$. By hypothesis, there exists a net $(T_\alpha) \subset \mathcal{K}(\mathcal{G}_v(U),F)$ such that $T_\alpha \xrightarrow{\tau_c} T_f$. Now, corresponding to each α, we have $f_\alpha \in \mathcal{H}_v^c(U;F)$ such that $T_{f_\alpha} = T_\alpha$ by Proposition 2.8(b). Using Theorem 2.7, we get $f_\alpha \xrightarrow{\tau_{\mathcal{M}}} f$. Hence, $\overline{\mathcal{H}_v^c(U;F)}^{\tau_{\mathcal{M}}} = \mathcal{H}_v(U;F)$.

Conversely, for $T \in \mathcal{L}(\mathcal{G}_v(U),F)$, there exists $f \in \mathcal{H}_v(U,F)$ such that $T = T_f$ by Theorem 2.3. Consequently, by hypothesis, we can find a net $\{f_\alpha\} \subset \mathcal{H}_v^c(U;F)$ such that $f_\alpha \xrightarrow{\tau_{\mathcal{M}}} f$. Thus, $(T_{f_\alpha}) \subset \mathcal{K}(\mathcal{G}_v(U),F)$ by Proposition 2.8(b) and $T_\alpha \xrightarrow{\tau_c} T_f = T$ by Theorem 2.7. □

Writing $\mathcal{H}_K^\infty(V; E) \equiv \mathcal{H}_v^c(V; E)$ for $v \equiv 1$, the final result of this section characterizes the compact approximation property for the space E in terms of $\mathcal{H}_v^c(V; E)$, vis-à-vis $\mathcal{H}_K^\infty(V; E)$, as follows:

Theorem 3.4 *Let E be a Banach space. Then, for each Banach space F, the following are equivalent:*

(i) *E has the compact approximation property.*
(ii) $\overline{\mathcal{H}_v^c(V; E)}^{\tau_\mathcal{M}} = \mathcal{H}_v(V, E)$, *for each open subset V of F and weight v on V.*
(iii) $\overline{\mathcal{H}_K^\infty(V; E)}^{\tau_\mathcal{M}} = \mathcal{H}^\infty(V, E)$, *for each open subset V of F.*

Proof (i)⇒(ii): Assume that E has the compact approximation property. Then, by taking $F = \mathcal{G}_v(V)$ in Theorem 2.10(iii), $\overline{\mathcal{K}(\mathcal{G}_v(V), E)}^{\tau_c} = L(\mathcal{G}_v(V), E)$. Thus, $\overline{\mathcal{H}_v^c(V; E)}^{\tau_\mathcal{M}} = \mathcal{H}_v(V, E)$ by Proposition 3.2.

(iii): Follows from (ii) by taking $v \equiv 1$.

(iii)⇒(i): cf. Theorem 5 of [16]. □

4 The Topology $\tau'_\mathcal{M}$ on $\mathcal{H}_v(U; F)$

Analogous to the topology $\tau_\mathcal{M}$, we introduce another locally convex topology $\tau'_\mathcal{M}$ on $\mathcal{H}_v(U, F)$. It is generated by the family $\{q_{\overline{\alpha},\overline{A}} : \overline{\alpha} = (\alpha_j) \in c_0^+,\ \overline{A} = (A_j),\ A_j$ being finite subset of U for each $j\}$ of semi-norms given by

$$q_{\overline{\alpha},\overline{A}}(f) = \sup_{j\in\mathbb{N}}(\alpha_j \sup_{x\in A_j} v(x)\|f(x)\|).$$

Concerning this topology, we have

Proposition 4.1 *For a weight v on an open subset U of a Banach space E, we have:*

(i) $\tau_0 \leq \tau_\mathcal{M} \leq \tau'_\mathcal{M} \leq \tau_{\|.\|_v}$ *on* $\mathcal{H}_v(U, F)$.
(ii) $\tau'_\mathcal{M}|\mathcal{B} = \tau_0|\mathcal{B}$ *for any* $\|\cdot\|_v$*- bounded set* $\mathcal{B}$.

Proof (i) Clearly, $\tau_\mathcal{M} \leq \tau'_\mathcal{M}$. In view of (2), it suffices to prove $\tau'_\mathcal{M} \leq \tau_{\|.\|_v}$ which follows from the inequality, $q_{\overline{\alpha},\overline{A}}(f) \leq \|\overline{\alpha}\|_\infty \|f\|_v$.

(ii) The proof is analogous to the one given in [27]. However, for the sake of completeness, we outline the same. Let $\mathcal{B}$ be a bounded set in $(\mathcal{H}_v(U, F), \|\cdot\|_v)$. Then, there exists a constant $M > 0$ such that $\|f\|_v \leq M$, for every $f \in \mathcal{B}$. Consider a $\tau'_\mathcal{M}$ -continuous semi-norm q given by

$$q(f) = \sup_{j\in\mathbb{N}}(\alpha_j \sup_{x\in A_j} v(x)\|f(x)\|),\ f \in \mathcal{H}_v(U, F)$$

where $(\alpha_j) \in c_0^+$ and (A_j) is a sequence of finite subsets of U. Fix $\epsilon > 0$ arbitrarily. Then, there exists $m_0 \in \mathbb{N}$ such that

$$\alpha_j < \frac{\epsilon}{2M}, \quad \forall j > m_0.$$

Write $K = \bigcup_{j \leq m_0} A_j$. Then, K is a compact subset of U. Note that

$$\sup_{j \leq m_0} (\alpha_j \sup_{x \in A_j} v(x) \|(f - g)(x)\|) \leq L \; \|\overline{\alpha}\|_\infty p_K(f - g)$$

where $L = \sup_{x \in K} v(x)$. Thus,

$$p(f - g) < \epsilon \text{ whenever } p_K(f - g) < \delta$$

for $f, g \in \mathcal{B}$, where $\delta = \frac{\epsilon}{2L\|\overline{\alpha}\|_\infty}$. This completes the proof. □

For $f \in \mathcal{H}_v(U; F)$, let us define $S_n f(x) = \sum_{k=0}^n \frac{1}{m!} \hat{d}^m f(0)(x)$ and $C_n f(x) = \frac{1}{n+1} \sum_{k=0}^n S_k f(x)$. Then, $\|C_n(f)(x)\|_v \leq \|f\|_v$ for each $f \in \mathcal{H}_v(U; F)$ and $n \in \mathbb{N}$, cf. [27].

As a consequence of the above proposition, we derive the following result similar to Proposition 2.6

Proposition 4.2 *Let E and F be Banach spaces. For a radial weight v on a balanced open subset U of E with $\mathcal{P}(E) \subset \mathcal{H}_v(U)$, the space $\mathcal{P}(E; F)$ is $\tau'_{\mathcal{M}}$-dense in $\mathcal{H}_v(U; F)$.*

Proof Let $f \in \mathcal{H}_v(U, F)$. Then, the set $\{C_n(f) : n \in \mathbb{N}_0\}$ is a $\| \cdot \|_v$-bounded in $\mathcal{H}_v(U, F)$. As $C_n f \to f$ in $(H(U, F), \tau_0)$, the result follows by Proposition 4.1 (ii). □

Using the above proposition, we prove

Theorem 4.3 *Let v be a radial weight on a balanced open subset U of a Banach space E such that $\mathcal{P}(E) \subset \mathcal{H}_v(U)$. Then, for each Banach space F, the following are equivalent:*

(a) $v^{\frac{1}{i}-1} I_U \in \overline{\mathcal{H}_v^c(U; E)}^{\tau'_{\mathcal{M}}}$ for each $i \in \mathbb{N}$, where $I_U : U \to E$ is the inclusion mapping.
(b) $\overline{\mathcal{H}_v^c(U; F)}^{\tau'_{\mathcal{M}}} = \mathcal{H}_v(U, F)$.

Proof (a)⇒ (b): Let $f \in \mathcal{H}_v(U, F)$ and q be a $\tau'_{\mathcal{M}}$-continuous semi-norm given as

$$q(f) = \sup_{j \in \mathbb{N}} (\alpha_j \sup_{x \in A_j} v(x) \|f(x)\|)$$

where $(\alpha_j) \in c_0^+$ and (A_j) is a sequence of finite subsets of U. Then, by Proposition 4.2, there exists $P \in \mathcal{P}(E; F)$ such that

$$q(f - P) < \frac{\epsilon}{2} \tag{3}$$

Write $P = P_0 + P_1 + \cdots P_m$, $P_i \in \mathcal{P}({}^i E, F)$, $0 \leq i \leq m$. Fix i, $1 \leq i \leq m$ arbitrarily. Define $K = \bigcup_{j\in\mathbb{N}} \{(\alpha_j \sup_{x\in A_j} v(x))^{\frac{1}{i}} y : y \in A_j\} \bigcup\{0\}$. Then, K is a compact subset of U, cf. [27, Proposition 4.4], and there exists a $\delta > 0$ such that

$$\|P_i(x) - P_i(y)\| < \frac{\epsilon}{2m}, \text{ for each } x \in K, y \in E \text{ with } \|x - y\| < \delta \tag{4}$$

Since $q_i(f) = \sup_{j\in\mathbb{N}}((\alpha_j)^{\frac{1}{i}} \sup_{x\in A_j} v(x)\|f(x)\|)$ is a $\tau'_{\mathcal{M}}$-continuous semi-norm, there exists $f_i \in \overline{\mathcal{H}_v^c(U; E)}^{\tau'_{\mathcal{M}}}$ by (a) such that

$$q_i(v^{\frac{1}{i}-1} I_U - f_i) = \sup_{j\in\mathbb{N}}((\alpha_j)^{\frac{1}{i}} \sup_{x\in A_j} v(x)\|v^{\frac{1}{i}-1}(x) - f_i(x)\|) < \delta \tag{5}$$

Let $g_i = v^{i-1} P_i \circ f_i$. Clearly $g_i \in \mathcal{H}_v^c(U; F)$. Note that

$$(\alpha_j \sup_{x\in A_j} v(x))^{\frac{1}{i}} \|x - v^{1-\frac{1}{i}}(x) f_i(x)\| \leq (\alpha_j)^{\frac{1}{i}} \sup_{x\in A_j} v(x)\|v^{\frac{1}{i}-1}(x) - f_i(x)\|.$$

Therefore, by (4) and (5), we have

$$q(P_i - g_i) = \sup_{j\in\mathbb{N}} \|P_i((\alpha_j \sup_{x\in A_j} v(x))^{\frac{1}{i}} x) - P_i(\alpha_j \sup_{x\in A_j} v(x))^{\frac{1}{i}} v^{1-\frac{1}{i}}(x) f_i(x))\| < \frac{\epsilon}{2m}.$$

Write $g = g_0 + g_1 + g_2 + \cdots + g_m$, where $g_0 = P_0$. Then, $g \in \mathcal{H}_v^c(U; F)$ with $p(f - g) < \epsilon$, thereby proving (b).

(b)⇒(a): Since $\sup_{x\in A_j} v^{\frac{1}{i}}(x)\|x\| < \infty$ for each $i \in \mathbb{N}$ by Lemma 3.1, $\|v^{\frac{1}{i}-1} I_U\|_v < \infty$. Thus, (a) follows. $\square$

Remark 4.1 (a). The above result is more general than [16, Theorem 5]; indeed, for $v \equiv 1$, $\tau_{\mathcal{M}} \equiv \tau'_{\mathcal{M}} \equiv \tau_\gamma$; (b). Since $\tau_{\mathcal{M}} \leq \tau'_{\mathcal{M}}$, $\overline{\mathcal{H}_v^c(U; F)}^{\tau_{\mathcal{M}'}} \subset \overline{\mathcal{H}_v^c(U; F)}^{\tau_{\mathcal{M}}} = \mathcal{H}_v(U, F)$ and so the implication $(a) \Rightarrow (b)$ is true for $\tau_{\mathcal{M}}$ also. However, it would be interesting to know the non-constant weights for which $\tau_{\mathcal{M}} \equiv \tau'_{\mathcal{M}}$.

5 Weighted (LB)-Spaces and Approximation Properties

Let Λ be a directed set and $\{(X_\alpha, \tau_\alpha) : \alpha \in \Lambda\}$ be a family of locally convex spaces such that for $\alpha \leq \beta$, $X_\alpha \subset X_\beta$, $X = \bigcup_{\alpha\in\Lambda} X_\alpha$ and $I_{\alpha,\beta} : X_\alpha \to X_\beta$ be the continuous inclusion maps with $I_{\alpha,\beta} \circ I_\alpha = I_\beta$. In this chapter, we consider the *inductive limit* τ as the finest Hausdorff locally convex topology for which each inclusion map

$I_\alpha : X_\alpha \to X$ is continuous. We write $(X, \tau) = \varinjlim_{\alpha\in\Lambda}(X_\alpha, \tau_\alpha)$. If Λ is countable and each X_α is a Banach space, then (X, τ) is said to be an *(LB)-space*.

Let $\{(Y_\alpha, \tau_\alpha) : \alpha \in \Lambda\}$ be a family of locally convex spaces such that for each $\alpha \geq \beta$, $\pi_{\alpha,\beta} : Y_\alpha \to Y_\beta$ is continuous linear map and $\pi_{\alpha,\beta} \circ \pi_\beta = \pi_\alpha$, where π_α are the canonical mappings from $Y = \{(x_\alpha)_{\alpha\in\Lambda} : \pi_{\alpha,\beta}(x_\beta) = x_\alpha$ for each $\alpha \leq \beta\}$ to Y_α. The space Y endowed with the weakest topology on Y such that all the canonical mappings π_α are continuous is the *projective limit* of the above system and is written as $(Y, \tau) = \varprojlim_{\alpha\in\Lambda}(Y_\alpha, \tau_\alpha)$. A projective limit $(Y, \tau) = \varprojlim_{\alpha\in\Lambda}(Y_\alpha, \tau_\alpha)$ is said to be *reduced* if each $\pi_\alpha(Y)$ is dense in Y_α for each $\alpha \in \Lambda$.

Proposition 5.1 ([33]) *Let $(Y, \tau) = \varprojlim_{\alpha\in\Lambda} Y_\alpha$ be a reduced projective limit such that each Y_α has the approximation property. Then, Y has the approximation property.*

For the theory of projective and inductive limits, we refer to [30, 31, 33].

Let us now consider inductive limit of weighted spaces of holomorphic functions. Assume that $V = \{v_n\}$ is a countable decreasing family, i.e., $v_{n+1} \leq v_n$ for each n, of radial rapidly decreasing weights on E. Corresponding to V, inductive limit of weighted spaces is defined as $VH(E) = \bigcup_{n\geq 1} \mathcal{H}_{v_n}(E)$ endowed with the locally convex inductive topology $\tau_{\mathcal{I}}$. Since the closed unit ball B_{v_n} of each $\mathcal{H}_{v_n}(E)$ is τ_0 compact, $VH(E)$ is complete by Mujica's completeness theorem, namely,

Theorem 5.2 ([39]) *Let $(E, \tau) = \varinjlim_{n\in\mathbb{N}} E_n$ be an (LB)-space, and suppose that there exists a locally convex Hausdorff topology $\tilde{\tau} < \tau$ on E such that the closed unit ball B_n of each E_n is $\tilde{\tau}$-compact. Then,*

$$F = \{u \in E' : u|B_n \textit{ is } \tilde{\tau}\textit{-continuous for each } n \in \mathbb{N}\}$$

endowed with the topology of uniform convergence on the sets B_n, is a Fréchet space such that the evaluation mapping $J : E \to F'$ given by $J(x)(u) = u(x)$ for each $x \in E$ and $u \in F$, is a topological isomorphism from E onto F'_i (the inductive dual of F) and hence E must be complete.

The predual of $VH(E)$ defined as

$$VG(E) = \{\phi \in VH(E)' : \phi|B_{v_n} \text{ is } \tau_0\text{-continuous for each } n \in \mathbb{N}\ \}$$

is endowed with the topology of uniform convergence on the sets B_{v_n}. Also, $VG(E) = \varprojlim_{n\in\mathbb{N}} G_{v_n}(E)$ is a reduced projective limit, cf. [6]. Combining this fact with Propositions 2.9 and 5.1, we get

Theorem 5.3 *Let $V = \{v_n\}$ denote a countable decreasing family of radial rapidly decreasing weights on E. Then, E has the approximation property if and only if $VG(E)$ has the approximation property.*

Following [6], a family V of radial rapidly decreasing weights satisfies *condition* (A) if for each $m \in \mathbb{N}$, there exist $D > 0$, $R > 1$, and $n \in \mathbb{N}$, $n \geq m$ such that

$$\|P^j f(0)\|_n \leq \frac{D}{R^j}\|f\|_m$$

for each $j \in \mathbb{N}$ and $f \in \mathcal{H}_{v_m}(U)$.

For the final result of this section, we make use of the following result proved in [28]

Theorem 5.4 *If $V = \{v_n\}$ is a family of weights satisfying condition (A), then the sequence of spaces $\{\mathcal{Q}^m E)\}_{m=1}^{\infty}$ forms an $\mathcal{S}$-absolute decomposition for $VG(E)$ with respect to the topology of uniform convergence on B_{v_n}'s for each n.*

Finally, we have

Theorem 5.5 *Let $V = \{v_n\}$ denote a countable decreasing family of radial rapidly decreasing weights on E satisfying condition (A). Then, E has the compact approximation property if and only if $VG(E)$ has the compact approximation property.*

Proof It follows directly from Propositions 2.12, 2.15 and Theorem 5.4. □

Note 5.1 As Proposition 5.1 is not known to be true for the compact approximation property, Theorem 5.5 cannot be derived for the family V which does not satisfy condition (A). However, for $V = \{v\}$, the above result holds, though the singleton family of weights does not satisfy condition (A), cf. [28, Remark 4.4].

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