

Preface

Model theory is a branch of mathematical logic that studies mathematical structures through sentences of a suitable formal language concerning the elements of structures. Since mathematicians rarely exploit the precise syntactical structure of sentences, model theory gives new techniques to mathematics. These have been used very successfully to settle some outstanding conjectures in hardcore mathematics.

Therefore, model theory is a subject in its own right. It has its own deep concepts and is rich in techniques. Currently, it is a very active and challenging area of research. The main purpose of this book is to usher young researchers into this beautiful, challenging and useful subject.

About the Book

This book is an exposition of the most basic ideas of model theory. It is primarily aimed at lower undergraduate students who would like to work in model theory. Knowledge of logic will be helpful, though not essential, for this book. This book is a book on model theory and not mathematical logic. So, we completely avoid proof theoretic approach. Throughout the book, the style is semantic except for introducing languages and interpretations (i.e. structures) formally. This enables us to get into the subject rather quickly.

Chapters 1–6 constitute the core of model theory which all researchers should start with. In Chap. 7, we have presented model theory of valued fields. It also contains the full proof of Ax-Kochen theorem on Artin’s conjecture on the field of p -adic reals.

Because we have put equal stress on applications, a good background in algebra is required. In Appendices A–C, we give necessary background material from set theory and algebra that is required for this book. Some background material on

algebra has also been presented in the main text as we go along. It is hoped that this will make the book self-contained.

Ideally, the book should be covered in two semesters. The first two chapters contain most of the basic concepts and basic results to get started. These two chapters contain standard materials that are traditionally covered in the first introduction to the subject. It can also be taught to senior undergraduate students.

Chapter 1 mainly concentrates on basic concepts. It includes first-order language, its terms and formulas and its structures, homomorphisms, embeddings and elementary embeddings, Skolemization of a theory, definability, etc. In order to handle definable equivalence classes, a brief introduction of many-sorted logics, imaginary elements and elimination of imaginaries is presented. In the first reading, if time does not permit, sections on many-sorted logics and elimination of imaginaries can be skipped.

Chapter 2 contains most of the introductory techniques and results. Ultraproduct of structures and Łoś fundamental lemma, compactness theorem and its consequences, upward Löwenheim Skolem theorem, quantifier elimination and model completeness are some of the most basic results presented in the chapter. These are used to present the model theory of dense, linearly ordered sets without end points, torsion-free divisible abelian groups and ordered divisible abelian groups, algebraically closed fields and real closed fields. The chapter concludes with some applications in algebra and geometry such as Hilbert Nullstellensatz, Ax's theorem on polynomials, Chevalley's projection lemma on algebraically closed fields and its real counterpart Artin–Seidenberg theorems on real closed fields, solution of Hilbert's seventeenth problem, etc.

Both Chaps. 1 and 2 end up with a large number of exercises. They are an integral part of the subject. Several concepts are introduced in the exercises. Readers should work out all the exercises. Much of the material presented in the exercises are used later.

Chapters 3–6 can be termed as the beginning of modern model theory and require a bit of sophistication. In Chap. 3, we make a systematic study of types. Types are used to define most of the modern concepts and are essential for the development of modern model theory. Chapters 4 and 5 form the bedrock of modern model theory. In Chap. 4, we introduce important subclasses of structures and theories. Of particular interest are topics on saturated structures and stable theory. We introduce Morley rank and Morley degree as well as forking independence in Chap. 4. In Chap. 5, we introduce indiscernibles and prove Morley categoricity theorem. In Chap. 6, we initiate the study of strong types. Strong types are equivalence classes of the so-called bounded, invariant, equivalence relations. We introduce mainly Lascar strong types and Kim–Pillay strong types. Strong types are important for stable theories, simple theories, independence, etc. These are also interesting mathematical objects. To illustrate this, we show their connection with descriptive set theory.

There are many important topics such as stability theory, simple theory and independence, NIP theories, etc. that are not covered in this book. This is primarily

because we want this to remain an introductory graduate level text book to get started in model theory.

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