A Game Theory Approach for Deploying Medical Resources in Emergency Department

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Abstract. Emergency department need a decision support tool to advise the critical medical resources (such as doctors, nurses, or beds) to urgent patients in the different service time requirements. This study proposes a framework for emergency response service that incorporates two game theory models designed to deploy response medical resources when raising three threat advisory levels. First, the interactions between a group of weekly patients and a response agent of emergency department are modeled as a non-cooperative game, after which a threat, vulnerability, and consequence value (i.e., TVC) of each type of patients for emergency event is derived from Nash equilibrium of game. Second, four TVC values of emergency events are utilized for computation of the Shapely value for each type of patient. The deployment of emergency medical resources is carried out based on their expected marginal contribution. Then, the model scheduled daily physicians, nurses, and beds in emergency department. The experimental results show that proposal model is feasible as a method to improve efficiency in emergency department.

Keywords: Emergency response · Nash equilibrium · Shapley value · Medical resources scheduling

1 Introduction

Emergency Department (ED) overcrowding gradually becomes a serious problem which reduces quality of emergency care, increased costs of patients, and decreased physician job satisfaction [6, 18]. However, the more emergency medical resources (such as doctors, nurses, and beds) deploy, the more patients need to care and hence the price to pay is higher. Mass deployments are probably to waste resources. On the contrary, the fewer medical resources deployed, the easier patients to be delayed, and the more difficult it is to respond. The robustness of a completely emergency decision process depends upon a balance between the medical resources requirements of and the urgent of patients. Therefore, there exists in ED a dilemma between overcrowding of visiting patients and the capability of medical resources. The current systems lack

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specific measures for rational decision-making, as well as the capacity to apply mathematical models to capture the interactions between patients and medical resources. The ED administrator should have the tools to measure the strength of overcrowding patients and the resistance capability of the response medical resources. A rating system can be built to assist with decision making by considering the utilities of the moves that the overcrowding patient and the medical resource can take [11].

Game theory tools provide analytical techniques that are already applied in many other research areas, where multiple agents compete and interact with each other within a specific system. In most multiple agent interactions, the overall outcome depends on the choices made by all self-interested agents. The goal is to make choices that optimize the outcome [2]. Game theory also focuses on these adversarial risk analyses may lead to far more effective allocations of limited response resources than current multiplying threat, vulnerability, and consequence (TVC) analyses. Risk analysis and game theory are also support each other and deeply complementary so that they can improve risk assessors to avoid producing potentially irrelevant or misleading TVC risks scores [1].

The proposed model is applied to deploy medical resources for emergency responses in ED. Two game-theoretic models are constructed, representing the two stages needed for economical deployment of the available resources. In the first step, the interactive movements between a group of weekly patients and a response agent (i.e., administrator) of emergency department are modeled and analyzed as a non-cooperative game, after which the TVC value of each type of patients for emergency event is derived from Nash equilibrium of game. This value quantified threat, vulnerability, and consequence of an emergency for visiting patient. In the second step, the interactions of four types of emergency event of patient in a whole administration region are likened to the playing of a cooperative game. Four TVCs of patients for emergency events are utilized to compute each type of emergency's Shapley value. Then, the Shapley value assists in setting priorities for medical resources allocation. Finally, the administrator deploys three types of medical resources for the three shifts working scheduling per day to improve efficiency and enhance patient care in emergency department.

2 Literature Review

The efficiency and cost of emergency systems depend on the performance of deployment and working scheduling of available medical resources. However, constructing optimized work timetables for those resources are NP-hard problem [19, 20]. Luscombe and Kozan [12] proposed a resource allocation model which is heuristic approach to find near-optimal solutions. Their ED model provided the patient appropriate bed and scheduled medical resources with a tabu search. Some simulation-based models analysis the patient requirements and minimize the patient average length of stay so as to produce the appropriate medical resources allocations, such as doctors, nurses, and beds in ED [3, 8]. Although they utilized simulated data to search for the optimal parameters to match the real data, but these optimal approaches could not satisfy the sudden overcrowding patients, particularly when available medical resources are limited in ED. Gupta and Ranganathan [17] surveyed several optimization approaches such as genetic algorithms (GA), the Bayesian search method (BS), as well as random search (RS) and greedy allocation (GS) methodologies. These algorithms cannot be applied to the multi-crisis management problem because they do not provide individual rationality, since some of the members of the population become dominant as the algorithm progresses. However, using game theory, scenarios that optimize multiple competing objectives can be modeled.

Jacobson et al. [7] indicated there is a trade-off between prioritizing urgency jobs and prioritizing higher expected payoff for emergency patient in mass-casualty incidents. An efficient emergency response need to assign the appropriate resources to the urgency of a single patient or a group of patient's requirements. Game theory can model interactions between self-interested agents (e.g., job or resource) and analysis as to which strategies can be designed that will maximize the benefits of an agent in a multiple agent system. Many of the applications of game theory have been to analyze the negotiation and coordination of multiple agents. Game theory can be a useful tool for building future generations of mixed game theory and decision theory [16]. In a non-cooperative game, each player (or self-interested agent) tries to utilize resources at minimum cost and the coordination is not enforced externally but is self-enforcing. All players optimize their decisions which maximize their payoffs in a non-cooperative game. N.E. is a solution concept for a non-cooperative game which identifies a prediction of the game outcome such that every player in the game is satisfied with respect to every other player.

In a cooperative game, the self-interested agents implement their joint actions to form a specification of the coalition. No players can separate from the coalition and take a joint action that makes all of them better off [14]. The Shapley value is a solution concept for cooperative games which computes the power index of an individual for cost allocation [13]. The cooperative game provides a suitable model for the design and analysis of response agent deployment, and it has been shown that the famous Shapley value rule satisfies many nice fairness properties [21]. The Shapely value also identifies a socially fair, good quality allocation for all agents in a multi-agent system (MAS). Here, the individual fairness for each player is optimal and the average fairness of the MAS is high. The social optimality property ensures that each player in the game receives the best utility for himself and for the complete MAS. A power index in the form of the Shapley value is applied to calculate the marginal contribution among agents and achieve mutually agreeable division of cost for MAS deployment.

3 The Proposed Model

This paper employs two solution concepts of game theory to set up a threat level framework for deploying three types of emergency medical resources (e.g., doctors, nurses, and beds) in an ED for working scheduling and dispatch. Two games are constructed, which represent the two stages needed for the economical deployment of emergency medical resources. One is the TVC game, the other is the deployment medical resource game.

A simplified workflow chart describing the principles of optimal medical resource deployment is shown in Fig. 2. First step plays four non-cooperative games for medical, surgical, pediatric, and independent emergency event. In each game, interactions between a group of weekly patients and a response agent (or administrator) of emergency department (ED) are modelled as a two-person, zero-sum game, for each type of emergency event. After considering some of the information needed to measure the efficiency of emergency medical services, such as available resources (e.g., doctors, nurses, and beds), resources requirements, emergency priorities (e.g., five levels of triage categories), mean length of stay per patient (LOS) [4], and the number of patients. This stage simultaneously calculates four payoff matrices which combines the payoffs of each player's strategies. Then, the four payoff matrices respectively obtain expected payoff of a group of weekly patients and four TVCs of emergency events derived from N.E (shown in Fig. 1).

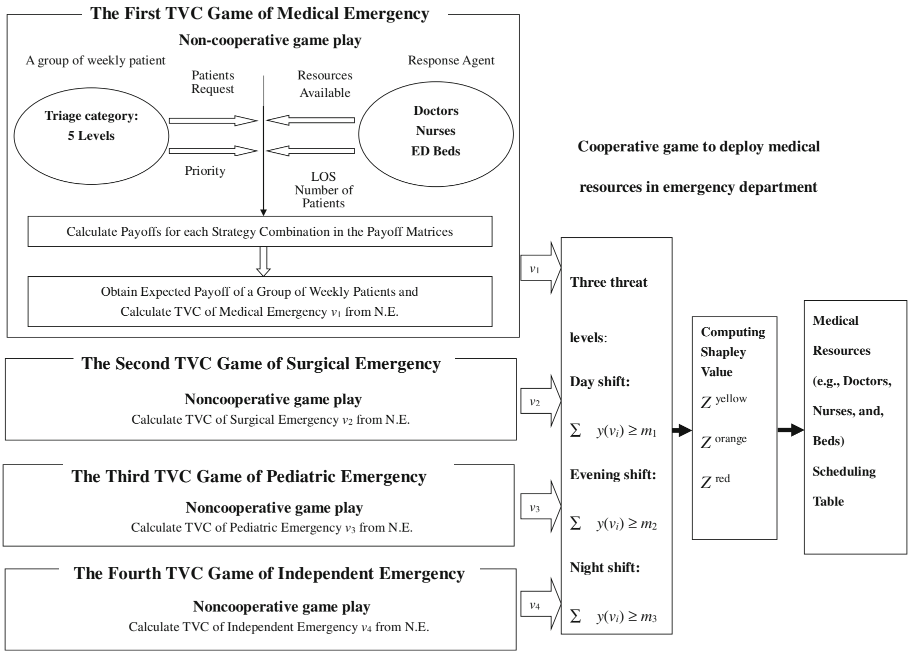

Fig. 1. Optimal resources scheduling workflow of two game models for hospital emergency department

In the second step, four TVCs of emergency events are applied to compute three thresholds so as to confirm three threat levels (i.e., day shift, evening shift, and night shift) in one day. A cooperative game model is applied to establish a rational emergency medical resources allocation by using the Shapley values as the indices of power. Given the three threat levels (i.e., day shift, evening shift, and night shift) the proposed model produces the specific Shapley value vector. Finally, a daily shift schedule of three type of medical resources (i.e., doctors, nurses, and beds) in ED is derived from the total number of each type of medical resources multiple by the Shapley value vector.

3.1 Model Assumptions

Certain assumptions must be made when treating various patients and emergency medical resources response management scenario as a game theory framework. The assumptions are listed below.

1. When patients arriving at the hospital emergency department, a triage nurse will assess and categorized them into 5 levels of urgency — Category 1 (critical), 2 (emergency), 3 (urgent), 4 (standard) and 5 (non-urgent) [8]. According to their categories, patients follow specific procedures and are directed to different treatment areas. ED nurse gives patient directing initial medical care and services required. And then doctors contact with the patient, diagnosis, and examination should be followed by decisions, often required to be made as soon as possible in serious cases. Four types of emergencies (i.e., medical, surgical, pediatric, and independent emergency) may occur simultaneously in different patients in a time-overlapped manner, and resources requests and emergency priorities will be received by the response agent of ED (shown in Table 1).

Table 1. Emergency priorities, types, and the daily resource requests

Triaged category	Priority (1–5)	Doctors				Nurses	Beds
		Internist	Surgeon	Pediatricians	Others		
Level 1	5	13	2	3	1	19	19
Level 2	4	52	11	17	8	87	87
Level 3	3	32	7	10	7	55	55
Level 4	2	2	0	1	1	4	4
Level 5	1	1	0	0	0	1	1
Total		100	20	30	16	166	166

2. This study assumes a group of weekly patients is player 1 and a response agent of ED is player 2. Hence, two game players can utilize a static single step. The first model is a normal form game for modelling the interactions between two players. Player 2 has a different number of resources available, such as doctors, nurses, and beds for four types of patients (shown in Table 2). These four types of patients include medical, surgical, pediatric, and other independent emergencies, which are provided a good standard of treatment and care by three types of medical resources. In TVC game there is complete information regarding the number of patient visits per week.

Table 2. Doctors, nurses and beds available for 4 types of patients

Type of patients	Medical resources		
	Doctors	Nurses	Beds
Medical emergency	20	60	42
Surgical emergency	6	18	42
Pediatric emergency	4	12	42
Independent emergency	28	18	42
Total	58	108	168

3. The weekly mean length of stay (LOS) [4] per triage category patient indicates the time from each type of patient arrival to time of discharge from the ED, calculated as a weekly average (shown in Table 3). Longer length of stay may reflect inefficient use of emergency resources, and length of stay is a key factor of patient satisfaction.

3.2 TVC Game

Four TVC games are applied in the first step of the proposed model, which are designed to obtain the TVC of the i^{th} type of emergency event (e.g., medical, surgical, pediatric, or other independent emergency), i = 1, 2, 3, 4. In each non-cooperative game, this study assumes that a group of weekly patients is player 1 and a response agent of ED is player 2. The two players' behaviors are captured with a two-person game theory model. Player 1 generates five levels of urgency patients. Player 2 is a stewardship [9] who is responsible for providing excellent care for these patients within the constraints of medical resources available. This study assumes a group of weekly patients and a response agent of ED to be rational players, a set of noncooperative players $I = \{I_1, I_2\}$, where I_1 is a group of weekly patients and I_2 is a response agent of ED. The parameters for determining the threat, vulnerability, and consequence (TVC) measures are defined in the following paragraphs.

A group of weekly patients is player 1. Five categories of urgency patients could happen simultaneously as a result of the player 1's actions. S_1 denotes the set of strategies available to player 1: $S_1 = \{u_1, u_2, u_3, u_4, u_5\}$ = {category 1 (critical), 2 (emergency), 3 (urgent), 4 (standard) and 5 (non-urgent)}. ED medical resources are available for categories of urgency patients. W denotes the set of resource requirements for five categories of urgency patients in an ED. $W = \{w_1, w_2, \ldots, w_n\}$. The variable $w_{j,k}$ denotes the number of resources required by a u_j triaged category from resource d_k. Moreover, the priority for each category of urgency patient is related to the resource requirements. Typically, in a priority-based system, a high priority patient needs more resources, and a lower priority patient needs fewer resources. Each patient is assigned a priority P on a scale of 1 to 5 indicating the severity of the urgency patients, which is used as a weight in the payoff function to facilitate the calculation of gains. p_j denotes the priority of the j^{th} patient p_j = 1, 2, 3, 4, 5 where 1 is lowest and 5 is highest level (shown in Table 1).

Response agent is player 2, which in turn have varied resources to prepare for five triage category patients who will generate four types of emergency events. In this game, player 2 is responsible for providing ED patients with three types of medical resources (i.e., three strategies). S_2 denotes the sets of player 2 strategies: $S_2 = \{d_1, d_2, d_3\}$ = {doctors, nurses, and beds}. O denotes the set of medical resources available in ED. $O = \{o_1, o_2, o_3\}$. o_k denotes the number of resources available at resource d_k; $k \in 1, 2, 3$. Length of emergency department stay (LOS) is a measure of medical care efficiency, and it is often a mainly source of patient complaint and frequently the target of interventions. $t_{i,j}$ denotes the weekly mean length of stay of j^{th} triage category patients in the i^{th} type of emergency events in the ED. Response agent should offer more medical resources to serve the patients when the number of patients in need increases and the average LOS of patient takes long time.

Table 3. Weekly mean length of stay per triage category patient with four types of emergencies

(Minutes) Triage categories	Medical emergency	Surgical emergency	Pediatric emergency	Independent emergency (the others)
Level 1	185	188	165	105
Level 2	163	145	115	99
Level 3	162	123	105	89
Level 4	125	117	120	82
Level 5	104	99	115	75
Total	739	672	620	450

Two players will simultaneously make their strategic decisions. A 5 × 3 payoff matrix for the each TVC game is created based on two players' strategies and interactions as seen in Table 4. The payoff to player 1 for choosing a particular strategy when player 2 makes his selection can be represented as a gain by player 1 (+) or a loss for player 2 (–). In this model, a summation of the losses of player 2 is depicted, and player 1 tries to maximize this loss. Player 2 tries to minimize the losses. Player 1 gains a profit from player 2's effort of resource responses. Player 2 pays as a result of player 1's multi-emergency events and various triage patients. This game assumes that each player aims to achieve as high a payoff for him or her as possible.

Table 4. Payoff matrix for the TVC game in i^{th} type of emergency event (e.g., medical emergency)

Player 1 (patients)	Player 2 (response agent)		
	d_1 (doctor)	d_2 (nurse)	d_3 (bed)
u_1 (level 1)	$\left(\frac{w_{i,1}}{o_{all,1}-w_{1,1}}\right)t_{i,1}p_1$	$\left(\frac{w_{i,2}}{o_{all,2}-w_{1,2}}\right)t_{i,1}p_1$	$\left(\frac{w_{i,3}}{o_{all,3}-w_{1,3}}\right)t_{i,1}p_1$
u_2 (level 2)	$\left(\frac{w_{i,1}}{o_{all,1}-w_{2,1}}\right)t_{i,2}p_2$	$\left(\frac{w_{i,2}}{o_{all,2}-w_{2,2}}\right)t_{i,2}p_2$	$\left(\frac{w_{i,3}}{o_{all,3}-w_{2,3}}\right)t_{i,2}p_2$
u_3 (level 3)	$\left(\frac{w_{i,1}}{o_{all,1}-w_{3,1}}\right)t_{i,3}p_3$	$\left(\frac{w_{i,2}}{o_{all,2}-w_{3,2}}\right)t_{i,3}p_3$	$\left(\frac{w_{i,3}}{o_{all,3}-w_{3,3}}\right)t_{i,3}p_3$
u_4 (level 4)	$\left(\frac{w_{i,1}}{o_{all,1}-w_{4,1}}\right)t_{i,4}p_4$	$\left(\frac{w_{i,2}}{o_{all,2}-w_{4,2}}\right)t_{i,4}p_4$	$\left(\frac{w_{i,3}}{o_{all,3}-w_{4,3}}\right)t_{i,4}p_4$
u_5 (level 5)	$\left(\frac{w_{i,1}}{o_{all,1}-w_{5,1}}\right)t_{i,5}p_5$	$\left(\frac{w_{i,2}}{o_{all,2}-w_{5,2}}\right)t_{i,5}p_5$	$\left(\frac{w_{i,3}}{o_{all,3}-w_{5,3}}\right)t_{i,5}p_5$

The payoff for the j^{th} strategy for player 1 when player 2 chooses the k^{th} strategy to the response can be formulated as

$$\pi_1 = \sum\nolimits_{j=1}^{5} \sum\nolimits_{k=1}^{3} \left(\frac{w_{i,\text{k}}}{o_{all,k} - w_{j,\text{k}}}\right) t_{i,\text{j}} \times p_j, \quad i \in 1,2,3,4 \tag{1}$$

where $o_{all,k}$ is the total number of medical resources available of type k in the whole ED (such as total number of nurses who can care for the patient). The proposed model assumes that the non-cooperative game is a zero-sum game; therefore, the payoff function of player 2 is given by

$$\pi_2 = -\sum_{j=1}^{5}\sum_{k=1}^{3}\left(\frac{w_{i,k}}{o_{all,k} - w_{j,k}}\right)t_{i,j} \times p_j, \quad i \in 1,2,3,4 \tag{2}$$

Player 1 and 2's expected payoff is computed when they use mixed strategies r and q, respectively. If the game has no pure strategy Nash Equilibrium (N.E.), a mixed N.E. pair (r^*, q^*) exist in the game, which also is an optimal strategy [5, 10]. The mixed N.E. for the probability vector is $r^* = \{r^*(u_1), r^*(u_2), r^*(u_3), r^*(u_4), r^*(u_5)\}$ with actions $\{u_1, u_2, u_3, u_4, u_5\}$ by player 1 and the vector $q^* = \{q^*(d_1), q^*(d_2), q^*(d_3)\}$ with actions $\{d_1, d_2, d_3\}$ by player 2. The player 1's expected payoff for a N.E. (pure or mixed strategy) is defined as its TVC of the i^{th} type of emergency event. This study defines the v_i as a TVC of the i^{th} type of emergency event in a week, given by

$$v_i = \sum_{j=1}^{5}\sum_{k=1}^{3} r^*\left(u_j\right)q^*(d_k)\pi_1\left(u_j^*, d_k^*\right), \quad u_j^*, d_k^* \in N.E. \tag{3}$$

Therefore, v_i is derived from the expected payoff of two players' optimal strategies which represents the TVC of the i^{th} type of emergency event in the first game model. The next proposed model applies the value v_i to compute the Shapley value of each type of emergency event within the cooperative game.

3.3 The Deployment Game

This study likens the interaction of four types of emergency events (e.g., medical, surgical, pediatric, or other independent emergency) in a whole hospital ED administration to the playing of a cooperative game. A fair and efficient method is needed to decide the number and priority of medical resources to be deployed when the emergency threat level is raised. The Shapley value is a power index for cost allocation. The cooperative game provides a suitable model for the design and analysis of resources deployment, and it has been shown that the famous Shapley value rule satisfies many fairness properties. Thus, in the proposed model, the Shapley value is applied to create an optimal cost allocation for the deployment of ED resources for working scheduling.

This research defines y: $V \rightarrow R+$ as a one-to-one function by assigning a positive real number to each element of v and $y(0) = 0$, $V = \{v_1, v_2, v_3\}$. The four emergency events for medical resources deployment is based on the concept of the threat level. Three emergency threat levels in one day (e.g., day shift, evening shift, and night shift) are L= $\{l_1, l_2, l_3\}$, where $0 < m_1 < m_2 < m_3$ represents the corresponding threshold values. Given the output vector of four TVCs of the emergency events, the threat level L of the HSAS response region is equal to l_f if the sum of the TVCs of emergencies is greater than or equal to m_f, f = 1, 2, or 3.

$$L = \begin{cases} l_1 & if \sum_{i=1}^{N} y(v_i) \geq m_1 \\ l_2 & if \sum_{i=1}^{N} y(v_i) \geq m_2 \\ l_3 & if \sum_{i=1}^{N} y(v_i) \geq m_3 \end{cases} \tag{4}$$

where $m_1 = v_{Mini} + m_{in}, m_2 = m_1 + m_{in}, m_3 = m_2 + m_{in}, m_{in} = \left(\frac{v_{Max} - v_{Mini}}{4}\right)$

Four TVCs of emergency events can be grouped into three threat levels according to the average value of the interval m_{in} of the threshold. This is divided by three threat levels from the maximum TVC v_{Max} to the minimum v_{Min} value.

Four types of emergency events (e.g., medical, surgical, pediatric, or other independent emergency) can be modeled as a 4-person game with $X = \{1, 2, 3, 4\}$, which includes the set of players and each subset $V \subset N$ and where $v_i \neq 0, \forall_i \in V$ is called a coalition. The coalition of X emergency event groups in the m^{th} threshold of the threat level and each subset of the X (coalition) represents the observed threat pattern for different threat levels of L. The aggregate value of the coalition is defined as the sum of the TVCs of the emergency events,$y(C) = \sum_{i \in C} y(v_i)$ and is called a coalition function. Each emergency event coincides with one or another given m thresholds of the threat level. Therefore, the different priorities for four types of medical resources deployment are derived from three threshold values. Based on the emergency threat for each type of resource with respect to others and the effect of the threshold values on three threat levels, the Shapley value represents the relative importance of each emergency. Now let $y(C) = \sum_{i \in C} y(v_i), v_i \in V, C \subset X$ be the value of the coalition C with cardinality c. The Shapley value of the i^{th} element of the emergency vector is defined by:

$$\omega(i) = \sum_{\substack{C \subset X \\ i \in c}}^{n} \frac{(c-1)!(n-c)!}{n!} [y(C) - y(C - \{i\})] \quad (5)$$

$$\Rightarrow \omega(i) = \sum_{\substack{C' \subset X \\ i \in c'}}^{n} \frac{(c-1)!(n-c)!}{n!} \quad (6)$$

Equation (5) can be simplified to Eq. (6) because the term $y(C) - y(C - \{i\})$ will always have a value of 0 or 1, taking the value 1 whenever C' is a winning coalition. If C' is not a winning coalition, the terms $C - \{i\}$ and $y(C)$ are 0 [15]. Hence, the Shapley value is $\omega(i)$, where C' denotes the winning coalitions with $\sum v_i \geq m_f, i \in C'$. The Shapley value of the i^{th} type of emergency event output indicates the relative TVC for the thresholds l_f (i.e., threat levels). Therefore, a Shapley value represents the strength of emergency event which ED resources should give patient medical care and services required in one day.

We applied Shapley values of i^{th} type of emergency event to compute the number of k^{th} type of resources allocation in the threat level l_f. The numbers of resources of type k allocated to the i^{th} emergency event are defined by

$$e_k(i) = \omega(i) \times o_{all,k} \quad (7)$$

where $o_{all,k}$ is the total number of medical resources available of type k in the whole ED (such as total number of nurses who can care for the patient). The allocated numbers of the i^{th} medical resources $e_k(i)$ are derived from the Shapley value of the i^{th} emergency event $\omega(i)$ multiplied by $o_{all,k}$ the total number of resources available of type k. Finally, the response agent of ED can deploy three types of medical resources to improve efficiency and enhance patient care in emergency department.

4 Simulation Experiments

This study hypothesizes that a response agent of ED possesses a different number of resources available for four types of patients. Given the three threat levels (e.g., day shift, evening shift, and night shift), response agent provides medical resources scheduling and rostering per day in ED. The simulation employs Tables 1, 2 and 3, which show information regarding the resource types and the availability, emergency requests, emergency priorities, and length of stay (LOS) per triage category patient.

This experiment first modeled the noncooperative game and generated simulation sets of TVC measures for the four emergency events in a week. The payoff matrix of the zero-sum TVC game for each emergency event is modeled by Eqs. (1)–(2) and Table 4. Then, four TVC values of emergency events are calculated by Eq. (3) (shown in Fig. 2). Second, four TVCs of the emergency events are utilized to compute the Shapley value of each emergency event (shown in Fig. 3). This study assumes three threat levels for the emergency resources response: night shift, evening shift, and day shift. We adopted the Matlab tool to compute the Shapley value of each emergency event based on the three threat levels. Then, a simulation creates three minimum sets of medical resources (i.e., doctors, nurses, and beds) for working shifts per day in ED by Eqs. (4)–(6) (shown in Figs. 4, 5 and 6).

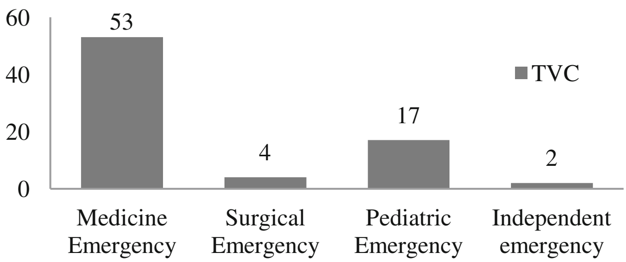

Fig. 2. Four TVCs of emergency events

As can see in Figs. 4, 5 and 6, the medicine emergency need more ED resources (e.g., internist, nurse, and beds) than surgical, pediatric, and independent emergency in day and evening shift. The proposed working schedules or rostering suggest that ED deploys most of physicians, nurses, and beds to care the patients of medicine emergencies. This provides the administrator of ED with a way to prioritize medical resources when allocating limited resources in case of multiple emergencies. It also provides more quantitative values for the various situations than human decision making can provide, and it more fairly deploys the three resources (i.e., doctors, nurses, beds) for the three threat levels. In addition, the administrator can use this model to quantitatively evaluate any emergency threat to response resources and easily discover where response resources are most at risk within the three threat levels (i.e., day shift, evening shift, and night shift). This function also can increase the response agent's vigilance and can secure the critical medical resources to "prevail" against urgency patients.

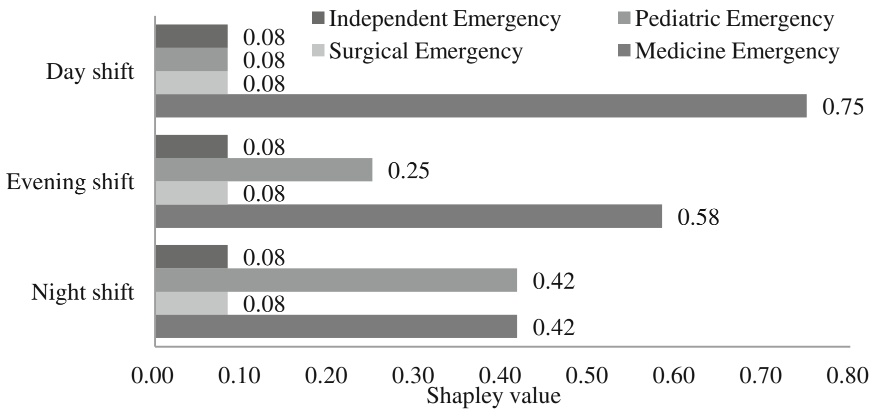

Fig. 3. Four shapley values of emergency events considering the three levels

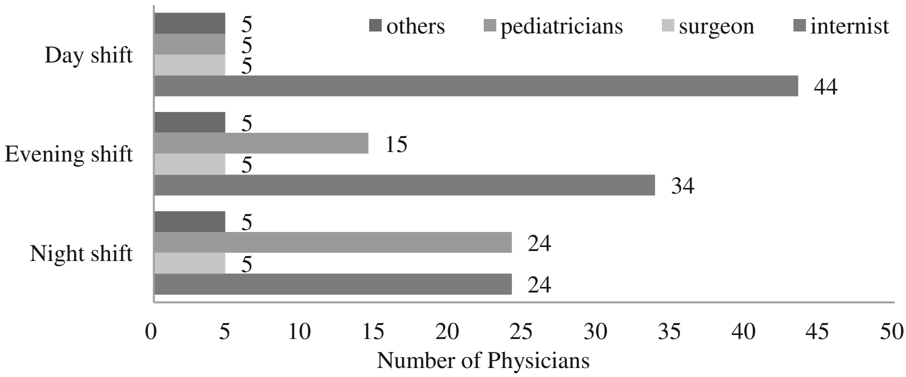

Fig. 4. Three types of ED doctors shift scheduling for the three threat levels

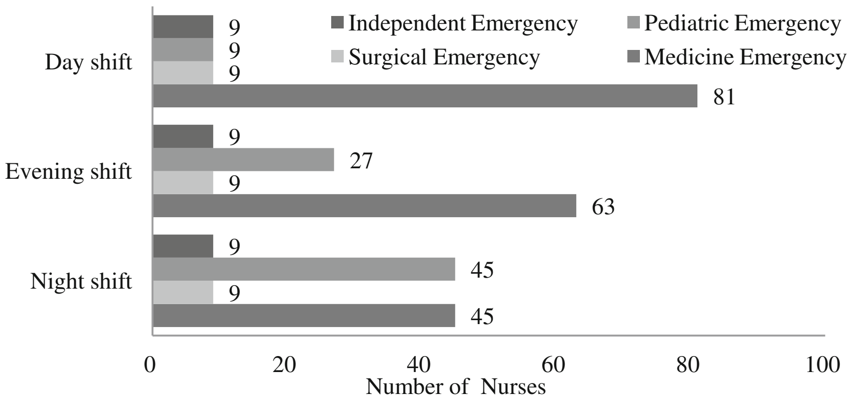

Fig. 5. Three types of ED nurses shift scheduling for the three threat levels

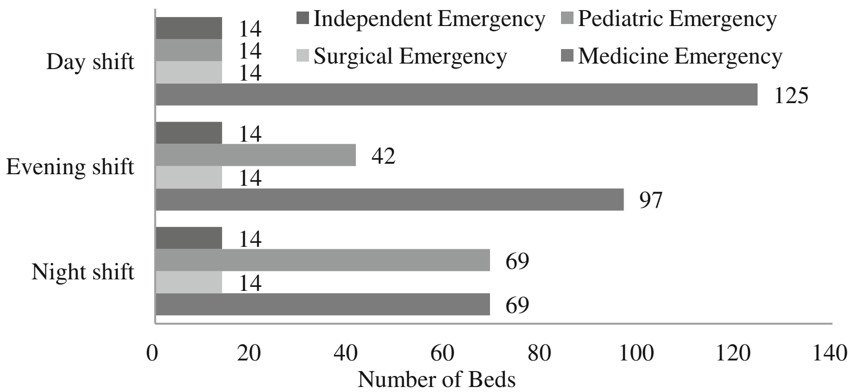

Fig. 6. Three types of ED beds shift scheduling for the three threat levels

5 Conclusions

In the proposed framework, two game theoretic models are applied to develop the optimal deployment of emergency medical resources and to provide three work shifts scheduling per day in emergency department. The first model creates a game strategy using the Nash equilibrium to find the TVC of each emergency event from a non-cooperative game. The second model uses all TVC values of the emergency events to compute the Shapley value for each emergency event given three emergencies levels (i.e., day shift, evening shift, and night shift). The experiments help us to identify the framework connecting the Nash equilibrium and Shapley values, which will enable administrator to prioritize medical resource deployment for the three working shifts in hospital emergency department.

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