

Preface

Topology deals with the concept of continuity and properties unchanged under continuous deformations. In a real system, a continuous deformation, requiring small changes, would involve modes or excitations that cost low energies ($\rightarrow 0$). Since studies of low energy excitations are at the heart of traditional condensed matter physics (CMP), it is natural to expect that topology would play a role in the development of CMP. Curiously, it did not.

An early application of topology in CMP goes back to the use of Morse theory for van Hove singularities. Despite all its beauty and actions, it was not a hit – too much effort for too little gain. However, it did produce something of a ripple, which got amplified further by several disturbing results, like the Aharonov-Bohm effect in quantum mechanics, flux quantization in superconducting rings, the Abrikosov lattice in type-II superconductors, and so on. These hinted at a need to think afresh beyond calculus-based physics. While the new era saw topological ideas fruitfully exploited in the high energy physics context, the condensed matter physics community, on the other hand, remained hesitant. One had to wait for the Berezinski-Kosterlitz-Thouless transition involving vortices in a two dimensional xy magnet to appear on the scene, and then the use of homotopy theory for classification of defects in ordered media, to realize that topology could not be kept out of the reckoning. With the discovery of the quantum Hall effect and its various avatars, and their study via topological invariants, the language of topology became indispensable to CMP. The theory of topological insulators was the shot in the arm CMP was waiting for.

There was a parallel stream involving statistical mechanics. Although the idea of knots in ether as matter (or atoms), e.g., sodium as two linked rings because of the two D lines, got a quick burial in the nineteenth century, but still knot theory survived as a distinct field of topology. The discovery of knot and link structures in polymers and DNA by electron microscopy made studies of knots important in the chemical and biological domains. The study of integrable statistical mechanical models based on hydrogen bonded crystals, like ice, (vertex models, Yang-Baxter equation) in synergy with more modern developments in knot theory produced knot invariants and polynomials sought by the topologists. This was an example of physics helping mathematics. With braids as intertwined paths of two dimensional quantum particles, also called

directed polymers, braid groups led to exotic particle statistics, which are found in condensed matter systems. Such exotic nonabelian particles can be exploited in quantum computation, raising hopes of a secret rendezvous of Alice and Bob.

With so much to gain, topology needs to be an essential toolkit for physicists in general and condensed matter physicists in particular. But, to the best of our knowledge, the subject is not covered in standard physics courses. No doubt, there are authoritative books on topology and geometry in physics, but such are mostly for high energy physicists. It was in this context that a school was organized to introduce topology and its applications in condensed matter problems, at a pedagogic level, developed almost from scratch. This entailed introduction of topology with mathematical rigor while remaining accessible to physicists, side by side with the use of topology in quantum mechanics, statistical physics, and solid state physics.

At a time when mathematics is getting sidelined in physics curricula, the SERC school held in Kolkata in Nov-Dec 2015 was an attempt to halt, if not change, the trend. Moreover, this school tried to focus on a holistic approach to several frontier areas, bridging the language barrier among the disciplines of physics, and between physics and mathematics. Our ultimate hope is that a cross-fertilization will motivate newer research areas in this genre.

This book is a compilation of the lectures delivered in the SERC school. We thank all the contributors for coming and giving the lectures in 2015 in Kolkata, and for their extra effort to write these pedagogic lecture notes. We thank the referees for their meticulous work that improved the readability of the chapters. A summary of the book and guidelines on the usage of the book for courses are given separately in the following pages.

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Topology and Condensed Matter Physics

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2017, XXVIII, 507 p. 148 illus., 80 illus. in color.,

Hardcover

ISBN: 978-981-10-6840-9