

Preface

Random ordinary differential equations (RODEs) are ordinary differential equations (ODEs) that include a stochastic process in their vector field. They seem to have had a shadow existence to Itô stochastic differential equations (SODEs), but have been around for as long as if not longer and have many important applications. In particular, RODEs play a fundamental role in the theory of random dynamical systems.

In the older engineering and physics literature, a simpler kind of RODE is investigated with the vector field being chosen randomly rather than depending on a stochastic process. Such RODEs are still of interest in the uncertainty quantification community. They are a special case of those considered in this book, but will not be treated separately.

RODEs, unlike SODEs, can be analysed pathwise with deterministic calculus, but require further treatment beyond that of classical ODE theory. Specifically, since the driving process in a RODE has at most Hölder continuous sample paths, the solution sample paths are continuously differentiable, but the sample paths of the derivative are no more than Hölder continuous in time. The resulting vector field after insertion of the driving stochastic process is at most Hölder continuous in time, no matter how smooth the vector field is in its original variables. Thus, the solutions of RODEs do not have sufficient smoothness to have Taylor expansions in the usual sense.

Taylor expansions are a very basic tool in numerical analysis. They allow one to derive one-step numerical schemes for ODEs of arbitrary high order. In practice, such Taylor schemes are rarely implemented, but are used instead as a theoretical comparison for determining the convergence orders of other schemes that have been derived by more heuristic methods. This theory does not apply directly to RODEs or SODEs. Although classical numerical schemes for ODEs can be used pathwise for RODEs, they rarely attain their traditional order.

The situation with RODEs is similar to that for Itô SODEs, the sample paths of which are just Hölder continuous and not even differentiable. For Itô SODEs, stochastic Taylor expansions can be derived using an iterated application of the Itô formula, the stochastic chain rule, in integral form. These stochastic Taylor

expansions were the starting point for the derivation of consistent higher order numerical schemes for SODEs. A similar approach will be used in this book for RODEs.

A major motivation for writing this book is to make more widely known recent results on the derivation of higher order numerical schemes for RODEs. Another is to make RODEs themselves and the closely associated theory of random dynamical systems better known too. A particular motivation of personal interest to us is to show how RODEs are being used in the biological sciences, where non-Gaussian and bounded noise are often more realistic than the conventionally used Itô calculus.

Auburn, USA
Wuhan, China
March 2017

Xiaoying Han
Peter E. Kloeden

Random Ordinary Differential Equations and Their
Numerical Solution

Han, X.; Kloeden, P.E.

2017, XVII, 250 p. 21 illus. in color., Hardcover

ISBN: 978-981-10-6264-3