Preface

In the theory of statistical estimation, the asymptotic properties such as the consistency, asymptotic normality, and asymptotic efficiency have been discussed under usual regularity conditions, and in particular, it is well known that the maximum likelihood estimator (MLE) has such properties. But, in the non-regular case when the regularity conditions do not necessarily hold, it is seen that the property of estimator depends on the irregularity. In the book, we treat truncated exponential families of distributions as a typical situation when both of the regular and non-regular structures exist and clarify how they affect the estimation of a natural parameter and truncation parameters. Such families include a (upper-truncated) Pareto distribution which is widely used in various fields such as finance, physics, hydrology, astronomy, and other disciplines. For a one-sided truncated exponential family (oTEF) with a natural parameter and a truncation parameter, we consider the estimation of a natural parameter with a truncation parameter as nuisance one. Then, the MLE of a natural parameter when a truncation parameter is known and the MLE of a natural parameter when a truncation parameter is unknown have been seen to have the same asymptotic normality. However, in the book it is shown that the asymptotic difference between them appears in the second order after a bias-adjustment, and it is defined as the notion of second-order asymptotic loss through the second-order asymptotic variances obtained from their stochastic expansions. The regular and non-regular structures of the oTEF are reflected in the second-order asymptotic variance of the latter MLE, which effects the loss. The corresponding results to the case of a oTEF are obtained in that of a two-sided truncated exponential family (tTEF) of distributions with a natural parameter and truncation parameters. We also conversely consider the estimation of a truncation parameter with a natural parameter as nuisance one for a oTEF using a bias-adjustment. The bias-adjusted MLE of a truncation parameter when a natural parameter is known and the bias-adjusted MLE of a truncation parameter when a natural parameter is unknown are constructed, and their asymptotic difference is clarified in a similar way to the case of the estimation of a natural parameter. The corresponding results to the case of a oTEF are obtained in the case of a tTEF. From the Bayesian viewpoint, such estimation is also discussed.

Further several examples including a truncated exponential, a truncated normal, a (upper-truncated) Pareto, a truncated beta, and a truncated Erlang (type) distributions are given. In some examples, related results to the uniformly minimum variance unbiased estimation are also described.

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