

Chapter 2

Robust Control of Stochastic Structures Using Minimum Norm Quadratic Partial Eigenvalue Assignment Technique

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Abstract The use of active vibration control technique, though typically more expensive and difficult to implement compared to the traditionally used passive control approaches, is an effective way to control or suppress dangerous vibrations in structures caused by resonance or flutter due to the external disturbances, such as winds, waves, moving weights of human bodies, and earthquakes. From control perspective, it is not only sufficient to suppress the resonant modes of vibration but also to guarantee the overall stability of the structural system. In addition, the no-spillover property of the remaining large number of frequencies and the corresponding mode shapes must be maintained. To this end, the state-space based linear quadratic regulator (LQR) and H_∞ control techniques and physical-space based robust and minimum norm quadratic partial eigenvalue assignment (RQPEVA and MNQPEVA) techniques have been developed in the recent years. In contrast with the LQR and H_∞ control techniques, the RQPEVA and MNQPEVA techniques work exclusively on the second-order model obtained by applying the finite element method on the vibrating structure, and therefore can computationally exploit the nice properties, such as symmetry, sparsity, bandness, positive definiteness, etc., inherited by the system matrices associated with the second-order model. Furthermore, the RQPEVA and MNQPEVA techniques guarantee the no-spillover property using a solid mathematical theory. The MNQPEVA technique was originally developed for the deterministic models only, but since then, further extension to this technique has been made to rigorously account for parametric uncertainty. The stochastic MNQPEVA technique is capable of efficiently estimating the probability of failure with high accuracy to ensure the desired resilience level of the designed system. In this work, first a brief

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review of the LQR, H_∞ , and MNQPEVA techniques is presented emphasizing on their relative computational advantages and drawbacks, followed by a comparative study with respect to computational efficiency, effectiveness, and applicability of the three control techniques on practical structures. Finally, the methodology of the stochastic MNQPEVA technique is presented and illustrated in this work using a numerical example with a representative real-life structure and recorded data from a California earthquake.

2.1 Introduction

Structural systems (e.g., high-rise buildings, bridges, wind turbines, aircraft, etc.) may undergo high amplitude vibration due to resonance or flutter when subjected to various dynamic loads (e.g., earthquake, wind, wave, moving weights of human bodies, etc.). If not suppressed, such high amplitude vibration may lead to collapse (e.g., failure of the Tacoma Narrows Bridge, Washington in 1940 [1]) or loss of functionality (e.g., wobbling of the Millennium Bridge, London in 2000 [2]) of the structures. Currently, there exist various strategies for vibration suppression, among which the most popular is the active vibration control (AVC). Large number of AVC techniques, for example, linear quadratic regulator (LQR) [3–5] and H_∞ [4–9] have been developed in the last three decades. The LQR and H_∞ techniques are based on the state-space representation of the dynamics of structural systems. In recent years, minimum norm quadratic partial eigenvalue assignment (MNQPEVA) and robust quadratic partial eigenvalue assignment (RQPEVA) techniques have been developed for AVC, which are based on the physical-space based representation of the dynamics of structural systems [10–12]. The MNQPEVA and RQPEVA techniques are computationally more efficient than the LQR and H_∞ techniques as (i) the system matrices in the state-space approach have twice the dimension of the system matrices in the physical-space based approach, (ii) the system matrices in the state-space approach lose the computationally exploitable properties, for example, symmetry, definiteness, sparsity, bandness, etc., inherited by the system matrices in the physical-space based approach, (iii) unlike the LQR and H_∞ techniques, the MNQPEVA and RQPEVA techniques require only partial information of the eigenpairs (i.e., eigenvalues and associated eigenvectors) to be shifted for the design of controlled/closed-loop systems, and (iv) the MNQPEVA and RQPEVA techniques ensure the *no-spillover property* in terms of sophisticated mathematical theory [13].

In practice, structural systems are plagued with uncertainties from several primitive sources including, but not limited to, variability in input parameters (e.g., material, geometric, loading) often modeled as *parametric uncertainty* [14] and system-level structural complexities typically modeled as *model form uncertainty* or *model uncertainty* [15]. If the available AVC techniques fail to account for the effects of these uncertainties, the designed control systems might not be able to reduce the vibration level of the structures. In the recent past, state-space approach-based H_∞ control technique has been extended to incorporate several sources of uncertainties

mentioned above [9, 16–22]. Recently, physical-space based stochastic MNQPEVA technique has been developed that is computationally more effective than the H_∞ technique for large-scale systems due to the reasons mentioned earlier in this section [23].

In this work, a brief review of three AVC strategies, viz., the LQR, the H_∞ , and the MNQPEVA techniques is presented first highlighting their relative computational advantages and drawbacks (see Sects. 2.2.1–2.2.3). Using a representative real-life structure, a comparison of the three techniques is presented next that (i) emphasizes on the computational efficiency and effectiveness of the MNQPEVA technique over the LQR and H_∞ techniques and (ii) provides motivation for extending the MNQPEVA technique to account for various uncertainties in structural systems (see Sect. 2.2.4). The comparative study is followed by a discussion on the stochastic MNQPEVA technique developed earlier in the literature [23] (see Sects. 2.3.1–2.3.2). Finally, the stochastic MNQPEVA technique is illustrated in this work through a numerical example with a representative real-life structure and recorded data from a California earthquake (see Sect. 2.3.3).

2.2 Brief Review and Comparison of LQR, H_∞ , and MNQPEVA Techniques

In this section, brief description of the LQR, H_∞ , and MNQPEVA control techniques are presented first followed by comparison of these three techniques illustrated using a numerical example. To understand the working principle of the three techniques, let us first consider the governing equation representing the dynamics of structural systems given by

$$\mathbf{M}\ddot{\boldsymbol{\eta}}(t) + \mathbf{C}\dot{\boldsymbol{\eta}}(t) + \mathbf{K}\boldsymbol{\eta}(t) = \mathbf{f}(t) \quad (2.1)$$

Equation (2.1) is obtained via the finite element (FE) discretization of the governing partial differential equations (PDEs) approximating the dynamic response of the physical structural system. In Eq. (2.1), the matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ denote the mass, damping, and stiffness matrices of the system, respectively, vector $\boldsymbol{\eta}(t)$ denotes the displacement response of the system at any instant t , $\mathbf{f}(t)$ denotes the dynamic load acting on the system at instant t , and the dot (·) denotes time derivative. Note that $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are real symmetric positive-definite matrices of size $n \times n$, where n is the degrees of freedom (dof) associated with the FE model of the system. Since $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are generated by FE technique, they are very large and often inherit nice structural properties, such as definiteness, bandness, sparsity, etc., which are useful for large-scale computing. The system is, in general, known as open-loop system. The dynamics of the open-loop system can also be represented in the linear state-space format as follows:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{E}\mathbf{d}(t) \\ \mathbf{z}(t) &= \mathbf{W}\mathbf{x}(t) \end{aligned} \quad (2.2)$$

where $\mathbf{A}$, $\mathbf{E}$, $\mathbf{W}$, $\mathbf{x}(t)$, $\mathbf{d}(t)$, and $\mathbf{z}(t)$ denote the system matrix, external disturbance matrix, output matrix, state vector, external disturbance vector, and output vector, respectively. The quantities $\mathbf{A}$, $\mathbf{E}$, $\mathbf{x}(t)$, and $\mathbf{d}(t)$ appearing in Eq. (2.2) are related to the quantities $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, $\boldsymbol{\eta}(t)$, and $\mathbf{f}(t)$ present in Eq. (2.1) via the relations given by

$$\begin{aligned}\mathbf{A} &= \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \\ \mathbf{E} &= \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{M}^{-1} \end{bmatrix} \\ \mathbf{x}(t) &= [\boldsymbol{\eta}(t), \dot{\boldsymbol{\eta}}(t)]^T \\ \mathbf{d}(t) &= [\mathbf{0}, \mathbf{f}(t)]^T\end{aligned}\quad (2.3)$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{0}$ denotes both the null matrix and the null vector. It will be clear from the context whether $\mathbf{0}$ refers to a null matrix or a null vector. The real matrices $\mathbf{A}$ and $\mathbf{E}$ that appear in Eqs. (2.2) and (2.3) are typically highly populated and have dimensions twice as that of the original system matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$. Therefore, more computation memory and time will be required, if the state-space approach (see Eq. (2.2)) is adopted instead of the physical-space based approach (see Eq. (2.1)), to estimate the displacement response $\boldsymbol{\eta}(t)$ (and/or velocity and acceleration responses) of a large-scale open-loop system (i.e., system matrices with large n).

2.2.1 Brief Review of LQR Technique

The objective of the traditional LQR technique is to design a control input $\mathbf{u}(t)$ for the nominal open-loop system given by

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) \\ \mathbf{z}(t) &= \mathbf{W}\mathbf{x}(t)\end{aligned}\quad (2.4)$$

such that the nominal closed-loop system given by

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}_L\mathbf{u}(t) \\ \mathbf{z}(t) &= \mathbf{W}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned}\quad (2.5)$$

is asymptotically stable, i.e., $\mathbf{x}(t) \rightarrow 0$ as $t \rightarrow \infty$ [3–5]. In Eq. (2.5), the matrices $\mathbf{B}_L$ and $\mathbf{D}$ denote the control matrix and the direct transmission matrix, respectively, and they are real valued. It is important to note that the control input $\mathbf{u}(t)$ designed via the traditional LQR technique does not account for the dynamic load $\mathbf{f}(t)$ as the nominal open-loop, and the nominal closed-loop systems are independent of the

external disturbance $\mathbf{d}(t)$ (see Eqs. (2.4) and (2.5)). Details pertaining to the design of $\mathbf{u}(t)$ following the traditional LQR technique are presented in Sect. 2.2.1.1.

2.2.1.1 Traditional LQR Technique

In traditional LQR technique, the control input $\mathbf{u}(t)$ is typically obtained by minimizing the quadratic cost function given as follows:

$$J = \int_{t=0}^{\infty} [\mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) + \mathbf{u}^T(t) \mathbf{R} \mathbf{u}(t)] dt \quad (2.6)$$

where $\mathbf{Q} = \mathbf{Q}^T \geq \mathbf{0}$ and $\mathbf{R} = \mathbf{R}^T > \mathbf{0}$ are the weight matrices associated with the norms $\|\mathbf{x}(t)\|_2$ and $\|\mathbf{u}(t)\|_2$, respectively. In practice, $\mathbf{Q} = \mathbf{W}^T \mathbf{W}$, $\mathbf{R} = \rho \mathbf{D}^T \mathbf{D} = \rho \mathbf{I}$, and $\mathbf{D}^T \mathbf{W} = \mathbf{0}$ are typically assumed. With these assumptions and $\rho = 1$, the cost function can be expressed as $J = \int_{t=0}^{\infty} \|\mathbf{z}(t)\|_2^2 dt$, where $\mathbf{z}(t) = \mathbf{W} \mathbf{x}(t) + \mathbf{D} \mathbf{u}(t)$. Note that, by minimizing J , two conflicting quantities $\|\mathbf{x}(t)\|_2$ (i.e., the response of the system) and $\|\mathbf{u}(t)\|_2$ (i.e., the input required for vibration control) are simultaneously minimized in the traditional LQR technique.

The form $\mathbf{u}(t) = -\mathbf{G}_L \mathbf{x}(t)$, where $\mathbf{G}_L$ denotes the feedback gain matrix, is typically chosen in the traditional LQR technique with the assumption that all the states $\{x_i(t)\}_{i=1}^{2n}$ are observable at any instant t . Note that the matrix $\mathbf{G}_L$ is a real valued matrix of dimension $m_L \times 2n$ with $m_L \leq 2n$. With $\mathbf{u}(t) = -\mathbf{G}_L \mathbf{x}(t)$, the dynamics of the nominal closed-loop system can be written as follows:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= (\mathbf{A} - \mathbf{B}_L \mathbf{G}_L) \mathbf{x}(t) \\ \mathbf{z}(t) &= (\mathbf{W} - \mathbf{D} \mathbf{G}_L) \mathbf{x}(t) \end{aligned} \quad (2.7)$$

Assuming (i) the pair $(\mathbf{A}, \mathbf{B}_L)$ be stabilizable and (ii) the pair $(\mathbf{A}, \mathbf{Q})$ be detectable, there exists a unique optimal control input $\mathbf{u}^*(t) = -\mathbf{G}_L^* \mathbf{x}(t)$. The unique optimal feedback gain matrix $\mathbf{G}_L^*$ is given by

$$\mathbf{G}_L^* = \mathbf{R}^{-1} \mathbf{B}_L^T \mathbf{X} = \frac{1}{\rho} \mathbf{B}_L^T \mathbf{X} \quad (2.8)$$

where $\mathbf{X} = \mathbf{X}^T \geq \mathbf{0}$ is the unique solution of the algebraic Riccati equation (ARE) given as follows:

$$\begin{aligned} \mathbf{A}^T \mathbf{X} + \mathbf{X} \mathbf{A} - \mathbf{X} \mathbf{B}_L \mathbf{R}^{-1} \mathbf{B}_L^T \mathbf{X} + \mathbf{Q} &= \mathbf{0} \\ \Rightarrow \mathbf{A}^T \mathbf{X} + \mathbf{X} \mathbf{A} - \frac{1}{\rho} \mathbf{X} \mathbf{B}_L \mathbf{B}_L^T \mathbf{X} + \mathbf{W}^T \mathbf{W} &= \mathbf{0} \end{aligned} \quad (2.9)$$

Existence of $\mathbf{u}^*(t)$ indicates that (i) the closed-loop system given by Eq. (2.5) or (2.7) is asymptotically stable irrespective of the stability of the open-loop system given by Eq. (2.4), and (ii) the decay in response of the closed-loop system is faster than the decay in response of the corresponding open-loop system. It should be noted here that the optimal control input $\mathbf{u}^*(t)$ and the associated optimal feedback gain matrix $\mathbf{G}_L^*$ can also be obtained by solving an alternate optimization problem involving linear matrix inequalities (LMIs) [24, 25], which is not presented here to save space.

2.2.1.2 Modified LQR Technique

The feedback gain matrix $\mathbf{G}_L$ obtained via the traditional LQR technique depends on the weight coefficient ρ (see Eq. (2.9)). Therefore, the required maximum control force, which is associated with the maximum value of control input $\mathbf{u}(t)$, is influenced by the choice of ρ . It can be observed from Eq. (2.6) that, by choosing large ρ , the cost function J can be heavily penalized for small increase in control input $\mathbf{u}(t)$ [3, 5, 26]. Hence, the maximum value of control input $\mathbf{u}(t)$ can be reduced by increasing the weight coefficient ρ , however, at the cost of increase in system response (i.e., increase in $\mathbf{x}(t)$).

The dynamics of a nominal closed-loop system, which is designed following the traditional LQR technique, subjected to some external disturbance $\mathbf{d}(t)$ can be written as follows:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= (\mathbf{A} - \mathbf{B}_L \mathbf{G}_L) \mathbf{x}(t) + \mathbf{E} \mathbf{d}(t) \\ \mathbf{z}(t) &= (\mathbf{W} - \mathbf{D} \mathbf{G}_L) \mathbf{x}(t)\end{aligned}\quad (2.10)$$

As mentioned earlier in Sect. 2.2.1, the effect of external disturbance $\mathbf{d}(t)$ (or dynamic load $\mathbf{f}(t)$) is not accounted for in the design of control input $\mathbf{u}(t)$ (and feedback gain matrix $\mathbf{G}_L$) by the traditional LQR technique. Therefore, the response (e.g., displacement, inter-story drift, velocity, acceleration, stress, etc.) of the closed-loop system given by Eq. (2.10) be within certain user-prescribed limit is not always guaranteed. In this regard, a new optimization problem, which can reduce the required control input and simultaneously constrain the closed-loop system response within certain user-prescribed limit, can be formulated as follows:

$$\begin{aligned}\max_{\rho} \quad & \rho \\ \text{s.t.} \quad & y_i(\mathbf{x}(t; \rho), \dot{\mathbf{x}}(t; \rho)) \leq y_{i_{cr}}; \quad i = 1, \dots, n_y\end{aligned}\quad (2.11)$$

In Eq. (2.11), $y_i(\mathbf{x}(t; \rho), \dot{\mathbf{x}}(t; \rho))$ represents the i th response quantity of interest with the associated critical value given by $y_{i_{cr}}$ and n_y denotes the number of response quantities. The term $\mathbf{x}(t; \rho)$, which is parameterized with respect to ρ , represents the state of the closed-loop system given by Eq. (2.10). To evaluate the term $y_i(\mathbf{x}(t; \rho), \dot{\mathbf{x}}(t; \rho))$, one needs to solve for $\mathbf{x}(t; \rho)$ and $\dot{\mathbf{x}}(t; \rho)$ using

$$\dot{\mathbf{x}}(t; \rho) = \left(\mathbf{A} - \frac{1}{\rho} \mathbf{B}_L \mathbf{B}_L^T \mathbf{X}(\rho) \right) \mathbf{x}(t; \rho) + \mathbf{E} \mathbf{d}(t) \quad (2.12)$$

where $\mathbf{X}(\rho)$ can be obtained by solving Eq. (2.9).

2.2.2 Brief Review of H_∞ Technique

The objective of the H_∞ technique is to design a control input $\mathbf{u}(t)$ for the open-loop system given by Eq. (2.2) such that the influence of external disturbance $\mathbf{d}(t)$ on the output $\mathbf{z}(t)$ of the closed-loop system is minimized. Assuming all the states $\{x_i(t)\}_{i=1}^{2n}$ are observable at any instant t , the form $\mathbf{u}(t) = \mathbf{G}_H \mathbf{x}(t)$ is typically chosen in state-feedback H_∞ technique. Note that the matrix $\mathbf{G}_H$ is the feedback gain matrix of dimension $m_H \times 2n$ with $m_H \leq 2n$. With the control input $\mathbf{u}(t) = \mathbf{G}_H \mathbf{x}(t)$, the dynamics of the closed-loop system can be written as follows:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= (\mathbf{A} + \mathbf{B}_H \mathbf{G}_H) \mathbf{x}(t) + \mathbf{E} \mathbf{d}(t) \\ \mathbf{z}(t) &= (\mathbf{W} + \mathbf{D} \mathbf{G}_H) \mathbf{x}(t) \end{aligned} \quad (2.13)$$

where $\mathbf{B}_H$ denotes the control matrix and the definitions of $\mathbf{A}$, $\mathbf{E}$, $\mathbf{W}$, $\mathbf{D}$, $\mathbf{x}(t)$, and $\mathbf{z}(t)$ are same as defined earlier in Sect. 2.2.

In the Laplace domain, the influence of the external disturbance $\mathbf{d}(s)$ on the output $\mathbf{z}(s)$ of the closed-loop system can be characterized by the transfer function matrix which is given as follows:

$$\mathbf{T}_{zd}(s) = (\mathbf{W} + \mathbf{D} \mathbf{G}_H) (s\mathbf{I} - \mathbf{A} - \mathbf{B}_H \mathbf{G}_H)^{-1} \mathbf{E} \quad (2.14)$$

In this regard, the design objective of H_∞ technique is to obtain a feedback gain matrix $\mathbf{G}_H$ which minimizes the H_∞ -norm of the transfer function matrix denoted by $\|\mathbf{T}_{zd}(s)\|_\infty$. It should be noted here that it is computationally difficult to obtain $\|\mathbf{T}_{zd}(s)\|_\infty$ as compared to obtain a $\gamma > 0$ for which $\|\mathbf{T}_{zd}(s)\|_\infty < \gamma$ [9]. Therefore, in practice, the feedback gain matrix $\mathbf{G}_H$ is obtained by solving the optimization problem given as follows:

$$\begin{aligned} \min_{\gamma} \quad & \gamma \\ \text{s.t.} \quad & \|\mathbf{T}_{zd}(s)\|_\infty < \gamma \\ & \gamma > 0 \end{aligned} \quad (2.15)$$

Assuming (i) the pairs $(\mathbf{A}, \mathbf{B}_H)$ and $(\mathbf{A}, \mathbf{E})$ be stabilizable, (ii) the pair $(\mathbf{A}, \mathbf{W})$ be observable, (iii) $\mathbf{D}^T \mathbf{D} = \mathbf{I}$, and (iv) $\mathbf{D}^T \mathbf{W} = \mathbf{0}$; the constraint $\|\mathbf{T}_{zd}(s)\|_\infty < \gamma$ in Eq. (2.15) is satisfied if and only if, for a given $\gamma > 0$, there exists a solution $\mathbf{Y}$ of the ARE given as follows:

$$\mathbf{A}^T \mathbf{Y} + \mathbf{Y} \mathbf{A} - \mathbf{Y} \left(\mathbf{B}_H \mathbf{B}_H^T - \frac{1}{\gamma^2} \mathbf{E} \mathbf{E}^T \right) \mathbf{Y} + \mathbf{W}^T \mathbf{W} = \mathbf{0} \quad (2.16)$$

where $\mathbf{Y} = \mathbf{Y}^T \geq \mathbf{0}$. If the optimal solution of the problem given by Eq. (2.15) be γ^* , then the corresponding feedback gain matrix is given by

$$\mathbf{G}_H^* = -\mathbf{B}_H^T \mathbf{Y} \quad (2.17)$$

where $\mathbf{Y}$ is the solution of the ARE given by Eq. (2.16) for $\gamma = \gamma^*$. Note that the feedback gain matrix $\mathbf{G}_H$ can also be obtained by solving an alternate optimization problem involving LMIs [24], however, not shown here to save space. Since the H_∞ technique is based on indirect minimization of $\|\mathbf{T}_{zd}(s)\|_\infty$, the resulting optimal closed-loop system will exhibit minimum response; hence, no alternate optimization formulation, which constrains the system response within certain user-prescribed limit, is required.

A comparison of the feedback gain matrices obtained via the LQR and H_∞ techniques indicates that the $\mathbf{G}_H^*$ is different from the $\mathbf{G}_L^*$ because the H_∞ technique accounts for the effect of $\mathbf{d}(t)$ whereas the LQR technique does not. Both $\mathbf{G}_H^*$ (see Eqs. (2.16)–(2.17)) and $\mathbf{G}_L^*$ (see Eqs. (2.8)–(2.9)) will be same if $\gamma \rightarrow \infty$ and $\rho = 1$. When $\|\mathbf{T}_{zd}(s)\|_\infty < \infty$, $\|\mathbf{z}(t)\|_2 < \infty$ for any $\|\mathbf{d}(t)\|_2 < \infty$, hence, $\|\mathbf{x}(t)\|_2 < \infty$ for any $\|\mathbf{d}(t)\|_2 < \infty$ implying that the closed-loop system associated with the H_∞ technique (see Eq. (2.13)) is stable. In this case, the closed-loop system given by Eq. (2.13) satisfies the objective of LQR technique, i.e., $\mathbf{x}(t \rightarrow \infty) \rightarrow 0$ for $\mathbf{d}(t) = 0$.

2.2.3 Brief Review of MNQPEVA Technique

The open-loop system given by Eq. (2.1) typically exhibits high dynamic response when some of the damped natural frequencies of the system are close to the dominant range of loading spectrum. The damped natural frequencies of the system are related to the eigenvalues λ associated with the quadratic pencil $\mathbf{P}_o(\lambda) = (\lambda^2 \mathbf{M} + \lambda \mathbf{C} + \mathbf{K})$. For an open-loop system with n dof, the set of eigenvalues denoted by $\{\lambda_i\}_{i=1}^{2n}$ can be obtained by solving the quadratic eigenvalue problem (QEP) $\mathbf{P}_o(\lambda) \mathbf{x} = \mathbf{0}$, where $\{\mathbf{x}^{(i)}\}_{i=1}^{2n}$ are the right eigenvectors of the system. Let $\{\lambda_i\}_{i=1}^{2p}$ with $2p \ll 2n$ be the set of problematic eigenvalues related to the set of damped natural frequencies which are close to the dominant range of loading spectrum. For vibration control of the open-loop system, the set of eigenvalues $\{\lambda_i\}_{i=1}^{2p}$ needs to be replaced by a suitably chosen set $\{\mu_i\}_{i=1}^{2p}$. The technique to replace the set $\{\lambda_i\}_{i=1}^{2p}$ by the set $\{\mu_i\}_{i=1}^{2p}$ is known as quadratic partial eigenvalue assignment (QPEVA) technique [5, 13, 27]. It should be noted at this point that, for large-scale systems (i.e., large n), only a small number of eigenvalues and eigenvectors can be computed using the state-of-the-art computational techniques (e.g., Jacobi–Davidson methods [28, 29]) or measured in a vibration laboratory [30–32]. Therefore, the computational and engineering challenges

associated with QPEVA technique for AVC is to design a control system based only on the knowledge of the set of eigenvalues $\{\lambda_i\}_{i=1}^{2p}$ and the set of associated eigenvectors $\{x_{o_i}\}_{i=1}^{2p}$. In the recent past, few algorithms [10–12, 27] have been developed that can implement the QPEVA technique with only the knowledge of $\{\lambda_i\}_{i=1}^{2p}$ and $\{x_{o_i}^{(i)}\}_{i=1}^{2p}$. In addition, these algorithms [10–12, 27] ensure the *no-spillover property*, i.e., the rest of the eigenvalues $\{\lambda_i\}_{i=2p+1}^{2n}$ and the associated eigenvectors of the system remain unchanged even after the application of control force. The QPEVA technique, as developed in the literature [10], is presented in Sect. 2.2.3.1.

2.2.3.1 Traditional MNQPEVA Technique

The QPEVA technique typically assumes a control force of the form $\mathbf{B}\mathbf{u}(t)$, where $\mathbf{B}$ is a real-valued control matrix of dimension $n \times m$ with $m \leq n$ and the control input vector $\mathbf{u}(t) = \mathbf{F}^T \dot{\boldsymbol{\eta}}(t) + \mathbf{G}^T \boldsymbol{\eta}(t)$. The matrices $\mathbf{F}$ and $\mathbf{G}$ are real-valued feedback gain matrices of dimensions $n \times m$. With the application of the control force $\mathbf{B}\mathbf{u}(t)$, the governing equation of closed-loop system can be written as follows:

$$\mathbf{M}\ddot{\boldsymbol{\eta}}(t) + (\mathbf{C} - \mathbf{B}\mathbf{F}^T)\dot{\boldsymbol{\eta}}(t) + (\mathbf{K} - \mathbf{B}\mathbf{G}^T)\boldsymbol{\eta}(t) = \mathbf{f}(t) \quad (2.18)$$

Using only the knowledge of $\{\lambda_i\}_{i=1}^{2p}$ and $\{x_{o_i}^{(i)}\}_{i=1}^{2p}$, the objective of QPEVA technique is to find the feedback matrices $\mathbf{F}$ and $\mathbf{G}$ for a given $\mathbf{B}$, such that the set of eigenvalues of the closed-loop system is $\{\{\mu_i\}_{i=1}^{2p}, \{\lambda_i\}_{i=2p+1}^{2n}\}$ with the eigenvectors $\{x_{o_i}^{(i)}\}_{i=2p+1}^{2n}$ corresponding to $\{\lambda_i\}_{i=2p+1}^{2n}$ remain unchanged (i.e., no-spillover property is guaranteed). Assuming (i) $\{\lambda_i\}_{i=1}^{2p} \cap \{\mu_i\}_{i=1}^{2p} \cap \{\lambda_i\}_{i=2p+1}^{2n} = \emptyset$ (ii) $0 \notin \{\lambda_i\}_{i=1}^{2p}$, (iii) $\{\lambda_i\}_{i=1}^{2p}$ and $\{\mu_i\}_{i=1}^{2p}$ are closed under complex conjugation, and (iv) partial controllability of the pair $(\mathbf{P}_o(\lambda), \mathbf{B})$ with respect to $\{\lambda_i\}_{i=1}^{2p}$, the feedback matrices $\mathbf{F}$ and $\mathbf{G}$ satisfying the no-spillover property can be obtained as follows:

$$\begin{aligned} \mathbf{F}(\boldsymbol{\Gamma}) &= \mathbf{M}\mathbf{X}_{o_{2p}}\boldsymbol{\Lambda}_{o_{2p}}\mathbf{Z}^{-T}\boldsymbol{\Gamma}^T \\ \mathbf{G}(\boldsymbol{\Gamma}) &= -\mathbf{K}\mathbf{X}_{o_{2p}}\mathbf{Z}^{-T}\boldsymbol{\Gamma}^T \end{aligned} \quad (2.19)$$

where $\boldsymbol{\Lambda}_{o_{2p}} = \text{diag}\{\lambda_i\}_{i=1}^{2p}$, $\boldsymbol{\Lambda}_{c_{2p}} = \text{diag}\{\mu_i\}_{i=1}^{2p}$, $\mathbf{X}_{o_{2p}} = [\mathbf{x}_o^{(1)}, \mathbf{x}_o^{(2)}, \dots, \mathbf{x}_o^{(2p)}]$, and $\mathbf{Z}$ is the unique solution of the *Sylvester equation* given by

$$\boldsymbol{\Lambda}_{o_{2p}}\mathbf{Z} - \mathbf{Z}\boldsymbol{\Lambda}_{c_{2p}} = -\boldsymbol{\Lambda}_{o_{2p}}\mathbf{X}_{o_{2p}}^T\mathbf{B}\boldsymbol{\Gamma} \quad (2.20)$$

In Eqs. (2.19) and (2.20), $\boldsymbol{\Gamma} = \{\gamma_i\}_{i=1}^{2p}$ is a parametric matrix of dimension $m \times 2p$, and the set $\{\gamma_i\}_{i=1}^{2p}$ is closed under complex conjugation in the same order as the set $\{\mu_i\}_{i=1}^{2p}$.

Exploiting the parametric nature of the feedback matrices $\mathbf{F}$ and $\mathbf{G}$ (see Eq. (2.19)), optimization problem has been formulated in literature [10] that results in feedback matrices with minimum norm. The technique to implement QPEVA with minimum norm feedback matrices is known as MNQPEVA technique, and the associated optimization problem is given by

$$\min_{\mathbf{\Gamma}} \left(\frac{1}{2} [\|\mathbf{F}(\mathbf{\Gamma})\|_F^2 + \|\mathbf{G}(\mathbf{\Gamma})\|_F^2] \right) \quad (2.21)$$

where $\|\cdot\|_F$ denotes the Frobenius norm of a matrix.

2.2.3.2 Modified MNQPEVA Technique

The MNQPEVA formulation presented in Sect. 2.2.3.1 results in feedback matrices $\mathbf{F}$ and $\mathbf{G}$ with minimum norm for a chosen set $\{\mu_i\}_{i=1}^{2p}$. The dynamic responses (e.g., displacement, inter-story drift, velocity, acceleration, stress, etc.) of the closed-loop system given by Eq. (2.18) depend on the choice of $\{\mu_i\}_{i=1}^{2p}$. In engineering practice, manually choosing a set $\{\mu_i\}_{i=1}^{2p}$, which will ensure that the dynamic responses of the closed-loop system are within user-prescribe limit, can be challenging and might require iterations. As a way out, a nested optimization problem can be formulated that results in an optimal set $\{\mu_i\}_{i=1}^{2p}$ for which $\mathbf{F}$ and $\mathbf{G}$ have minimum norm and the dynamic responses of the closed-loop system are within user-prescribe limit. The outer optimization problem of the nested optimization scheme can be formulated as follows:

$$\begin{aligned} \min_{\mathbf{q}} \quad & g(\mathbf{q}) \\ \text{s.t.} \quad & y_i(\mathbf{q}) \leq y_{icr}; \quad i = 1, \dots, n_y \\ & l_{q_j} \leq q_j \leq u_{q_j}; \quad j = 1, \dots, 2p \end{aligned} \quad (2.22)$$

where $\mathbf{q}$ is the set of control input parameters associated with the set $\{\mu_i\}_{i=1}^{2p}$. The objective function $g(\mathbf{q})$ is obtained by solving an inner optimization problem given by

$$g(\mathbf{q}) = \min_{\mathbf{\Gamma}} \left(\frac{1}{2} [\|\mathbf{F}(\mathbf{q}, \mathbf{\Gamma})\|_F^2 + \|\mathbf{G}(\mathbf{q}, \mathbf{\Gamma})\|_F^2] \right) \quad (2.23)$$

Note that the optimization problem given in Eq. (2.23) is same as the traditional MNQPEVA problem but now expressed as a function of $\mathbf{q}$ (see Eq. (2.21)). In Eq. (2.22), the term $y_i(\mathbf{q})$ represents the i th response quantity of interest with the associated critical value given by y_{icr} . The terms l_{q_j} and u_{q_j} represent the lower and upper bounds, respectively, on the j th decision variable q_j .

Remark: Relative computational advantages/drawbacks of the LQR, H_∞ , and MNQPEVA techniques are summarized below.

1. The matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ possess computationally exploitable properties, such as the symmetry, positive definiteness, sparsity, bandness, etc., which are assets for large-scale computing. The MNQPEVA technique works exclusively on the second-order model (see Eq. (2.18)), and hence can take full advantage of these computationally exploitable properties. The state-space based LQR and H_∞ techniques, on the other hand, require a transformation of the second-order model (see Eq. (2.1)) to a first-order one (see Eq. (2.2)) due to which matrix $\mathbf{A}$ loses all these computationally exploitable properties. Note that the symmetry of matrix $\mathbf{A}$ can be preserved by transforming the second-order model to the generalized state-space form, but not the definiteness [29, Sect. 11.9.3]. Additionally, if $\mathbf{M}$ is ill-conditioned, computation of the matrices $\mathbf{A}$ and $\mathbf{E}$ will be problematic.
2. The dimensions of matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$, and matrix products $\mathbf{B}\mathbf{F}^T$, $\mathbf{B}\mathbf{G}^T$ are $n \times n$ (see Eq. (2.18)) whereas the dimensions of matrices $\mathbf{A}$, $\mathbf{E}$ and matrix products $\mathbf{B}_L\mathbf{G}_L$, $\mathbf{B}_H\mathbf{G}_H$ are $2n \times 2n$ (see Eqs. (2.10) and (2.13)). For large n , solving the dynamic equations associated with the LQR and H_∞ techniques (e.g., Eqs. (2.7), (2.10), and (2.13)) will be a formidable task even with the state-of-the-art large-scale matrix computation techniques.
3. The LQR and H_∞ techniques require solutions of AREs involving matrices of dimensions $2n \times 2n$ (see Eqs. (2.9) and (2.16)). If n is very large, it is difficult to obtain solutions of AREs as no computationally viable methods have still been developed for the solution of a very large ARE. The MNQPEVA technique, on the other hand, will not suffer computationally due to large n as it requires a solution of a Sylvester equation involving matrices of dimensions $2p \times 2p$ with $p \ll n$ (see Eq. (2.20)).
4. The LQR and H_∞ techniques ensure the stability of closed-loop systems by reassigning all the eigenvalues of the associated open-loop systems. In contrast, the MNQPEVA technique ensures the stability of a closed-loop system by reassigning only a few problematic eigenvalues to suitably chosen ones by the users. In addition, the MNQPEVA technique guarantees the no-spillover property by means of a sophisticated mathematical theory based on the orthogonal properties of the eigenvectors of the associated quadratic model [29, Sect. 11.9.1].

The computational advantages of the MNQPEVA technique make it a very suitable candidate for practical applications, such as vibration control of large-scale structures.

2.2.4 Numerical Comparison of LQR, H_∞ , and MNQPEVA Techniques

A comparison of the MNQPEVA technique (see Sect. 2.2.3) with the LQR (see Sect. 2.2.1) and H_∞ (see Sect. 2.2.2) techniques is presented in this section using a 60 dof system (see Fig. 2.1). This 60 dof system is a simplified representation of a 2D frame of a 60-story steel building. The floor masses $m_i = 4 \times 10^5$ kg and the floor heights $h_i = 4$ m for $i = 1, \dots, 60$. The distribution of the stiffness along the height of the system is presented in Table 2.1. Rayleigh type damping matrix with $\xi_{1,5} = 0.02$, where $\xi_{1,5}$ denote the damping ratios associated with the first and fifth modes of the system, is assumed for this example problem. The system is subjected to wind load that is typically expected in the west coast area of USA. The wind time-histories at different levels of the 60 dof system are simulated using the NatHaz online wind simulator [33, 34] with basic wind speed of 52 m/s and exposure category D for the system [35]. When subjected to the wind load, the open-loop system exhibits high dynamic responses which exceed the user-prescribed limits (see Table 2.2).

The comparison among the LQR, H_∞ , and MNQPEVA techniques is based on two criteria—(i) *economy* and (iii) *computation time*. A technique is deemed as the most economical one if it requires the least actuator capacity for control of the same

Fig. 2.1 Schematic representation of the 60 dof system subjected to wind load

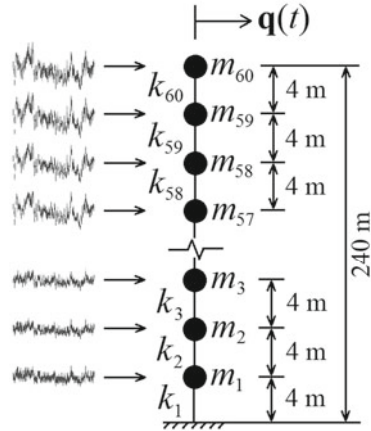


Table 2.1 Distribution of the stiffness along the height of the 60 dof system

Range of dof	Stiffness (GN/m)	Range of dof	Stiffness (GN/m)	Range of dof	Stiffness (GN/m)	Range of dof	Stiffness (GN/m)
1–9	1.12	28–30	0.61	40–42	0.39	52–54	0.21
10–15	0.97	31–33	0.54	43–45	0.35	55–57	0.16
16–21	0.80	34–36	0.48	46–48	0.31	58–60	0.14
22–27	0.70	37–39	0.43	49–51	0.26		

open-loop system (viz., the 60 dof system) and simultaneously satisfy some safety criterion associated with the response of the closed-loop system. For this example problem, the safety criterion is based on the maximum displacement response and the maximum inter-story drift response of a system. A closed-loop system is deemed safe if (i) the maximum displacement response denoted by y_1 satisfies $y_1 \leq y_{1_{cr}} = 0.600$ m, and (ii) the maximum inter-story drift response denoted by y_2 satisfies $y_2 \leq y_{2_{cr}} = 0.0033$. The computation time associated with any control technique is defined as the time required by that technique to find the optimal feedback matrix (e.g., $\mathbf{G}_L$ for LQR; $\mathbf{G}_H$ for H_∞) or matrices (e.g., $\mathbf{F}$ and $\mathbf{G}$ for MNQPEVA). The comparison is presented in two stages. In the first stage, only the traditional formulations of the LQR, H_∞ , and MNQPEVA techniques are compared with respect to the two criteria mentioned above. In the second stage, the modified LQR and modified MNQPEVA techniques are compared. For comparison purpose, the control matrices B_L , B_H , and B are chosen to be equivalent, i.e., (i) they satisfy $m_L = m_H = m = 2$, and (ii) they comply with the location of actuators at floor levels 5 and 60. Also, the matrices $\mathbf{W}$ and $\mathbf{D}$ are assumed as $\mathbf{W} = [\mathbf{I}_{2n \times 2n}, \mathbf{0}_{2 \times 2n}]^T$ and $\mathbf{D} = [\mathbf{0}_{2n \times 2n}, \mathbf{I}_{2 \times 2}]^T$, respectively, which satisfy the conditions $\mathbf{D}^T \mathbf{W} = \mathbf{0}$ and $\mathbf{D}^T \mathbf{D} = \mathbf{I}$. As mentioned earlier in Sect. 2.2.1.1, $\mathbf{Q} = \mathbf{W}^T \mathbf{W}$ is chosen in this example problem for the LQR technique. For comparison purpose, the results for the traditional LQR technique are obtained for two values of ρ (viz., $\rho = 1.0$ and $\rho = 0.5$). For MNQPEVA technique (both traditional and alternate formulations), only the first pair of eigenvalues $\{\lambda_{1,2}\}$ associated with the first damped natural frequency of the open-loop system are shifted. All the comparison results are presented in Table 2.2. Note that all computations in this section are performed in 32-bit MATLAB [36] using a single core in a computer with Intel(R) Core(TM)2 Duo processor having speed of 2 GHz and RAM of 3 GB.

It can be observed from Table 2.2 that the dynamic response of the 60 dof system can be reduced using all the three control techniques; however, the amount of reduction in response depends on the control technique adopted. Comparison of the traditional formulations of the LQR, H_∞ , and MNQPEVA techniques (i.e., Stage-1) indicates that the LQR technique with $\rho = 1.0$, and the H_∞ technique cannot guarantee the safety of the system. Among the traditional formulations, the MNQPEVA technique emerges as the best technique because it is (i) more economic in terms of the actuator capacity required at floor levels 5 and 60 and (ii) computationally more efficient (see Table 2.2). Note that the traditional MNQPEVA technique has been implemented in this example problem by shifting only the first pair of eigenvalues of the system from $\{-0.0218 \pm 1.0917j\}$ to $\{-0.0800 \pm 1.1700j\}$, where $j = \sqrt{-1}$. The results of Stage-2 indicate that the modified MNQPEVA technique turns out to be the most economical one, however, at the cost of higher computational time (see Table 2.2). It should be remarked here that the difference in computational time for the different techniques presented in Sects. 2.2.1, 2.2.2 and 2.2.3 depends on several factors, for example, formulations of the optimization problems, size of the system defined by the degrees of freedom, and the underlying space on which the problem needs to be constructed (i.e., state-space based LQR and H_∞ problem, and physical-space based MNQPEVA problem). It can be concluded from the results presented

Table 2.2 Comparison of LQR, H_∞ , and MNQPEVA techniques for a 60 dof system

Parameters for comparison	User-prescribed limit	Open-loop system	Closed-loop systems					
			Stage-1: Traditional			Stage-2: Modified		
			LQR with $\rho = 1$	LQR with $\rho = 0.5$	H_∞	MNQPEVA	LQR	MNQPEVA
y_1 (m)	0.6000	0.7952	0.6323	0.5926	0.6310	0.5748	0.6000	0.5885
y_2	0.0033	0.0042	0.0035	0.0033	0.0035	0.0033	0.0033	0.0033
Actuator capacity at floor 5 (MN)	–	–	0.0576	0.0884	0.0617	0.0668	0.0820	0.0628
Actuator capacity at floor 60 (MN)	–	–	0.8289	1.1141	0.8588	1.0605	1.0615	0.9975
Computation time (s)	–	–	0.1519	0.1537	87.5085	0.1277	38.4176	99.0121
Optimization algorithm	–	–	–	–	Interior-point	Quasi-Newton line search	Interior-point	Active-set

in Table 2.2 that the MNQPEVA technique performs better than the LQR and H_∞ techniques in terms of the economy and safety of the closed-loop systems, hence, justify the motivation for the development of a computationally effective stochastic MNQPEVA technique for economic and resilient design of closed-loop systems.

2.3 Control of Stochastic Systems Using MNQPEVA Technique

In this section, a variant of the modified MNQPEVA problem, termed as minimum norm quadratic partial eigenvalue assignment—minimum mass closed-loop system (MNQPEVA-MMCLS) problem, is presented first followed by a stochastic formulation of the MNQPEVA-MMCLS problem. The stochastic MNQPEVA-MMCLS problem is based on optimization under uncertainty (OUU). The optimal feedback matrices obtained by solving the stochastic MNQPEVA-MMCLS problem will lead to economic and resilient design of the stochastic structures. The computational challenge in solving the stochastic MNQPEVA-MMCLS problem is how to efficiently estimate a low probability of failure with high accuracy. An algorithm based on agglomerative hierarchical clustering technique (AHCT) and importance sampling (IS) scheme is presented next that can efficiently and accurately estimate very low probability of failure. Finally, the stochastic MNQPEVA-MMCLS technique is illustrated using a numerical example.

2.3.1 Stochastic Formulation of MNQPEVA-MMCLS Problem

The MNQPEVA-MMCLS problem minimizes the mass of the closed-loop system in addition to minimizing the norm of feedback matrices. The mass is minimized in order to reduce the material cost and inertia induced effects. Being a variant of the modified MNQPEVA problem, the deterministic MNQPEVA-MMCLS problem is also a nested optimization problem. This nested optimization scheme ensures both *economic design* and *safety* of the closed-loop system. The deterministic MNQPEVA-MMCLS problem, as developed in the literature [23], is given by

$$\begin{aligned} \min_{\mathbf{q}} \quad & a \left(\frac{g(\mathbf{q})}{g_{target}} - 1 \right)^2 + (1 - a) \left(\frac{\mathcal{M}(\mathbf{q})}{\mathcal{M}_{avg}} \right)^2 \\ \text{s.t.} \quad & y_i(\mathbf{q}) \leq y_{i_{cr}}; \quad i = 1, \dots, n_y \\ & l_{q_j} \leq q_j \leq u_{q_j}; \quad j = 1, \dots, n_q \end{aligned} \quad (2.24)$$

where $\mathbf{q}$ is a vector of structural (material and geometric) and control input parameters, a is a positive real-valued weight coefficient, $\mathcal{M}(\mathbf{q})$ is the total mass of the

closed-loop system for given $\mathbf{q}$, and $g(\mathbf{q})$ has the same form as given in Eq. (2.23). The terms g_{target} and $\mathcal{M}_{avg}$ in Eq. (2.24) are normalizing factors which are used to eliminate any bias due to difference in magnitude of the two terms while numerically carrying out the optimization scheme. In addition, g_{target} serves as a target value for $g(\mathbf{q})$. The terms $y_i(\mathbf{q})$, y_{icr} , l_{qj} , u_{qj} , n_y , and n_q in Eq. (2.24) are defined earlier in Sect. 2.2.3.2; however, $\mathbf{q}$ in Eq. (2.24) contains structural parameters in addition to the control input parameters related to the set $\{\mu_i\}_{i=1}^{2p}$.

The first step toward formulation of stochastic MNQPEVA-MMCLS problem is the characterization of various uncertainties, for example, *parametric uncertainty* [14] and *model uncertainty* [15], using the most suitable probabilistic model. The conventional parametric probabilistic models [14, 37, 38] can be used to characterize parametric uncertainties, whereas random matrix theory (RMT) based probabilistic models would be suitable for characterization of model uncertainties [15, 39–41]. Currently, there exists hybrid probabilistic formalism that can couple the conventional parametric model with the RMT-based model [42]. The stochastic MNQPEVA-MMCLS problem formulation, as developed in the literature [23], accounts for uncertainties in structural system parameters (e.g., material and geometric) and control input parameters by introducing a random vector $\mathbf{q}$, which collectively represent all random parameters associated with the structural system and control input. Using $\mathbf{q}$ and applying the concept of OUU [43], the stochastic MNQPEVA-MMCLS problem can be written as

$$\begin{aligned} \min_{\mu_q} \quad & \mathbb{E} \left[a \left(\frac{g(\mathbf{q})}{g_{target}} - 1 \right)^2 + (1 - a) \left(\frac{\mathcal{M}(\mathbf{q})}{\mathcal{M}_{avg}} \right)^2 \right] \\ \text{s.t.} \quad & P[y_i(\mathbf{q}) \leq y_{icr}] = 1 - P_{f_{y_i}} \geq P_{y_i}; \quad i = 1, \dots, n_y \\ & P[l_{qj} \leq \mathbf{q} \leq u_{qj}] \geq P_{q_j}; \quad j = 1, \dots, n_q \end{aligned} \quad (2.25)$$

where μ_q is the vector of design variables representing the mean vector of $\mathbf{q}$. The terms P_{y_i} and P_{q_j} are the user-defined lowest probabilities to be satisfied for the i th constraint and the j th input bounds, respectively. Note that $(1 - P_{y_i})$ is the prescribed probability of failure for $y_i(\mathbf{q})$, where failure is defined by $y_i(\mathbf{q}) > y_{icr}$. The term $P_{f_{y_i}}$ denotes the current probability of failure for $y_i(\mathbf{q})$ given μ_q .

An OUU problem is typically solved after simplifying it to a *robust optimization* (RO) problem where the objective function and the constraints are expressed as weighted sum of mean and variance of respective quantities neglecting the higher order statistics [43–48]. The algorithms for solving RO typically assume statistical independence among the random input parameters and require a priori knowledge of the probability density functions (pdfs) of constraints and objective function for proper selection of *weight coefficients* for mean and variance. In RO, the probabilistic constraint associated with the i th random response $y_i(\mathbf{q})$ is satisfied by shifting the support of the pdf $p_{y_i(\mathbf{q})}$ of $y_i(\mathbf{q})$ to the left of y_{icr} without changing the shape of $p_{y_i(\mathbf{q})}$ (see Fig. 2.2a). The constraint on $P_{f_{y_i}}$ can, however, be satisfied via a combination of change in shape and shift in support of $p_{y_i(\mathbf{q})}$ (see Fig. 2.2b) by considering the set of distribution parameters $\{\theta_k\}_{k=1}^{n_\theta}$ of the joint pdf p_q of $\mathbf{q}$ as the set of design variables for the MNQPEVA-MMCLS problem. The term n_θ denotes the number of distribution

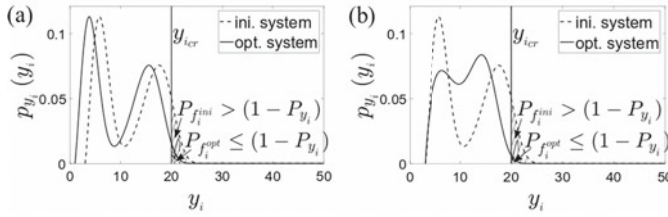


Fig. 2.2 Pictorial presentation of constraint satisfaction for probability of failure: **a** existing methods and **b** proposed method (adopted from [23])

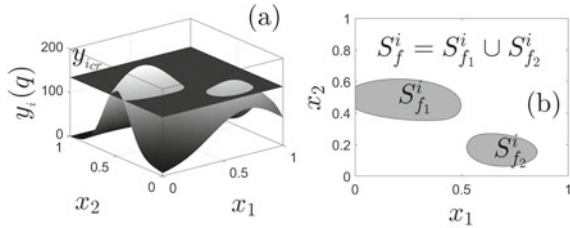
parameters required to define the joint pdf $p_{\mathbf{q}}$. This idea can be implemented using families of pdfs for which the set of distribution parameters include shape parameters (e.g., Beta distribution). It should, however, be noted that the selection of the family of pdfs should be carried out with proper judgment so that the selected pdf complies with the manufacturing/construction specifications.

Using the idea of satisfying the constraint on $P_{f_{y_i}}$ via a combination of change in shape and shift in support of $p_{y_i}(\mathbf{q})$, the formulation of stochastic MNQPEVAP-MMCLS problem, as developed in [23], can be written as follows:

$$\begin{aligned}
 \min_{\{\theta_k\}_{k=1}^{n_\theta}} \quad & \mathbb{E} \left[a \left(\frac{g(\mathbf{q}|\{\theta_k\}_{k=1}^{n_\theta})}{g_{target}} - 1 \right)^2 + (1-a) \left(\frac{\mathcal{M}(\mathbf{q}|\{\theta_k\}_{k=1}^{n_\theta})}{\mathcal{M}_{avg}} \right)^2 \right] \\
 \text{s.t.} \quad & P[y_i(\mathbf{q}|\{\theta_k\}_{k=1}^{n_\theta}) \leq y_{i_{cr}}] = 1 - P_{f_{y_i}} \geq P_{y_i}; \quad i = 1, \dots, n_y \\
 & l_{\theta_k} \leq \theta_k \leq u_{\theta_k}; \quad k = 1, \dots, n_\theta
 \end{aligned} \tag{2.26}$$

where $g(\mathbf{q}|\{\theta_k\}_{k=1}^{n_\theta})$ and $\mathcal{M}(\mathbf{q}|\{\theta_k\}_{k=1}^{n_\theta})$ are same as mentioned earlier in the deterministic MNQPEVA-MMCLS problem formulation (see Eq. (2.24)), however, now expressed as functions of $\{\theta_k\}_{k=1}^{n_\theta}$. Note that, in accordance with the deterministic MNQPEVA-MMCLS problem, the stochastic MNQPEVA-MMCLS problem is also a nested optimization problem. In Eq. (2.26), the terms l_{θ_k} and u_{θ_k} represent the lower and upper bounds on the k th decision variable, respectively. The optimality of the solution of Eq. (2.26) depends on the accuracy of the estimate of $P_{f_{y_i}}(\{\theta_k\}_{k=1}^{n_\theta})$, $\forall i$. Efficient yet accurate estimation of $P_{f_{y_i}}(\{\theta_k\}_{k=1}^{n_\theta})$, $\forall i$ is computationally difficult because the estimate depends on the knowledge of failure domain $S_f^i = y_i(\mathbf{q}) \geq y_{i_{cr}}$ which might be disjointed and complicated in shape (see Fig. 2.3), and not known explicitly as a function of system parameters (for example, available only through large-scale finite element simulations). A way out from this computational difficulty is presented in the following section.

Fig. 2.3 Schematic representation of **a** variation of $y_i(\mathbf{q})$ with $\mathbf{q}$ and **b** disconnected S_f^i associated with $y_i(\mathbf{q})$ where $\mathbf{q} = [x_1, x_2]^T$ (adopted from [23])



2.3.2 Estimation of Probability of Failure

An algorithm developed in [23] for efficient yet accurate estimation of probability of failure P_f has been presented in this section. The algorithm is based on (i) AHCT for identification of disjointed and complicated shaped failure domain S_f and (ii) IS scheme for computation of P_f .

Unknown S_f can be identified efficiently by applying AHCT on digitally simulated failure samples as (i) unlike the k -means and other clustering techniques, the AHCT does not assume the failure samples to be independent and identically distributed and (ii) the time complexity of AHCT ($\mathcal{O}(N^3)$) is much less compared to the time complexity of divisive hierarchical clustering technique ($\mathcal{O}(2^N)$) for large sample size N [49, Sect. 14.3.12]. Based on certain distance measures among the available samples (e.g., Euclidean, Minkowski, Cosine, etc.) and among clusters (e.g., single linkage (SL), complete linkage (CL), average linkage (AL), etc.), the samples are sequentially grouped together in AHCT, which is pictorially depicted via a dendrogram [49, Sect. 14.3.12]. The efficiency of the identification process can be increased further by applying SL-based AHCT as (i) it has lesser time complexity ($\mathcal{O}(N^2)$) as compared to the CL- and AL-based AHCTs ($\mathcal{O}(N^2 \log(N))$) [50] and (ii) it is more stable and has better convergence criteria [51]. The algorithmic steps for identification of S_f are presented in Algorithm 1 (see Steps 1–2).

After S_f is identified, estimate of P_f can be obtained by simulating samples in S_f . Note that the accuracy of the estimate depends on the sample size in S_f . Towards this estimation process, IS scheme is preferred over the conventional Monte Carlo simulation (MCS) technique as MCS will require large number of samples in S to obtain sufficient number of samples in S_f . An IS estimate of an order statistic of $f(\mathbf{q})$ representing a function of the random vector $\mathbf{q}$ is obtained by first generating samples from the domain of interest S_f using an auxiliary pdf $g_{\mathbf{q}}(\mathbf{q})$ defined over S_f and then correcting for the bias using original pdf $p_{\mathbf{q}}(\mathbf{q})$ [52–54]. To ensure almost sure convergence of the IS scheme, the chosen $g_{\mathbf{q}}(\mathbf{q})$ should comply with (i) $\forall \mathbf{q} \in S_f, g_{\mathbf{q}}(\mathbf{q}) > 0$ if $p_{\mathbf{q}}(\mathbf{q}) > 0$ and (ii) $p_{\mathbf{q}}(\mathbf{q}) < k g_{\mathbf{q}}(\mathbf{q})$ with $k \gg 1$ [54]. Also, the IS estimator has finite variance when $\int_{S_f} f^2(\mathbf{q}) \frac{p_{\mathbf{q}}^2(\mathbf{q})}{g_{\mathbf{q}}(\mathbf{q})} d\mathbf{q} < \infty$ [54]. One should note here that the traditional IS scheme is restrictive as it requires the information of “exact” S_f to define the support of $g_{\mathbf{q}}(\mathbf{q})$. When exact S_f is unknown, the support of $g_{\mathbf{q}}(\mathbf{q})$ can be defined over a domain $S_S \supseteq S_f$ to employ the IS scheme. The IS estimate of $P[\mathbf{q} \in S_f]$ is given by

$$P[\mathbf{q} \in S_f] = \frac{1}{N_{S_f}} \sum_{i=1}^{N_{S_f}} \frac{p_{\mathbf{q}}(q^{(i)})}{g_{\mathbf{q}}(q^{(i)})} \mathbb{I}_{S_f} \quad (2.27)$$

where N_{S_f} is the number of samples generated within S_f . To obtain the estimate of P_f associated with a S_f composed of several disconnected parts, the IS scheme given by Eq. (2.27) needs to be applied repeatedly over each disconnected failure domain (S_{f_j}) and summed over all j . The algorithmic steps to obtain IS estimate of P_f are presented in Algorithm 1 (see Steps 3–4).

Algorithm 1: Estimation of probability of failure

Input: (i) n -D Euclidean domain representing support S of $\mathbf{q}$, (ii) sample budget N_{ex} for domain exploration, (iii) failure criteria defined by $y_{i_{cr}}, \forall i$, (iv) tolerance ε , (v) sample budget N for IS scheme, and (vi) joint pdf $p_{\mathbf{q}}(\mathbf{q})$.

Step 1: Explore the failure domain $S_f \in S$ by first obtaining N_{ex} samples in S and then segregating the failure samples using the criteria $y_i > y_{i_{cr}}, \forall i$. For failure domain exploration without any bias, assume the elements $\mathbf{q}_j$ of $\mathbf{q}$ to be statistically independent and digitally simulate the N_{ex} samples using $\mathbf{q}_j \sim U[S_{min_{q_j}}, S_{max_{q_j}}], \forall j$ where $S_{min_{q_j}}$ and $S_{max_{q_j}}$ represent the possible minimum and maximum bounds on $\mathbf{q}_j$, respectively, to be chosen for numerical purpose.

Step 2: Identify n_{S_f} parts of a disjointed S_f by first obtaining a dendrogram using SL based AHCT and then slicing the dendrogram using a cutoff value associated with the normalized incremental height of the dendrogram.

Step 3: Using auxiliary pdfs $g_{\mathbf{q}}^j(\mathbf{q})$ for $j = 1, \dots, n_{S_f}$; obtain a total of N new samples from the n -D boxes bounding the disjointed parts of S_f with tolerance ε . The sample size N_j associated with the j -th bounding box is proportional to the size of the box so that $\sum_{j=1}^{n_{S_f}} N_j = N$.

Step 4: Use the IS scheme $P_f = \sum_{j=1}^{n_{S_f}} \frac{1}{N_j} \sum_{i=1}^{N_j} f(q^{(i)}) \frac{p_{\mathbf{q}}(q^{(i)})}{g_{\mathbf{q}}^j(q^{(i)})} \mathbb{I}_{S_{f_j}}$ to estimate probability of failure.

Output: IS estimate of probability of failure P_f .

Example: Let $\mathbf{q} = [\mathbf{x}_1, \mathbf{x}_2]^T$ be a random vector with two statistically independent random variables $\mathbf{x}_1 \sim \text{Beta}(4, 3, 0, 1)$ and $\mathbf{x}_2 \sim \text{Beta}(6, 2, 0, 1)$. The failure domain S_f is assumed to be disconnected with 2 parts: $S_{f_1} = \{\mathbf{q} : \frac{(x_1 - 0.090)^2}{0.075^2} + \frac{(x_2 - 0.120)^2}{0.100^2} \leq 1\}$ and $S_{f_2} = \{\mathbf{q} : \frac{(x_1 - 0.900)^2}{0.075^2} + \frac{(x_2 - 0.070)^2}{0.050^2} \leq 1\}$ (see Fig. 2.4a). The true probability of failure for this problem is 3.763×10^{-6} . Using Algorithm 1 with (i) $S = [0, 1] \times [0, 1]$, (ii) $N_{ex} = 500$, (iii) failure criteria mentioned above (S_{f_1} and S_{f_2}), (iv) $\varepsilon = 0.1$, (v) $N = 600$, and (vi) $p_{\mathbf{q}}(\mathbf{q}) = p_{\mathbf{x}_1}(x_1)p_{\mathbf{x}_2}(x_2)$, the IS estimate of P_f is obtained as 3.696×10^{-6} with 1.77% error. Note that the cutoff value for the normalized incremental height of the dendrogram is taken as 0.1. For this problem,

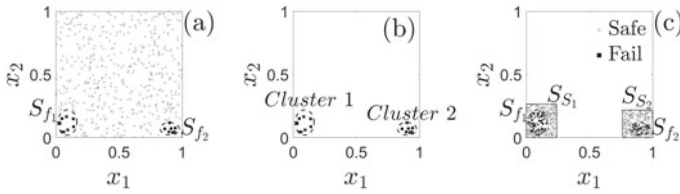


Fig. 2.4 Various attributes of the example problem: **a** domain exploration (Step 1), **b** failure domain identification (Step 2), and **c** sample generation from bounding boxes (Step 3) (adopted from [23])

Steps 1, 2, and 3 of Algorithm 1 are depicted in Fig. 2.4a, b, and c, respectively. A probabilistic convergence study conducted with 1000 different sets of $N_{ex} = 500$ exploratory samples indicates that the accuracy of Algorithm 1 increases significantly with sample budget N for IS scheme [23].

2.3.3 Numerical Illustration of Stochastic MNQPEVA-MMCLS Problem

The methodology presented in Sects. 2.3.1–2.3.2 for design of stochastic closed-loop system is numerically illustrated here using a 2D frame of a 4-story steel building (see Fig. 2.5a, adopted from [23]) subjected to the fault-normal component of ground acceleration recorded during the 1994 Northridge earthquake at the Rinaldi receiving station. The absolute maximum value of the ground acceleration record is scaled to $0.4g$ where g is the acceleration due to gravity.

In this example, only the material properties, viz., density ρ and Young's modulus E , are assumed to be random to reduce the computation cost due to dimensionality. All other parameters are assumed to be deterministic and their values are $w = 2800$ kg/m, $\xi_1 = \xi_2 = 0.02$, $b_1 = 0.7754$ m, $h_1 = 0.3190$ m, $t_{f_1} = 0.0229$ m, $t_{w_1} = 0.0381$ m, $b_2 = 0.4922$ m, $h_2 = 0.3781$ m, $t_{f_2} = 0.0330$ m, $t_{w_2} = 0.0445$ m, $b_3 = 0.4826$ m, $h_3 = 0.3938$ m, $t_{f_3} = 0.0254$ m, $t_{w_3} = 0.0381$ m, $l_1 = 7.31$ m, $l_2 = 5.49$ m, and $l_3 = 3.66$ m, where w is the dead load, $\{\xi_i\}_{i=1}^2$ are the damping ratios associated

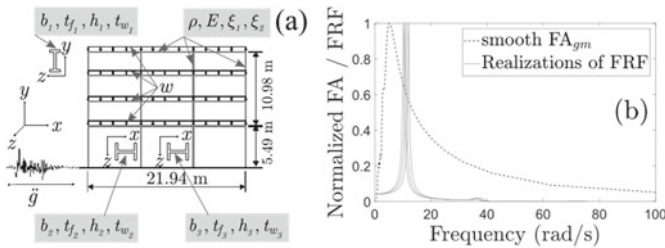


Fig. 2.5 Plots depicting **a** the 2D steel frame structure and **b** a few FRFs of the frame along with the normalized smooth FA of the ground motion (adopted from [23])

with the first two modes of vibration, and $\{b_i\}_{i=1}^3$, $\{h_i\}_{i=1}^3$, $\{t_f\}_{i=1}^3$, $\{t_w\}_{i=1}^3$, and $\{l_i\}_{i=1}^3$ are the geometric properties that represent the breadth, height, flange thickness, web thickness, and length of the I-sections, respectively, used as structural elements of the frame. It is assumed that $\boldsymbol{\rho} \sim \text{Beta}(\alpha_1, \beta_1, c_{l_1}, c_{u_1})$, $\mathbf{E} \sim \text{Beta}(\alpha_2, \beta_2, c_{l_2}, c_{u_2})$, and $\boldsymbol{\rho}$, $\mathbf{E}$ are statistically independent. The bounds of the distribution parameters $(\{\alpha_j\}_{j=1}^2, \{\beta_j\}_{j=1}^2, \{c_{l_j}\}_{j=1}^2, \{c_{u_j}\}_{j=1}^2)$ are presented in Table 2.3. An initial design of the stochastic 2D frame using the deterministic values mentioned earlier and assuming $\alpha_1 = 3$, $\beta_1 = 2$, $c_{l_1} = 7000 \text{ kg/m}^3$, $c_{u_1} = 8600 \text{ kg/m}^3$, $\alpha_2 = 2$, $\beta_2 = 3$, $c_{l_2} = 180 \text{ GPa}$, and $c_{u_2} = 240 \text{ GPa}$ indicates that the fundamental damped natural frequency ω_{dn_1} of the structure lies within the dominant range of the spectrum of the ground motion (see Fig. 2.5b, adopted from [23]).

To reduce the vibration level of the structure using MNQPEVA-based AVC, $\{\lambda_{1,2}\}$ associated with ω_{dn_1} of the structure need to be replaced by new target eigenvalues $\{\mu_{1,2} = r \pm jm\}$ where $j = \sqrt{-1}$. For this purpose, the control matrix $\mathbf{B}$ is chosen as $\mathbf{B}_{100 \times 2} = \begin{bmatrix} B_{1,1} = 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & B_{2,52} = 1 & 0 & \dots & 0 \end{bmatrix}^T$ that comply with the placement of actuators at the roof and first floor of the structure. As mentioned earlier, control input parameters are plagued with various uncertainties which will propagate to $\{\mu_{1,2} = r \pm jm\}$, and therefore, $\{r = \text{Re}(\mu_i), m = \text{Im}(\mu_i), i = 1, 2\}$ have been assumed to be random with $\mathbf{r} \sim \text{Beta}(\alpha_3, \beta_3, c_{l_3}, c_{u_3})$ and $\mathbf{m} \sim \text{Beta}(\alpha_4, \beta_4, c_{l_4}, c_{u_4})$. The lower and upper bounds of the distribution parameters $(\{\alpha_j\}_{j=3}^4, \{\beta_j\}_{j=3}^4, \{c_{l_j}\}_{j=3}^4, \{c_{u_j}\}_{j=3}^4)$ are presented in Table 2.3.

In this example, the random variables $\boldsymbol{\rho}$, $\mathbf{E}$, $\mathbf{r}$, and $\mathbf{m}$ are assumed to be statistically independent. Introducing a random vector $\mathbf{q} = [\boldsymbol{\rho}, \mathbf{E}, \mathbf{r}, \mathbf{m}]^T$, we have $p_{\mathbf{q}}(\mathbf{q}) = p_{\boldsymbol{\rho}}(\boldsymbol{\rho})p_{\mathbf{E}}(\mathbf{E})p_{\mathbf{r}}(\mathbf{r})p_{\mathbf{m}}(\mathbf{m})$. Note that the set of design variables for this example is given by $\{\theta_k\}_{k=1}^{16} = \left\{ \{\alpha_j\}_{j=1}^4, \{\beta_j\}_{j=1}^4, \{c_{l_j}\}_{j=1}^4, \{c_{u_j}\}_{j=1}^4 \right\}$. For design, the maximum von Mises stress, denoted by $\sigma_{VM_{max}}$, is considered as the only response quantity y_1 of interest. Failure occurs when y_1 exceeds $\sigma_{yield} = 250 \text{ MPa}$ where σ_{yield} is the yield stress of steel. For this example, the domain of $\mathbf{q}$ obtained using the lower bound values of $\{c_{l_j}\}_{j=1}^4$ and the upper bound values of $\{c_{u_j}\}_{j=1}^4$ is explored using $N_{ex} = 50000$ samples and three disjointed parts of failure domain are detected using Algorithm 1. For design of stochastic closed-loop system using Eq. (2.26), a resilience level associated with $P_{y_1} = 0.9999$ is selected. The coefficient a is chosen as 0.5 to provide same weight to both the terms. For this example, the values of the normalization factors g_{target} and $\mathcal{M}_{avg}$ are assumed to be 8.3107×10^{13} and 42829, respectively. The optimization problem is solved in MATLAB [36] using genetic algorithm (GA) with 0.8 crossover fraction, 0.01 mutation rate, 1 elite count, 25 initial population, and tournament selection. Note that the probability of failure for the optimal closed-loop system $P_{f_{y_1}}^{opt}$ obtained by solving Eq. (2.26) is 0.0000 as compared to the probability of failure for the initial closed-loop system $P_{f_{y_1}}^{ini} = 0.0247$. For performance comparison, the distributions of $\sigma_{VM_{max}}$ of the closed-loop systems (in the form of normalized histograms obtained from 10000 samples) before and after optimization are presented in Fig. 2.6a. It can be observed from this figure that

Table 2.3 Lower and upper bounds of distribution parameters associated with pdfs of ρ , $\mathbf{E}$, $\mathbf{r}$, and $\mathbf{m}$ (adopted from [23])

Parameter	ρ			$\mathbf{E}$					$\mathbf{r}$				$\mathbf{m}$			
Distribution parameter	α_1	β_1	c_{l1} (kg/m ³)	c_{u1} (kg/m ³)	α_2	β_2	c_{l2} (GPa)	c_{u2} (GPa)	α_3	β_3	c_{l3}	c_{u3}	α_4	β_4	c_{l4} (rad/s)	c_{u4} (rad/s)
Lower bound	0	0	7000	8280	0	0	180	228	0	0	-1.00	-0.52	0	0	16.0	28.8
Upper bound	∞	∞	7320	8600	∞	∞	192	240	∞	∞	-0.88	-0.40	∞	∞	19.2	32.0

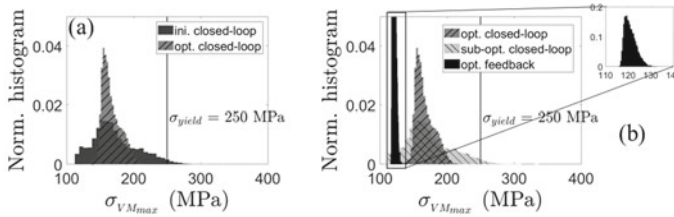


Fig. 2.6 Normalized histograms of **a** initial and optimal closed-loop systems and **b** optimal, sub-optimal closed-loop systems, and closed-loop system with optimal feedback (adopted from [23])

Table 2.4 Estimate of various statistics of σ_{VMmax} for various stochastic systems obtained through solving various optimization problems (adopted from [23])

Statistics	σ_{VMmax} for closed-loop systems			
	Initial	Optimal	Suboptimal	Optimal feedback
Mean	172.83	166.52	162.81	121.03
Standard deviation	34.62	14.03	29.32	2.59
Coeff. of variation	0.2003	0.0843	0.1801	0.0214

the optimal closed-loop systems are able to attain the required level of safety in the presence of uncertainties, whereas the initial closed-loop systems cannot. Also, it can be noted from Table 2.4 that the standard deviation of σ_{VMmax} for the optimal closed-loop system is quite less and hence good from design viewpoint.

A concern that might arise during the fabrication/construction of structural members is how to maintain the optimal probability distributions of ρ and $\mathbf{E}$ obtained by solving Eq. (2.26) as the state-of-the-art manufacturing process and/or construction technology are limited in their capabilities to produce material and geometric parameters with target distributions. Also, maintaining the optimal probability distributions of $\mathbf{r}$ and $\mathbf{m}$ might be difficult. To understand how the designed stochastic closed-loop system will behave under such circumstances, $\mathbf{q}_j \sim U(c_{l_j}^{opt}, c_{u_j}^{opt})$, $\forall j$ have been assumed (i.e., $\rho \sim U(c_{l_1}^{opt}, c_{u_1}^{opt})$, $\mathbf{E} \sim U(c_{l_2}^{opt}, c_{u_2}^{opt})$, $\mathbf{r} \sim U(c_{l_3}^{opt}, c_{u_3}^{opt})$, and $\mathbf{m} \sim U(c_{l_4}^{opt}, c_{u_4}^{opt})$), and another 10000 samples of $\mathbf{q}$ are generated. Note that the terms $c_{l_j}^{opt}$ and $c_{u_j}^{opt}$ denote the optimal lower and upper bounds of the support of the random variable $\mathbf{q}_j$, respectively. The uniform distribution has been chosen here as it has maximum entropy and can account for the maximum variability within these bounds. A comparison of the normalized histograms of the optimal and the new sub-optimal closed-loop systems indicates that the mean of σ_{VMmax} for both the cases is almost same while the standard deviation of σ_{VMmax} for the suboptimal case is higher compared to the optimal counterpart (see Fig. 2.6b). The probability of failure for the suboptimal case $P_{f_{s1}}^{subopt}$ turns out to be 0.0104 which exceeds the user-prescribed

value. The suboptimal case, nevertheless, still represents a significant improvement over the initial closed-loop system in restricting the response (see Fig. 2.6b). This is practically very appealing since the proposed methodology can be confidently adapted in the design of active controlled system to account for the effects of uncertainties.

Since the state-of-the-art manufacturing process is currently limited in their capability to produce material and geometric parameters, and structural complexities with target optimal pdfs, the optimization problem given in Eq. (2.26) is solved again with $a = 1$ to exclude the contribution from $\mathcal{M}(\mathbf{q})$. This approach will provide the optimal distribution of feedback matrices only and, hence, will allow the current manufacturing technologies to embrace the design methodology presented in Sect. 2.3.1. The results for the $a = 1$ case are presented in Fig. 2.6b and Table 2.4. It is observed that the probability of failure for both $a = 1$ and $a = 0.5$ cases is 0.0000; however, the mean and standard deviation of $\sigma_{VM_{max}}$ for $a = 1$ case are much lesser compared to that of $a = 0.5$ case.

2.4 Conclusions

1. A brief review with emphasis on computational advantages and/or drawbacks of the state-space based LQR and H_∞ techniques and the physical-space based MNQPEVA technique is presented in Sect. 2.2. A numerical study of the three techniques indicates that the MNQPEVA technique is computationally efficient and require less actuator capacity (in turn, economic) as compared to the LQR and H_∞ techniques for design of optimal closed-loop systems (see Sect. 2.2.4).
2. A methodology for economic and resilient design of active controlled stochastic structural systems has been presented in Sect. 2.3.1. The methodology is based on a nested optimization scheme termed as stochastic MNQPEVA-MMCLS problem. In this nested optimization scheme, the outer optimization problem minimizes the mean of a combination of norm of feedback matrices and mass of the stochastic closed-loop system. The inner optimization problem is the MNQPEVA problem. The output of the stochastic MNQPEVA-MMCLS problem is a set of optimal parameters of the probability density functions of random material properties, geometric features, control input, etc. Following the methodology, a design example of control system for a stochastic 2D frame of a 4-story steel building is presented in Sect. 2.3.3.
3. The difficulty in numerical implementation of the methodology lies in accurate yet efficient estimation of low probability of failure. Furthermore,

the numerical implementation becomes significantly challenging since this probability of failure is typically associated with disjointed and complicated shaped failure domain. An algorithm based on hierarchical clustering technique and importance sampling scheme is presented in Sect. 2.3.2 that can mitigate the computational difficulty associated with the estimation of low probability of failure over a disjointed and complicated domain.

4. The output of the proposed methodology is a set of optimal parameters of the probability density functions of the controllable and manufacturable closed-system system properties. This is proposed with a vision that it is likely to create a new research direction in the field of manufacturing. This new direction will strive to enable the future manufacturing/fabrication technologies to produce/build structural materials, components, and control units following the target optimal distributions. The current manufacturing/fabrication techniques are, however, limited in their capacity to produce or build such optimal material, structural, and control systems. To address this issue, two alternative solutions are recommended in Sect. 2.3: (i) suboptimal closed-loop systems and (ii) closed-loop systems with optimal feedback matrices. From design viewpoint, the second way out is better than the first one as it ensures the required resilience level while leading to economic design (in terms of the energy required for the control purposes).

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