Chapter 2
Dividing a Triangle in the Middle Ages: An Example from Latin Works on Practical Geometry

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Abstract: This chapter is concerned with an important question in geometry: the division (or dissection) of 2D figures. This question has been represented and developed in numerous mathematical traditions since Mesopotamian times. In particular, the great geometers of ancient Greece such as Euclid or Hero of Alexandria each tackled it in their own way. Many other developments were achieved in Islamic countries both in the East and West. Based on extracts from medieval Latin literature of the twelfth and thirteenth centuries with Plato of Tivoli, Fibonacci and Jordanus de Nemore, this chapter deals with just one basic geometric problem: 'dividing a triangle into two equal parts', from which the geometric constraints evolve. A broad historic introduction allows these problems to be placed in context; it is a great opportunity to introduce an historic perspective into mathematics teaching. When an author is quoted for the first time, their name is followed, in brackets, by the date of death when known or the proven period when they were active. When too little viable information is known, only the century will be given.

Keywords: Practical geometry, Division of triangle, Geometric construction, Decorative pattern, Geometrical magnitudes, Arithmetisation, Middle Ages, Abraham bar Hiyya, Abû l-Wafâ', Ibn Tâhir al-Baghdâdî, Fibonacci, Plato of Tivoli

Some Elements of Context

> It would be shameful for someone to practise whatever skill it might be and not know what it actually is, its genre, what it is about and all the other things that have gone before it. (Gundissalinus, 1903, p. 44)

Teaching mathematics in the secondary school (11–18 years) (and in particular in so-called difficult areas), I have always viewed my teaching in two complementary ways: scientific education and education for citizenship. In this vision I have deliberately introduced an historical perspective into my teaching. I notice that 11- to 14-year-old students have created a relatively false idea of what mathematics is, and of who the principal movers are. In general, a mathematician does not work isolated from all research or teaching contexts. S/he delves into the history of her/his subject, following in the footsteps of those who have gone before her/him agreeing or disagreeing with them. In the same way, the main mathematical results do not appear out of nothing. They germinate for a period of time before taking their place at the birth

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of a new discipline or in the elaboration of a new theory. The importance of the social and cultural environment of mathematical activities also needs to be clarified.

Indeed, numerous problems arise from everyday concerns. In general, science, and in particular mathematics follow the societies in which they develop. Conversely, they facilitate great progress within these societies.

The names of certain mathematicians hold a near monopoly in secondary classrooms. The majority of them go back to Ancient Greece, notably with Euclid (third century BCE) or to two mathematicians whose biographies are only little known if at all: Pythagoras and Thales. Thus we restrict the image of mathematics we pass on to our students. In this chapter, my idea is to show how simple problems of plane (two-dimensional) geometry are set out and solved in the history of mathematics and in the history of the social groups who have thought about them, whatever their origin.

Here, I concentrate only on the problems of dividing plane figures (see also Moyon, 2009). Dissecting or dividing a plane figure means dividing it up according to previously given constraints. These constraints are relative to the geometric properties of the transversals, or to the figures required and are related to the sizes with conditions linked to the sections resulting from the dissection. For example, divide a given parallelogram *ABCD* in the ratio m:n, by a line parallel to two of its sides (see Figure 2.1). The problem can also be to divide up a given triangle into nine equal triangles (see Figure 2.2).

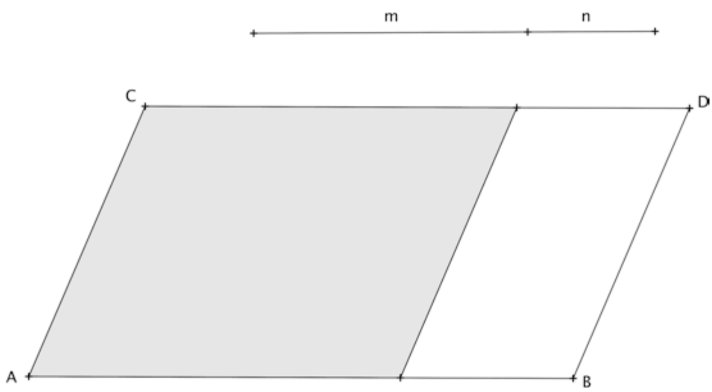

Figure 2.1. Division of the parallelogram.

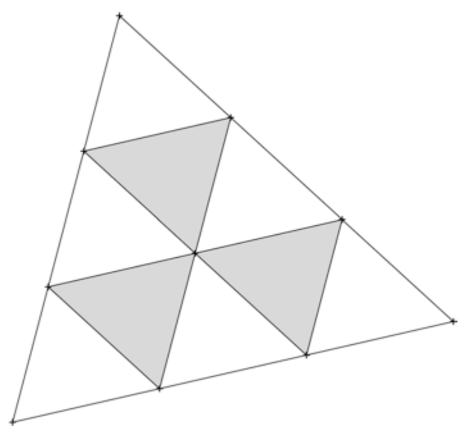

Figure 2.2. Division of a triangle into nine congruent ones.

A little history of these problems allows us to see the importance of a mathematical tradition too often reduced to a role of transmitter from Greek science to Latin Europe: that of Arab mathematics. The term "Arabic nature" makes absolutely no reference to precise geographical and cultural origins. Here we must understand that Arabic nature is concerned with the language used for the publication and the teaching of the science. Numerous seats of learning would emerge from the eighth century, in the Islamic countries which form an empire extending progressively from Samarkand to Saragossa (from East to West) and from the Pyrenees to Timbuktu (from North to South). Finally, with the arrival of Islam the Arab language is instrumental, in transferring ancient knowledge (essentially Greek and Indian) which was then the language of scientific communication (Gutas, 2005).

After an historical prologue, I will propose several problems with their solutions. All these relate to the division of the triangle and they are taken from Medieval Latin literature of which little is still known.

A Brief History

The problems of dividing plane figures are part of an ancient and recurrent theme in mathematics (Moyon, 2016). They occur in numerous traditions, from Mesopotamia (Frieberg, 2007) to the European Renaissance in several treatises on practical geometry, such as, for example, those by Christopher Clavius (fl. 1538–1612) or by Simon Stevin (fl. 1548–1620) (Moyon, 2017).

These problems are also widely dealt with in works from Islamic countries where they developed in two mutually enriching ways. The first lies in the inheritance of Greek mathematics with, notably, the translation of Euclid's work *On divisions of figures*, lost in its original Greek version (Hogendijk, 1993; Moyon, 2017). The second way grew from part of Arab scientific knowledge (scholarly geometry, problems of measuring and calculating areas and volumes, algebra) to solve pseudo down-to-earth problems linked to everyday topics such as surveying land, dividing up inheritances or architecture and decoration. Numerous authors tackle these problems in their works classified under mensuration either independently or in the form of a chapter (Moyon, 2011, 2012). Abû l-Wafâ' al-Bûzjanî (940–998), al-Karajî (d. 1023) or ᶜAbd al-Qâhir ibn Tâhir al-Baghdâdî (eleventh century) are examples from the Muslim East. Finally, for the Muslim West (Maghreb and Andalusia) we have Ibn al-Yâsamîn (d. 1204), Muhammad al-Mursî and Ibn al-Jayyâb, two mathematicians of the thirteenth century (Djebbar, 2007).

To understand how these problems of dividing figures can reflect cultural and social problems, I look at two examples here. The first is borrowed from *Kitâb fîmâ yahtâju ilayhi as-saniᶜ min aᶜmâl al-handasa* [Book on what is needed by the Craftsman for Geometric Constructions] by Abû l-Wafâ'. This problem deals with constructing a square from cutting out three identical squares (Abû l-Wafâ', 1979). The construction produces a decorative pattern that has inspired many artisans from Islamic countries (Figure 2.3) (Berggren, 2007).

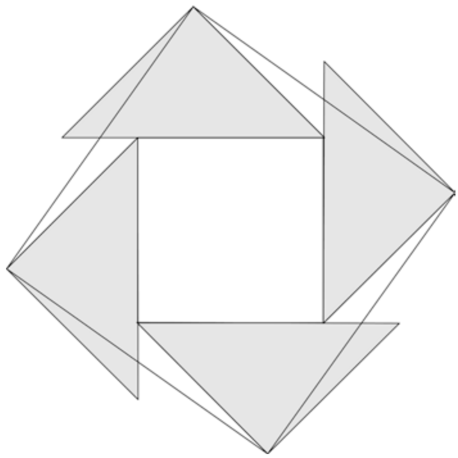

Figure 2.3. Decorative pattern that has inspired artisans.

The second example is the explanation of a mensuration problem from his *Risâla fî l-misâha* [Epistle on measurement] by Ibn Tâhir al-Baghdâdî:

> We divide two squares into two halves along the diagonals and we put each of them to one of the sides of the third square by placing the half-right [angle] of each triangle on one of the corners of the square. Consequently, part of the triangle projects from the side of the square at the adjacent corner of the square. Then we join the right angles of the triangles with straight lines. This will be the side of the square that we were looking for. So, from each large triangle, a small triangle is seen which we cut and replace on the other side. (Djebbar , 2009, pp. 24–25)
>
> For example, a piece of land twenty by thirty is to be shared between three brothers with a passageway two cubits wide. If one takes the pathway from the side measuring thirty, one needs to know the length of this route. (…) If the share was between two sons and one daughter, the share would be in fifths. If the share was between two daughters and a son the share would be in quarters. (Moyon, 2016, p. 13)

From the thirteenth century onwards Latin becomes the language for disseminating all the ancient inherited knowledge plus the knowledge of science in Islamic countries. Geometry, in particular the methods for dividing figures, is no exception. Indeed, mainly thanks to the important movement of translation linked to the *Al-Andalus* area of the twelfth century, Latin Europe would adopt some of the geometric knowledge available in Arabic in the region (Moyon, 2017). This was of Greek origin—Euclid, Apollonius (second century BCE), Archimedes (d. 212 BCE), Menelaus (first century BCE), Ptolemy (second century) amongst others, and their original Arab extensions and developments—the brothers Banû Mûsâ (ninth century), Thâbit ibn Qurra (d. 901) or Ibn al-Haytham (d. 1041), for example.

At the time when a great number of scholars of the twelfth century, such as Gerard of Cremona (d. 1187), Adelard of Bath (fl. 1116–1142) or even Plato of Tivoli (fl. 1132–1146) flocked from all over Europe to discover the science and philosophy written in Arabic, continuous scientific developments kept on being produced, notably amongst the Hebrews. For example, the mathematician Abraham Bar Hiyya (d. ca. 1145) wrote several works, one of which was the *Book on Mensuration and Arithmetic* (Lévy, 2001). There he puts forward several problems on the division of surfaces. While the author was still alive, this book was reworked and translated into Latin by Plato of Tivoli who has passed on to us the *Liber Embadorum* [*Book of Surfaces*] (Curtze, 1902).

When Leonardo Pisano, whom we know as Fibonacci, wrote his *Practica Geometriae* in 1220, a body of geometric knowledge became available in Europe (Boncompagni, 1862).

According to the preface of the second edition of his *Liber Abbaci* (1228), we know that he is the son of a customs official from Béjaïa (in modern day Algeria), later a trading post merchant in Pisa. We also learn that he is thought to have been in direct contact with Islamic mathematical practices thanks to frequent visits to Mediterranean countries, not only Algeria but also to Syria, Egypt, Greece, Sicily and southern France (Moyon & Spiesser, 2015). The *Practica Geometriae* is generally intended for two types of audience: a learned one for whom Fibonacci makes reference to the models of Euclidean geometry with demonstrations and another more mundane one, to fulfil the need of possible practitioners who might not have the necessary scholastic references. The position given to problems relating to the division of figures is important. In fact, he dedicates an entire section of his work to it. Its title *On the Division of Fields Amongst Co-Owners* would allow us to catch a glimpse of the practical application of this topic, whatever its origin might be. There seems to be a close link with the work of Abraham Bar Hiyya (1912) but no evidence to support this (Moyon, 2017).

My teaching activities rest mainly on the two preceding authors as they both provide a very full and didactic exposition of the chosen geometric topic. Nonetheless, I end this historic introduction with two other Latin authors of the thirteenth and fourteenth centuries. Jordanus de Nemore, about whom there is little biographical information, wrote a work on geometry: *Liber Philotegni* (Clagett, 1984). He includes some problems on the division of plane figures. Apparently this book was the set textbook in the Faculty of Arts. Finally, Jean de Murs, Master of Arts at the Sorbonne, wrote works in all disciplines of the *quadrivium*. These are four in number: arithmetic, geometry, astronomy and music. The specific term, *quadrivium*, only appears with Boetius at the start of the sixth century. With the disciplines of the *trivium* (grammar, dialectic, rhetoric) they were to form the teaching program of the medieval universities. In this context a work of practical geometry entitled *De Arte Mensurandi* [*On the Art of Mensuration*] is attributed to Jean de Murs. The book is largely dedicated to the problems of measuring but there are also some problems concerning dissection.

Cutting a Triangle into Two Equal Parts: Why and How

The wording of the problem is very simple: it concerns "dividing a triangle, of whatever sort, into two equal parts." This wording will be echoed in what follows when other constraints are added.

Initially, I place this problem in the context of an equal share. The hypothetical question, taking inspiration at the time from situations linked to contemporary life (sharing between co-owners, or between those who have rights following an inheritance), can be a better representation of the problem. Here let us follow several statements of Islamic tradition concerning the sharing of inheritances. As we have already seen, statements like the following can be found in a large number of mathematics books (notably with the use of algebraic tools) or geometry books (with reference to Euclid's *Elements*): "A man dies and leaves his two sons a triangular field. It is a question of determining, in the case of equal division, the share of the field which is granted to each of his two sons." The statement expressed in this way has no constraints and leaves the student free to explore the problem with their inherent knowledge of size and shapes.

Once the problem had been outlined, I instigated a class debate so they understood that it meant the areas had to be equal regardless of the shape of the figures. It is also important that the exchange helps the students understand there is no unique answer. They must also be guided to formulate several different shares before resolving them. This is a rough summary of a solution, where the leading ideas of a proof can nonetheless be outlined orally. This stage should allow the students to become aware of the richness and variety of mathematical procedure.

Then I posed three different problems while adding some of the restrictions which had been suggested by the students during the course of the earlier discussion. These three problems are just as presented in Fibonacci's *Practica Geometriae* as in Plato de Tivoli's *Liber Embadorum*, to give just two examples. They have been the object of either individual or collective research during which the students have been led to conjecture one or several constructions for the equal division. So I judged it appropriate to try out the suggested solutions with the help of a piece of dynamic geometry software, which in this context, aided autonomy, the students' initiative and so the formulation of hypotheses.

First Problem: Sharing a Triangle into Two Equal Parts by a Straight Line Passing Through One of Its Angles

This very easy problem facilitates a willing involvement on the part of all the students in the research activity. The division, that is the position of point *D* (see Figure 2.4), is a natural one for most, if not all, the students to find. The dynamic geometry software is therefore only used to support the resulting hypothesis. This experiment is nonetheless of value as an introduction to later constraints.

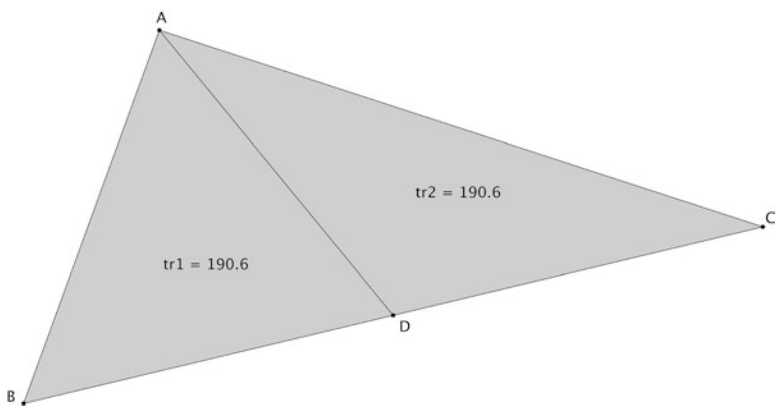

Figure 2.4. Point D divides the triangle into two equal areas.

The proof of the construction is also relatively easy to show. It allows one to revise the geometric properties of a triangle and the relative measurements associated with them. In fact, the student's reasoning is naturally established using the formula to calculate the area of the triangle, that is, half the product of the base and the height. Classroom experience shows that the student falls back on the literal calculation by using *h* for height and *b* for the relative base.

Measurements, Arithmetisation of Measurements and Numbers in Fibonacci's Work on Geometry

Reading Fibonacci's text proves very interesting. In fact, the mathematician from Pisa offers two demonstrations. One refers to Euclid's *Elements* and considers the figures as they are. The other is identical with that of the student considering how to calculate the area of each of the two triangles obtained. This extract allows the students to see that there is not one single right answer to a mathematical problem. Several solutions can be given for the same problem according to the range posed by the mathematician. In the first case, Fibonacci considers the area as a quantity. In the second case, the area is the result of a numerical calculation, that is to say an arithmetic quantity (Barbin, 2007). The distinction between the quantities and the numbers, with the practices inherent in the two quantities, is a dynamic of elementary school and lower secondary school. It is a double problem of an epistemological and of a didactic nature. The activity outlined here allows work to be carried out on this difficult distinction by linking the two approaches. Let us place it within more general pedagogical development. It happens after the so-called Pythagoras' theorem and some of its proofs. It is in this context that I compare the two of them.

The first demonstration is that of proposition 47 from Book I of Euclid's *Elements,* as presented by Vitrac (1990): "In a right angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides" (p. 282). It relies on the area method, otherwise known as the theory of proportion assigned to plane figures, developed by Euclid throughout the second part of Book I. The second demonstration seen in class is for its part, based on basic arithmetic and the formulae to calculate the area of triangles and trapezia from Figure 2.5. The area of the trapezium is half the product of the sum of the bases and the height, that is to say:

$$\frac{(a+b)\times(a+b)}{2}=\frac{a^2+b^2+2ab}{2} \tag{2.1}$$

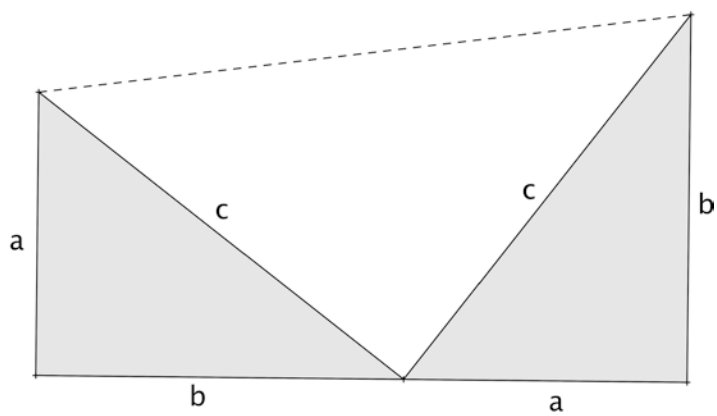

Figure 2.5. Diagram used in calculating the area of the trapezium.

The area of the trapezium is also the sum of the areas of the three triangles added together, that is:

$$\frac{ab}{2}+\frac{ab}{2}+\frac{c^2}{2} \tag{2.2}$$

Equating (2.1) and (2.2) gives the required result.

Furthermore, a reading of this extract from *Practica Geometriae* (Boncompagni, 1862) allows us to encounter the style of writing of one of the most important Latin mathematicians of the thirteenth century:

> Therefore, when you want to divide any triangle into two equal [parts] working from one of its angles, draw a line from this angle to the middle of the opposite side. And you will find what you are looking for.
>
> For example, we want to divide the triangle *ABG* into two equal parts from point *A* (see Figure 2.6).

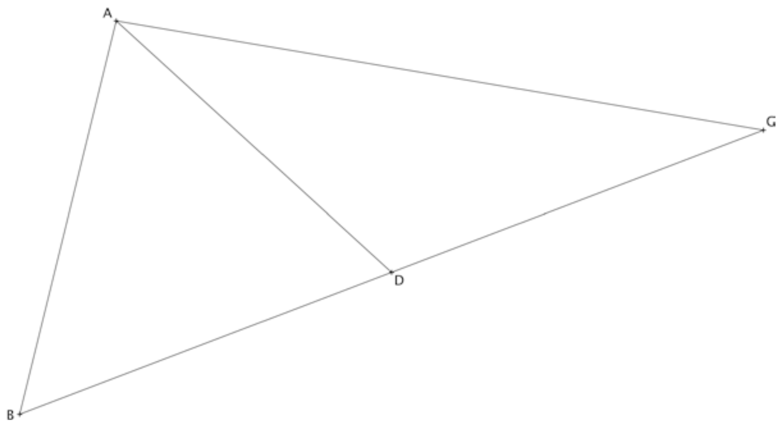

Figure 2.6. Diagram used in Fibonacci's proof.

> Let side *BG* be divided into two equal parts at point *D* and let a straight line *AD* be drawn. I say that the triangle *ABG* is divided into two equal parts.
>
> The two triangles *ABD* and *ADG* are equal to each other, since they are constructed on equal bases and have the same height which is the perpendicular from *A* to the line *BG*. Indeed, two triangles that are constructed each with the same height and the same base are equal according to the sixth Book [of Euclid]. This is why: *BD* is to *DG* just as the triangle *ABD* is to triangle *ADG*. Just as the base *BD* is equal to the base *DG*, therefore the two triangles *ABD* and *ADG* are equal to each other, which has been previously stated.
>
> Or, if we draw in the height coming from point *A* down to the line *BG*, it will in any case be the height of each of the two triangles *ABD* and *ADG*. The product of half the height and the bases *BD* and *DG* equals the product of half of this same height and the base *BG*.
>
> Similarly from the product of half the height and the bases *BD* and *DG* comes the area of the triangles *ABD* and *ADG*. Therefore it is shown that the triangle *ABD* is equal to triangle *ADG* (p. 110).

The reading of this passage should simplify the solution of the problem with two basic propositions from Euclid's *Elements*. Initially there is the first proposition from Book VI explicitly quoted by Fibonacci: "Triangles and parallelograms which are all of the same height are as equal to each other as their bases are." (Vitrac, 1994, p. 155) Fibonacci quotes the general result which relates to the notion of the link between the two measures whereas in the particular case of two equal bases, he could have quoted proposition 38 of Book I which is just as fundamental as the preceding one: "Triangles on equal bases and between the same parallels are equal." (Vitrac, 1994, p. 264) This result will be central to the following problem.

Second Problem: Divide a Triangle into Two Equal Parts by a Straight Line Passing Through a Point Situated on One of its Sides

Even if the problem seems no more difficult to understand than the previous one, the instruction needs an explanation. The students are asked to reformulate the problem with the help of a geometric figure on which the points are named. To achieve this, I took inspiration from the previous extract where the problem is directly followed by a demonstration, also known as an ecthesis (a statement of faith): "For example we wish to divide the triangle *ABG*. … Here that could be Let *ABC* be a triangle and *D* a point on [*AB*]. We want to divide the triangle in two equal parts by a straight line passing through *D*." Thus, with point *D* fixed, the student must find the position of point *G* so that the straight line (*DG*) cuts *ABC* into two equal parts: "trap1" and "trap2". Depending on the position of *D* (see Figures. 2.7 and 2.8), point *G* is found either on side [*BC*] or on side [*AC*]. In addition, I asked the students to describe the limiting position of *D* so that the point *G* might pass from side [*BC*] to side [*AC*]. To answer the question it is enough to revisit the previous problem.

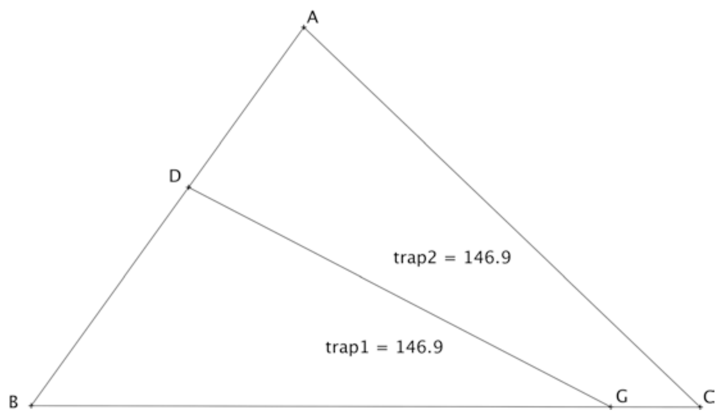

Figure 2.7. Finding point G on BC.

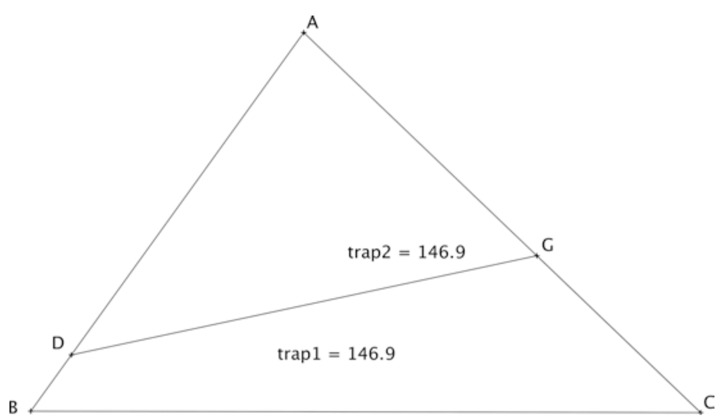

Figure 2.8. Finding point G on AC.

But then, can one determine precisely the position of G with the help of a universal geometric construction? To answer this question, I suggest reading an extract from *Liber Philotegni* by Jordanus de Nemore. Indeed, this medieval author gives, very clearly, an effective construction of the line that crosses the triangle which he then demonstrates:

> Draw a straight line from a designated point on one of the sides of the triangle, so that the triangle can be divided into two equal parts.
> Let ABC be a triangle and D a given point on the side AB. Draw a straight line from the middle of its side to the point C, call it CE. Draw the line CD. Then from E draw a straight line parallel to the straight line DC and label it EG. Join G to D by a straight line. EC is cut at T (see Figure 2.9).

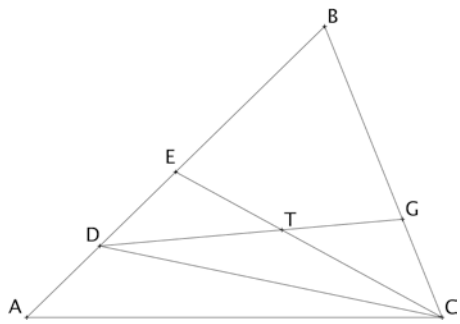

Figure 2.9. Diagram after construction lines are drawn.

Because the triangle CDE is equal to triangle DGC, [the] common [surface] [DTC] divided, the triangle ETD will be equal to triangle GTC. So, added to an equal [surface] [DTC] the surface isolated by the straight line DTG will be equal to the triangle EAC which is half the entire triangle, therefore DG divides the triangle into equal parts, and this is what was to be demonstrated (Clagett, 1984, p. 216).

The second part of the text, that is the demonstration, is nothing other than a game of compensation (or cut and paste) whose reasoning basically rests on proposition 38 of Book I of Euclid's *Elements* which we have already examined. To elucidate further the proof by Jordanus de Nemore, it is worth visualizing the process with the help of the following figures:

$CDE = DGC$ (see Figure 2.10)
$EDC - DTC = CDG - DTC$ (see Figure 2.11)
$ETD = GTC$
$ETD + ADTC = GTC + ADTC$ (see Figure 2.12)

$$\left.\begin{array}{l} EAC = ADGC \\ EAC = \dfrac{1}{2} ABC \end{array}\right\} ADGC = \frac{1}{2} ABC$$

This activity is a natural extension for students who are at ease with the construction of Jordanus de Nemore. Here we have bisected the triangle but it could also be divided in the ratio of 1/3 or 1/4 or any ratio p/q (p, q integers). Point E should then be placed on [AB] in the chosen ratio of p/q such that $\dfrac{AE}{AB} = \dfrac{p}{q}$.

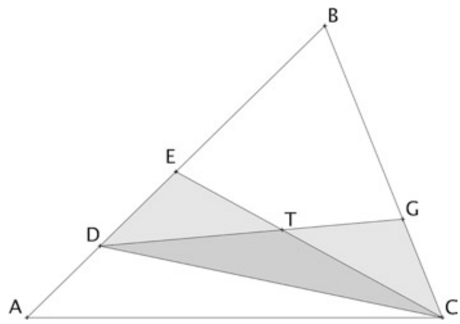

Figure 2.10. Showing *CDE* = *DGC*.

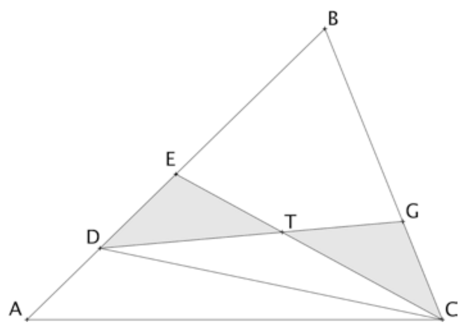

Figure 2.11. Showing *EDC* – *DTC* = *CDG* – *DTC*.

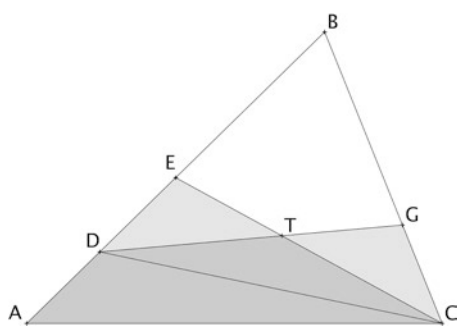

Figure 2.12. Showing *ETD* + *ADTC* = *GTC* + *ADTC*.

Third Problem: Divide a Triangle in Two Equal Parts by a Straight Line Parallel to One of Its Sides

The use of dynamic geometry software should not be a problem if the whole construction is well programmed. In the present case (see Figure 2.13) it is enough to vary the position of the straight line (*DE*) parallel to side (*BC*) until the two figures *tri* and *trap* are of equal area.

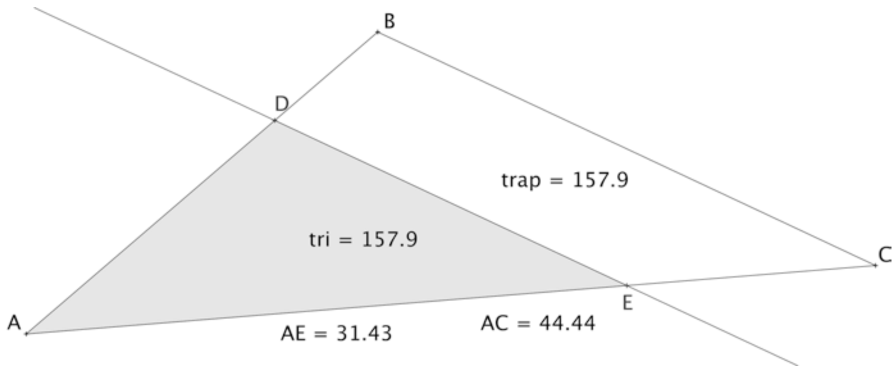

Figure 2.13. Adjusting the line DE until the triangle and trapezium have equal areas.

It is advisable to set the software to give values to one decimal place as this approximation is sufficient for our purposes. This prevents confusing students with too many decimal places or having areas that look equal to one square centimetre. This is fundamental to their heuristic nature. Indeed, the ratio between lengths *AE* and *AC*, which is irrational, is not easy for students to determine or even to estimate. In order to grasp it, a good understanding of the theorem of proportional lines, known as Thales' theorem (Vitrac, 1994), is necessary:

> If a straight line is drawn parallel to one of the sides of the triangle, it will cut the sides of the triangle proportionally; and if the sides of a triangle are cut proportionally, the straight line joining these two points of intersection will be parallel to the remaining side of the triangle. (p. 159)

A reading of Plato of Tivoli's text written in the twelfth century supports this with an exercise.

> For example, divide the line *AB* at *D* and the line *AC* at point *E* so that the square of line *AD* is half the square of line *AB* and the square of the line *AE* is half the square of the line *AC*. Then, drawing the line from point *D* to point *E*, the triangle *ABC* is divided in two equal [parts] of which the first will be the triangle *ADE* and the other will be the trapezium *DBCE.* (Curtze, 1902, pp. 131–132)

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