Chapter 2
The Physical Linked-Measure Works as Vortex with Linking to Turbulence

A physical linked-measure is mathematically consisted of a complex scalar, a complex vector and a bi-vector and is geometrically equivalent to a vortex. When the complex scalar means mass, the complex vector implies directed momentum and the bi-vector rotates angular momentum, with using the least action principle to the linked-measure, yielding energy-mass-momentum-angular momentum joint conservation. Hamilton equation and Lagrange equation are kept as the core of physics, leading fluid dynamics of energy way. As any periodic function can be expressed as a Fourier series, energy spectrum is suggested to be an analytical method. Combining the vortex dynamics with relativity and thermodynamics, Bekenstein-Hawking entropy and "no-hair theorem" of black hole are naturally derived. Applying to wingtip vortices, with adding wavelets, simplified ideal turbulence is described.

2.1 Introduction

During its glorious history, physics had contributed much knowledge of the human understanding to the universe. However, there are still some problems perplexing scientists, such as turbulence, which has been a difficult issue for more than a hundred years. It is well known that Navier-Stokes equations master the fluids including turbulence (Lesieur 1997; Ecke 2005) and the incompressible Navier-Stokes equations with conservative external field are the fundamental equations of fluid mechanics (Landau and Lifshits 1987; Davidson 2015). However, the Navier-Stokes equations are also kept as math-physical puzzle, while the equations act on the flow velocity in the case of incompressible homogeneous flows. Although the equations seem useful because they describe the physics of many models across weather change, ocean currents, water flow and air flow around the world, the solving Navier-Stokes equations still meet some difficult problems, whether in their full and simplified forms. There is an outstanding open problem in mathematics: how to determine an initial condition of the velocity field. There exists, in some senses, a unique solution of the Navier-Stokes equations starting with that initial condition and valid for all later

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times (Rosa 2006). In the studies of turbulence, from a mathematical perspective, it is fundamental to develop a rigorous background upon which to study the physical quantities of a turbulent flow, as the mathematical theory is related to the deterministic nature of chaotic systems assumed in dynamical system theory and is believed to hold in turbulence.

Meanwhile, vortex (Saffman 1992; Nitsche 2006) is an interesting path to approach turbulence (Chow et al. 1997), since wingtip vortices could become wingtip turbulence and cause damages in practice. With the wavelet applications, the vortex and turbulence may be linked (Farge 1992; Farge and Schneider 2006).

Since physics focuses on the fundamental mechanism of the nature and the universe, physical theories have to explain all the realities of the world (Penrose 2004). In order to simplify the problems and find solutions, here I introduce the physical linked-measure for characterizing natural vortex with the aim of leading to a solution of ideal turbulence theoretically, using the mathematical multi-vector method (Hestenes 2003; Doran and Lasenby 2003; Lasenby et al. 2004).

2.2 Methodology

Using multi-vector M_k ($k = 0, 1, 2, 3, 4$) as the total measure, where M_k is a multi-vector of grade k. $k = 0$ corresponds to scalar, $k = 1$ to vector, $k = 2$ to bi-vector, $k = 3$ to pseudo-vector and $k = 4$ to pseudo-scalar and the Clifford bases of four-dimensional space-time are generated by four orthonormal vectors $\{\gamma^\mu, \mu = 0, 1, 2, 3\}$ and spanned by 1 (1 scalar at grade 0), $\{\gamma^\mu\}$ (4 vectors at grade 1), $\{\sigma^k, \mathrm{i}\sigma^k\}$ (6 bi-vectors at grade 2), $\{\mathrm{i}\gamma^\mu\}$ (4 pseudo-vectors at grade 3) and i (1 pseudo-scalar at grade 4) orderly. Now we write the physical linked-measure as

$$\begin{aligned} M &= M_0 + M_1 + M_2 + M_3 + M_4 = \varphi + V + B + \mathrm{i}U + \mathrm{i}\theta = \psi + A + B \\ &= (\psi, A, B) \end{aligned} \tag{2.1}$$

where $\psi = \varphi + \mathrm{i}\theta$ constructs a complex scalar function (massive function), while $A = V + \mathrm{i}U$ forms a complex vector function (potential function) and $B = (1/2)B_{\mu\nu}\gamma^\mu \wedge \gamma^\nu$ as a unique bi-vector.

A physical linked-measure (Ye 2015) mathematically consists of a complex scalar, a complex vector and a bi-vector, which contain rich structural information. A linked-measure can be geometrically represented by a vortex, as shown in Fig. 2.1 (here is right-rotated system. Similarly, left-rotated system can be defined), where one scalar ψ, one vector A and one bi-vector B construct just one vortex.

Each M has its space-time conjugation, denoted by $\overline{M}$, as follows (revised conjugation is $\widetilde{M} = (\psi, \overline{A}, -B)$)

$$\overline{M} = -\mathrm{i}M\mathrm{i} = M_0 - M_1 + M_2 - M_3 + M_4 = \psi - A + B = (\psi, -A, B) \tag{2.2}$$

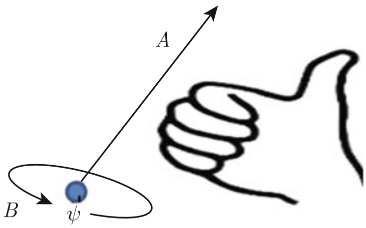

Fig. 2.1 Geometric representation of a linked-measure as vortex (*right-rotated* system)

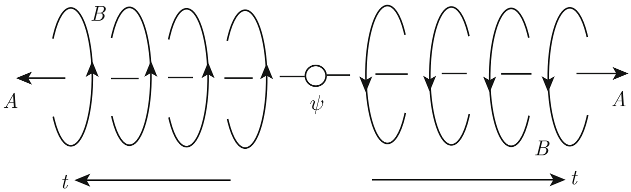

Fig. 2.2 The linked-measure of $M\overline{M}$

When M and $\overline{M}$ act to be $M\overline{M}$, leading to define $M^2 = |M\overline{M}|$, in which the two opposite directions offset so that scalar ψ and bi-vector B become main measure of M^2, as shown in Fig. 2.2.

In Fig. 2.2, if the right side denotes M and left side means $\overline{M}$, M and $\overline{M}$ construct right-rotated system, while M and $\overline{M}$ construct left-rotated system when A or B indicates inverse direction in M and $\overline{M}$. When time is put into the linked-measure, $M(s, t)$ is a fluid-type measure and a physical linked-measure $M(s, t)$ is related to space-time distribution of a complex scalar, a complex vector and a bi-vector, which is just suitable to apply into fluid dynamics.

The differential operators of one order derivatives can be introduced and defined as (Greek sub-indices μ, ν denote 1, 2, 3, 4)

$$\partial_\mu = \frac{\partial}{\partial x_\mu}; \quad \nabla = \gamma^\mu \partial_\mu \tag{2.3}$$

Then we keep the concepts of energy and momentum and define linked-energy E and linked-momentum p_μ with linking Hamilton function H and Lagrangian function L as follows.

$$H = \frac{\partial L}{\partial \dot{x}_\mu} \cdot x_\mu - L = p_\mu x_\mu - L = E(s, t) = \int_s e(t)\mathrm{d}t \tag{2.4}$$

$$L = p_\mu x_\mu - H; \quad p_\mu = \frac{\partial L}{\partial x_\mu} \tag{2.5}$$

where e(t) means density function of linked-measure in space. Now the energy-mass conservation is extended to energy-mass-momentum-angular momentum joint conservation.

So we obtain math-physical equations following Hamilton principle (Latin subindices i, j denote 1, 2, 3)

$$\frac{\partial H}{\partial p_i} = \frac{\mathrm{d}s_i}{\mathrm{d}t}; \quad \frac{\partial H}{\partial s_i} = -\frac{\mathrm{d}p_i}{\mathrm{d}t} \tag{2.6}$$

$$\delta \int_s L\mathrm{d}t = 0 \tag{2.7}$$

The Eq. (2.6) is the differential form and Eq. (2.7) is the integral form of the linked-field equations, which fit analytical tradition of physics.

Corresponding to the vorticity ω of a flow field with velocity distribution v, $\omega = \nabla \times v$, the Helmholtz vortex laws ($\nabla \cdot \omega = 0$) can be simplified to approach as

$$\partial_\mu M \to 0 \tag{2.8}$$

Differentiating from Navier-Stokes equations, where flow velocity is mainly concerned, the linked-measure focuses on energy, leading directly to fluid dynamics of energy way, while it also bypasses the difficulties of solving the Navier-Stokes equations.

Recalling $\psi = \varphi + \mathrm{i}\theta$ and $A = V + \mathrm{i}U$, both ψ and A are complex functions, which can be rewritten as

$$\psi = \varphi + \mathrm{i}\theta = r\mathrm{e}^{\mathrm{i}\alpha} = r(\cos\alpha + \mathrm{i}\sin\alpha) \tag{2.9}$$

$$A = V + \mathrm{i}U = R\mathrm{e}^{\mathrm{i}\beta} = R(\cos\beta + \mathrm{i}\sin\beta) \tag{2.10}$$

where (r, α) and (R, β) are polar coordinates.

Let's mention the complex potential A, which satisfies the Cauchy-Riemann equations

$$\frac{\partial V}{\partial x} = \frac{\mathrm{d}U}{\mathrm{d}y} = v_1 \tag{2.11}$$

$$\frac{\partial V}{\partial y} = -\frac{\mathrm{d}U}{\mathrm{d}x} = v_2 \tag{2.12}$$

where v_1 and v_2 can be understood as velocities, so that A is just the velocity potential.

The method above also supplies a generalized methodology of mathematical physics (Ye 2009).

In general, any periodic function g with a period of $2l$, i.e., $g(x) = g(x + 2l)$, can be expressed as a Fourier series as

$$g(x) = \frac{1}{2}a_0 + \sum_{n=0}^{\infty}(a_n \cos k_n x + b_n \sin k_n x) \tag{2.13}$$

where x is a spatial coordinate and $k_n = n\,\pi\,/l$ has been called the wavenumber. The Fourier coefficients are given by

$$a_n = \frac{1}{l}\int_{-l}^{l} g(x)\cos k_n x \mathrm{d}x \tag{2.14}$$

$$b_n = \frac{1}{l}\int_{-l}^{l} g(x)\sin k_n x \mathrm{d}x \tag{2.15}$$

and Parseval identity holds

$$\int_{-l}^{l} g^2(x)\mathrm{d}x = \frac{l}{2}a_0^2 + l\sum_{n=0}^{\infty}(a_n^2 + b_n^2) \tag{2.16}$$

Then the total energy can be obtained by integrating over the whole wavenumber space

$$\int_0^{\infty} E(k)\mathrm{d}k = l\sum_n g^2(k_n) \tag{2.17}$$

It is shown that $E(k)$ characterizes the energy, so that it is called as the energy spectrum, which provides the methodology of energy spectrum analysis.

2.3 Vortex Dynamics and Black Hole

Since a vortex can be geometrically represented by a linked-measure, the linked-measure matches following dynamic equation via Hamilton principle

$$\frac{\partial H}{\partial s_i} = \partial_\mu M = -\frac{\partial p_i}{\partial t} \tag{2.18}$$

Considering that Hamiltonian energy $H = E = kT$ in thermodynamics, where k is Boltzmann constant and T denotes temperature, it yields

$$\frac{\partial E}{\partial s_i} = k\frac{\partial T}{\partial s_i} = -\frac{\partial p_i}{\partial t} \tag{2.19}$$

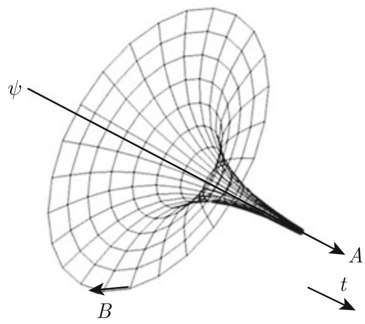

Fig. 2.3 *Black* hole as a vortex (*left-rotated* system)

Replacing flow-velocity way characterized by Navier-Stokes equations, Eqs. (2.18) and (2.19) provide the fluid dynamics of energy way, which could introduce another effective analysis for vortex movement, such as black hole, as shown in Fig. 2.3.

Visually, the process of a vortex rotates into the black hole can be simply viewed as rotated energy measure, in which B approaches to 0 but the total energy measure M keeps conservation when A and mass concentrate to the center of the black hole. Consequently, the center of the black hole forms a strong energy field, absorbing everything. However, all observed black holes in astronomy belong to universal vortices, not time-space singular points.

The theory of general relativity predicts that a sufficiently compact mass can deform space-time to form a black hole and the quantum field theory in curved space-time predicts that event horizons emit Hawking radiation, with the same spectrum as a black body of a temperature inversely proportional to its mass. The simplest static black holes have mass but neither electric charge nor angular momentum, which just fits the linked-measure with $B \to 0$.

However, following the no-hair theorem, a black hole achieves a stable condition after formation and it has only three independent physical properties: mass, charge and angular momentum. Any two black holes that share the same values of those properties, or parameters, are indistinguishable according to classical (non-quantum) mechanics. Those properties are special because they are visible outside a black hole.

Recalling the energy-mass-momentum relation in relativity

$$E^2 = m^2c^4 + p^2c^2 \tag{2.20}$$

we mention that the total energy should include the angular momentum part, so that we should have the form

$$E^2 = \psi^2(m) + A^2(p) + B^2(J) \tag{2.21}$$

According to "the first law of black hole mechanics" (Frolov and Novikov 1997; Ashtekar 2006; Papantonopoulos 2009),

$$dM = \frac{\kappa}{8\pi G} dA + \phi dQ + \omega dJ \tag{2.22}$$

where κ is the surface gravity of the black hole and Q is electric charge.

That is identical to the first law of thermodynamics

$$dE = \theta dS + \phi dQ + \omega dJ \tag{2.23}$$

where θ, ϕ and ω are black hole parameters, ϕ being the electrostatic potential at the horizon, and the thermodynamic entropy S is introduced as

$$dS = \frac{dE}{T} \tag{2.24}$$

Since $M = E$, combining Eqs. (2.22) and (2.23) yields the Bekenstein-Hawking entropy when $\theta = \kappa/(8\,\pi\,G) = c^3k/(4G\hbar)$

$$S = \frac{c^3 k}{4G\hbar} A \tag{2.25}$$

where A is the horizon area of the black hole and constants are the speed of light c, the Boltzmann constant k, Newton's constant G and the reduced Planck constant $\hbar$.

When $\psi(m) = \theta m$, $A(p) = \phi p$ and $B(J) = \omega J$, we derive

$$E^2 = (\theta m)^2 + (\phi p)^2 + (\omega J)^2 \tag{2.26}$$

and

$$dE = \theta dm + \phi dp + \omega dJ \tag{2.27}$$

Considering the potential, $A(p) = \phi p$, if ϕ comes from electromagnetic interaction ϕ-Q, we approach Eq. (2.28) in Planck units

$$Q^2 + \left(\frac{J}{m}\right)^2 + 1 = E^2 = M^2 \tag{2.28}$$

When $m \approx M$, it leads to famous "no-hair theorem"

$$Q^2 + \left(\frac{J}{M}\right)^2 \approx M^2 \tag{2.29}$$

where the black hole mass M equals to its total energy E. Black holes saturating that equation are called extremal. Solutions of Einstein's equations that violate that equation exist, but they do not possess an event horizon. Those solutions have so-called naked singularities that can be observed from the outside, and hence are deemed unphysical. The cosmic censorship hypothesis rules out the formation of such singularities, when they are created through the gravitational collapse of realistic matter.

As there are many vortices in the universe, including atmospheric vortices, water vortices and galaxies vortices, it is believed that the vortex dynamics simplified by linked-measure will be useful in physics.

2.4 Wingtip Vortices

Wingtip vortices are sometimes named contrails or lift-induced vortices because they also occur at points other than at the wingtips. Indeed, vorticity is trailed at any point on the wing where the lift varies span-wise (a fact described and quantified by the lifting-line theory) and it eventually rolls up into large vortices near the wingtip, at the edge of flap devices, or at other abrupt changes in wing planform. Three-dimensional lift and the occurrence of wingtip vortices can be approached with the concept of horseshoe vortex and described accurately with the Lanchester-Prandtl theory (Anderson 2001). In that point of view, the trailing vortex is a continuation of the wing-bound vortex inherent to the lift generation.

When viewed from the tail of the aircraft, looking forward in the direction of flight, there is one wingtip vortex trailing from the left-hand wing and circulating clockwise (M1), and the other trailing from the right-hand wing and circulating anti-clockwise (M2). The result is a region of downwash behind the aircraft, between those two vortices, as shown in Fig. 2.4.

Wingtip vortices are associated with induced drag, the imparting of downwash, and are a fundamental consequence of three-dimensional lift generation. Careful selection of wing geometry (in particular, aspect ratio), as well as of cruise conditions, is one of the most important designing and operational methods to minimize induced drag.

As the aircraft will keep symmetric mass m during the flight, we must have M1=M2. Supposing $E_1(k)$ and $E_2(k)$ as the energy spectra of M1 and M2 respectively, using the method of energy spectrum characterized by Eq. (2.17), the equilibrium condition of those two wingtip vortices becomes

$$\int_0^\infty E_1(k)\mathrm{d}k = \int_0^\infty E_2(k)\mathrm{d}k = l\sum_n g^2(k_n) \tag{2.30}$$

where $g(x)$ is a periodic function that can be expressed by Fourier series Eq. (2.13), which describes a stable scattering and circulating wingtip vortices in the tail of the aircraft.

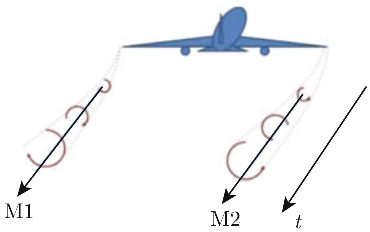

Fig. 2.4 The wingtip double-vortices

If there is no crosswind, those two wingtip vortices do not merge because they are circulating in opposite directions. They dissipate slowly and linger in the atmosphere long after the airplane has passed, approaching to laminar flow at last.

However, if there is crosswind, they are hazards to other aircrafts, known as wake turbulence, i.e., the wingtip vortices form the primary component of wake turbulence.

The wingtip turbulence occurrence is related to some conditions of aerodynamics and thermodynamics. Depending on ambient atmospheric humidity as well as the geometry and wing loading of aircraft, water may condense or freeze in the core of the vortices, making the vortices visible. When a wing generates aerodynamic lift, the air on the top surface has lower pressure relatively to the bottom surface. Air flows below the wing and out around the tip to the top of the wing in a circular fashion, and the pressure on the top of the wing is lower than that on the bottom, causing air to move around the edge of the wing from the bottom surface to the top, so that aerodynamic issues meet thermodynamic solutions.

In the situation of low pressure, vortex cores are regions. As a vortex core begins to form, the water in the air (i.e., in the region that is about to become the core) is in vapor phase, which means that the local temperature is above the local dew point. After the vortex core forms, the pressure inside it has decreased from the ambient value and so the local dew point T_c has dropped from the ambient value. Thus, in and of itself, a drop in pressure would tend to keep water in vapor form, where the initial dew point was already below the ambient air temperature and the formation of the vortex has made the local dew point even lower. However, as the vortex core forms, its pressure (and its dew point) is not the only property that is dropping. In other words, the vortex-core temperature is also dropping and in fact, it can drop by much more than the dew point does. Approximately, the formation of vortex cores is thermodynamically an adiabatic process. To put it simply, there is no exchange of heat. In such a process, the drop in pressure is accompanied by a drop in temperature, according to the equation (Green 1995)

$$\frac{T_f}{T_i} = \left(\frac{P_f}{P_i}\right)^{\frac{\gamma-1}{\gamma}} \tag{2.31}$$

where T_i and P_i are the absolute temperature and pressure at the beginning of the process (here, equal to the ambient air temperature and pressure); T_f and P_f are the absolute temperature and pressure in the vortex core (which is the end result of the process) and the constant γ is $7/5 = 1.4$ for air.

Thus, even though the local dew point inside the vortex cores is even lower than that in the ambient air, the water vapor may nevertheless condense, if the formation of the vortex brings the local temperature below the new local dew point.

Noticing that there are four most common Maxwell relations among temperature T, pressure P, volume V and entropy S as

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V; \quad \left(\frac{\partial T}{\partial V}\right)_P = -\left(\frac{\partial P}{\partial S}\right)_T;$$

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P; \quad \left(\frac{\partial T}{\partial p}\right)_V = \left(\frac{\partial V}{\partial S}\right)_T \tag{2.32}$$

We can also replace P with S in same space

$$\frac{T_f}{T_i} = \left(\frac{S_i}{S_f}\right)^{\frac{\gamma-1}{\gamma}} \tag{2.33}$$

So the temperature, energy and entropy are linked together. That is an issue concerning fluid mechanics and thermodynamics and that is only a simple preliminary exploration. In the case of wingtip vortices leading turbulence, it is important that the local temperature would be lower than the local dew point, so that the water vapor inside the vortices would indeed condense. In that circumstance, the local temperature in vortex cores may drop below the local freezing point, in which case ice particles will form inside the vortex cores.

2.5 Crosswind as Wavelets Leading to Ideal Turbulence

Wingtip turbulence indicates that there is a relation between vortex and turbulence (Pullin and Saffman 1998), within a fuzzy layer mixing vortex and turbulence, which can be simulated by introducing wavelets (Speziale 1998).

A wavelet is a wave-like oscillation with changed amplitude that begins at zero, increases and then, decreases back to zero. As a mathematical tool, wavelets can be used to extract information from many different kinds of data, including audio signals and video images. Sets of wavelets are generally needed to analyze data fully. A set of "complementary" wavelets will decompose data without gaps or overlap so that the decomposition process is mathematically reversible. Thus, sets of complementary wavelets are useful in wavelet based compression/decompression algorithms where it is desirable to recover the original information with minimal loss.

Instead of solving the Navier-Stokes equations over a highly refined mesh, we can use the wavelet decomposition of a low-resolution simulation to determine the loca-

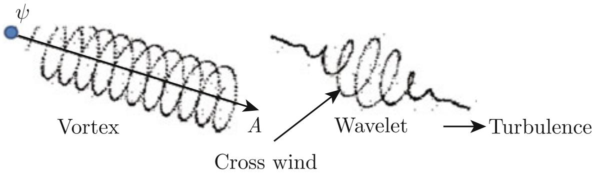

Fig. 2.5 Vortex meets wavelet leading to turbulence

tion and energy characteristics of missing high-frequency components. We then synthesize those missing components using a novel incompressible turbulence function and provide a method to maintain the temporal coherence of the resulting structures.

Visually, the crosswind acts onto bi-vector B, so that B could be changed by wavelets and the energy is kept in ψ and A. However, a part of the energy is scattered and disappeared by crosswind (wavelet), so that the total energy may be dissipated, then the energy spectrum is changed by wavelets, as shown in Fig. 2.5.

If the crosswind takes less energy than vortex itself, the wavelet vorticity $\omega = \nabla \times v$ looks weak, and much smaller than B, which produces stable fluid.

If the crosswind takes more energy as much as vortex itself, the wavelet vorticity $\omega = \nabla \times v$ shows the same as, even stronger than B, which leads to turbulence.

Generally, if the wavelets are so strong that cause the energy change of the vortex ($\nabla \times v \geq B$), the turbulence will occur. If the wavelets are weak ($\nabla \times v \ll B$), the vortex will keep stable dissipation and then merge into laminar flow.

Supposing the vortex has energy density $g(x)$, the results lead to

$$g(x) = \int_{R^n} \tilde{g}(x) f^*(x) \mathrm{d}^n x \tag{2.34}$$

The inner products $g(x)$ and $f^*(x)$ give wavelet coefficients

$$\tilde{g}(x) = \int_{R^n} \hat{g}(k) f^*(k) \mathrm{d}^n k \tag{2.35}$$

where R^n marks the real space.

Now the original energy spectrum $E(k)$ of $g(x)$ is changed by wavelets' one. Supposing $f^*(x)$ has energy spectrum $E^*(k)$. If $E^*(k)$ keeps much weaker corresponding to $E(k)$, the stable vortices can be kept and turbulence will not happen. However, if $E^*(k)$ has near equivalent strength at the same level of $E(k)$, the equilibrium will be destroyed and the turbulence will occur. So, we give the conditions of introducing the ideal turbulence as

$$E^*(k) \sim E(k); \quad \omega = \nabla \times v \succ B \tag{2.36}$$

Equation (2.36) means that wavelet energy spectrum $E^*(k)$ approaches the same level as vortex energy spectrum $E(k)$ and the wavelet vorticity $\omega = \nabla \times v$ shows dominance to vortex angular momentum measure B. In those conditions, we meet an ideal turbulence, which causes changes of the energy spectrum and structure of the vortex or vortices, leading to turbulence. Then the energy (or power) spectrum analysis provides a simplified methodology for studying fluids and turbulence, which is expected to introduce a new idea and develop useful analysis.

2.6 Discussion and Conclusion

In fluid mechanics or physics, there is still no perfect definition of turbulence. However, three main characteristic features can be listed as follows.

(1) Nonlinearity or irregularity. Differentiating from linear laminar flow, turbulence is nonlinear and random flow, which consists of a spectrum of different scales (i.e., irregularity). However, although turbulence seems chaotic, it is deterministic and can be studied via the Navier-Stokes equations or the energy spectra.

(2) Dissipation and diffusivity. Turbulent dissipation means that kinetic energy in the small dissipative vorticities is transformed into thermal energy. The small vorticities receive the kinetic energy from slightly larger vorticities and the slightly larger vorticities receive their energy from even larger vorticities and vice versa. The largest vorticities extract their energy from the mean flow. That process of transferring energy from the largest turbulent scales to the smallest is called the cascade process, in which the diffusivity increases. The increased diffusivity also increases the resistance and heat transfer in internal flows.

(3) Three-dimensional vorticity. Turbulence is also characterized by apparently random and chaotic three-dimensional vorticity, which usually dominates all other phenomena and results in increased energy dissipation and heat transfer and also increases the exchange of momentum in boundary layers.

The Reynolds number (Re) is not mentioned in this chapter. As the fluid dynamics of energy way and the energy spectrum analysis are not directly related to flow velocity, the Re that links to flow velocity is ignored. However, it keeps effective that turbulent flow occurs at high Re, such as the transition to turbulent flow in pipes occurs at $Re \simeq 2{,}300$ and in boundary layers at $Re \simeq 500{,}000$.

When a physical linked-measure is geometrically equivalent to a vortex, the methodology concentrates on the energy and the energy-mass conservation is extended to energy-mass-momentum-angular momentum joint conservation. By using the least action principle to the linked-measure, Hamilton equation and Lagrange equation are kept as the core of physics, leading fluid dynamics of energy way. Meanwhile, the energy (or power) spectrum is suggested to be the main analytical method, which looks useful for probing into the unified mechanism of fluids. Adding wavelets, simplified ideal turbulence can be described, which could introduce a new solution to approach turbulent flow.

Conclusively, the linked-measure as a simplified methodology may benefit for understanding vortex as well as turbulence, with focusing on energy construction and energy spectrum analysis, which could stimulate further considerations and studies.

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