Chapter 2
Infinite Impulse Response (IIR) Filter

2.1 Impulse-Invariant Mapping

The generalized transfer function of the system can be represented in Laplace transformation as given below:

$$H_a(s) = \sum_{k=1}^{k=N} \frac{A_k}{(S - p_k)}. \tag{2.1}$$

The corresponding impulse response of the causal system is obtained as

$$h_a(t) = \sum_{k=1}^{k=N} A_k e^{-jwt} u(t), \tag{2.2}$$

where $u(t)$ is the unit step function. Sampling the impulse response $h_a(t) = h(t)$, we get the discrete version of the system as given below:

$$h_a(nT_s) = h(n) = \sum_{k=1}^{k=N} A_k e^{-jwnT_s}, \tag{2.3}$$

for $n = 0\ 1 \ldots$. Taking z-transformation of the sequence $h(n)$, we get the following (Fig. 2.1):

$$\begin{aligned} H(z) &= \sum_{n=0}^{n=\infty} h(n) z^{-n} \\ &= \sum_{n=0}^{n=\infty} \sum_{k=1}^{k=N} A_k e^{-jwnT_s} \end{aligned}$$

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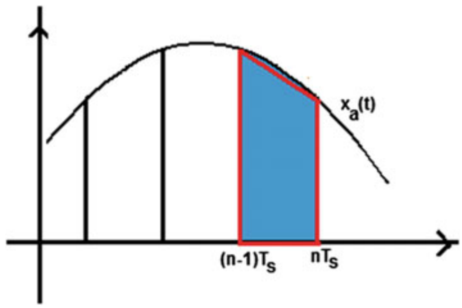

Fig. 2.1 Illustration on the computation of the area under the curve using Trapezoidal rule

$$= \sum_{k=1}^{k=N} A_k \sum_{n=0}^{n=\infty} e^{-jwnT_s} z^{-n}$$

$$= \sum_{k=1}^{k=N} A_k \sum_{n=0}^{n=\infty} (e^{-jwT_s} z^{-1})^n$$

$$= \sum_{k=1}^{k=N} A_k (1 - e^{-jwT_s} z^{-1})^{-1}$$

$$= \sum_{k=1}^{k=N} \frac{A_k}{1 - e^{-jwT_s} z^{-1}}.$$

By substituting $\frac{A_k}{(S-p_k)}$ with $\frac{A_k}{1-e^{-jwT_s} z^{-1}}$, we convert the continuous domain to discrete domain, i.e., we obtain the discrete sequence $h(n)$ from the continuous impulse response $h(t)$. This the impulse-invariant method of mapping S to Z domain.

2.2 Bilinear Transformation Mapping

The sampling frequency $F_s = \frac{1}{T_s}$ used in (2.3) needs to be fixed as greater than twice the maximum frequency content of $h(t)$ (Sampling theorem), which is not usually known. Suppose if F_s is chosen not satisfying the sampling theorem associated with the impulse response, overlapping in the spectrum occurs. In particular, if the spectrum of $h(t)$ is high-pass nature, it suffers a lot. This is circumvented using the technique known as bilinear transformation as described below. The area under the curve of the impulse response $x_a(t) = \frac{dh(t)}{dt}$ for $(n-1)T_s \leq t \leq nT_s$ is computed (refer Fig. 2.1) as

$$\int_{(n-t)T_s}^{nT_s} \frac{dh(t)}{dt} dt = (h((n-1)T_s) - h(nT_s)). \tag{2.4}$$

This is computed using the trapezoidal approximation (refer Fig. 2.1) as follows:

$$\frac{T_s}{2}(x_{nT_s} + x_{(n-1)T_s}). \tag{2.5}$$

Taking Laplace transformation on both sides of $x(t) = \frac{dh(t)}{dt}$, we get $X(s) = sH(s)$. Taking z-transformation of (2.4) and (2.5), we get the following: $H(z)(1 - z^{-1}) = \frac{T_s}{2} X(z)(1 + z^{-1})$. Thus equating the ratio $\frac{X(s)}{H(s)}$ with $\frac{X(z)}{H(z)}$, we get the following:

$$s = \frac{2}{T_s} \frac{1 - z^{-1}}{1 + z^{-1}}. \tag{2.6}$$

Substituting $s = jw$ and $z = e^{jw_d}$ in (2.6), we get $w = \frac{2}{T_s} tan(w_d/2)$, where w is the analog frequency and w_d is the digital frequency. This method of mapping s-domain to z-domain is called Bilinear transformation. Even when w tends to ∞, w_d tends to the value π. As π corresponds to the maximum frequency content of the signal after sampling, maximum frequency of the content of the signal is bounded to ∞. This is equivalent to obtaining the scaled down version of the spectrum of $h(t)$ such that maximum frequency is bounded to $\frac{F_s}{2}$, irrespective of actual value of the F_s. Thus overlapping of spectrum never occur. Hence, this is suitable for high-pass filtering. But the drawback is the shrinkage of the spectrum.

2.2.1 Frequency Pre-warping

The relationship between the digital frequency w_d and the analog frequency w (rad/sec) is linear ($w_d = wT_s$) in the case of impulse-invariant mapping. But if T_s is not properly chosen to obtain the discrete version of the analog filter, overlapping occurs. This is circumvented using Bilinear transformation given as $w = \frac{2}{Ts} tan(\frac{w_d}{2})$. This guarantees that even when the maximum analog frequency content of the impulse response is ∞, the corresponding digital frequency is bounded to π. But the relationship is nonlinear (refer Fig 2.2). Suppose if would like to design the low-pass filter with cutoff frequency w_c in rad/sec (equivalently $\frac{w_c}{F_s}$ in digital domain), we get the digital filter with cutoff frequency $2\frac{wT_s}{2} tan^{-1}(w)$. This is undesired property of Bilinear transformation. This is circumvented as follows.

- Suppose we need the low-pass filter with cutoff frequency w_c in rad/sec (equivalently $\frac{w_c}{F_s}$ in digital domain), we obtain the pre-warped frequency $pw_c = \frac{2}{Ts} tan (\frac{w_c T_s}{2})$. The plot between w_c and pw_c is given in the bandpass filter as shown in Fig. 2.3.

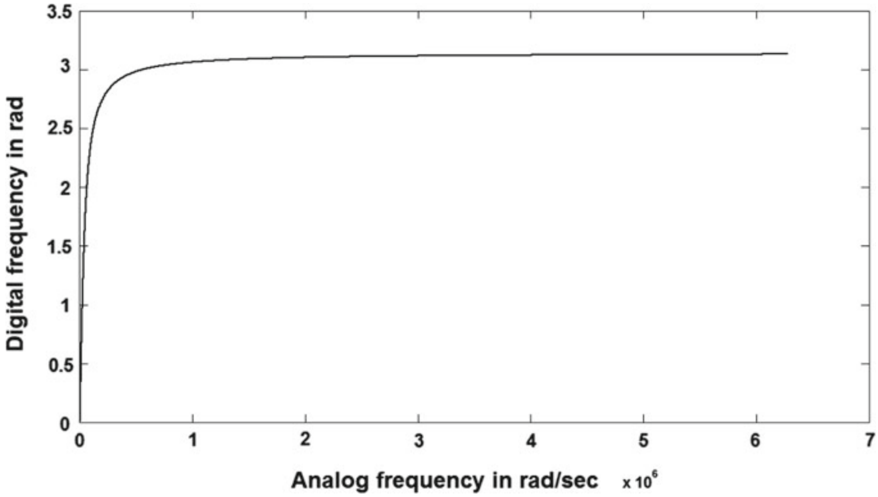

Fig. 2.2 Relationship between digital and analog frequency using bilinear transformation

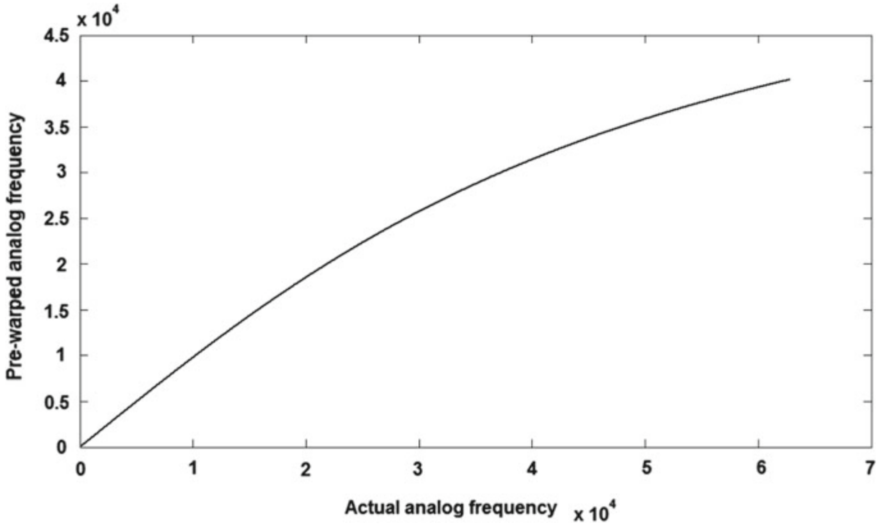

Fig. 2.3 Relationship between the actual analog frequency and the prewarped analog frequency (refer (2.2.1))

- Design the analog filter with the prewarped frequency pw_c in rad/sec.
- If the mapping is done from s to z using bilinear transformation for the designed analog filter, we get the digital filter with the desired cutoff frequency $\frac{w_c}{F_s}$.
- This is known as frequency pre-warping.

2.2.2 Design of Digital IIR Filter using Butterworth Analog Filter and Impulse-Invariant Transformation

- The generalized transfer function of the IIR analog low-pass filter is computed as follows:

$$H_a(s) = \pi_{k=1}^{k=\frac{N}{2}} \frac{B_k w_c^2}{S^2 + b_k w_c s + c_k w_c^2} \tag{2.7}$$

for N as even.

$$H_a(s) = \pi_{k=1}^{k=\frac{N-1}{2}} \frac{B_k w_c^2}{S^2 + b_k w_c s + c_k w_c^2} \frac{B_0 w_c}{s + c_0 w_c} \tag{2.8}$$

for N as odd.
- The magnitude response of the Butterworth filter is given as follows:

$$|H(jw)| = \frac{A}{[1 + (\frac{w}{w_c})^{2N}]^{\frac{1}{2}}}. \tag{2.9}$$

Refer Fig.2.4 for the typical magnitude response plot for various orders of the Butterworth filter with $w_c = 1$ rad/sec and $A = 1$.
- Given the magnitude response of the Butterworth filter at $w = 0$ (say A), magnitude of the transfer function is lesser than m at the stop band frequency (ws in rad/sec), and the order of the filter N is computed as follows: $N = \frac{\frac{A^2}{m^2} - 1}{2log(\frac{ws}{wc})}$, where wc is the cutoff of the Butterworth filter whose magnitude response is $\frac{A}{\sqrt{2}}$ at wc.
- For the typical value of N as even, the values for b_k are computed as $b_k = sin(\frac{(2k-1)\pi}{2N})$, $c_k = 1$, $B_k = A^{\frac{2}{N}}$ for $k = 1 \cdots \frac{N}{2}$.
- For the typical value of N as odd, the values for b_k are computed as $b_k = sin(\frac{(2k-1)\pi}{2N})$, $c_k = 1$, $B_k = A^{\frac{2}{N}}$ for $k = 1 \cdots \frac{N-1}{2}$ and $B_0 = 1$ and $c_0 = 1$.
- Mapping from the s-domain to z-domain ($H(S)$ to $H(z)$) is obtained by substituting the term of the form $\frac{A_k}{(S-p_k)}$ of $H_a(s)$ with $\frac{A_k}{1-e^{-jwT_s}z^{-1}}$. This is done by representing $\frac{B_k w_c^2}{S^2+b_k w_c s+c_k w_c^2}$ as the summation of two partial fractions for every k.
- Thus the digital Butterworth impulse-invariant filter $H(z)$ is obtained.

2.2.3 Design of Digital IIR Filter using Butterworth Analog Filter and Bilinear Transformation

- The magnitude response of the Butterworth filter is given as follows (2.9):

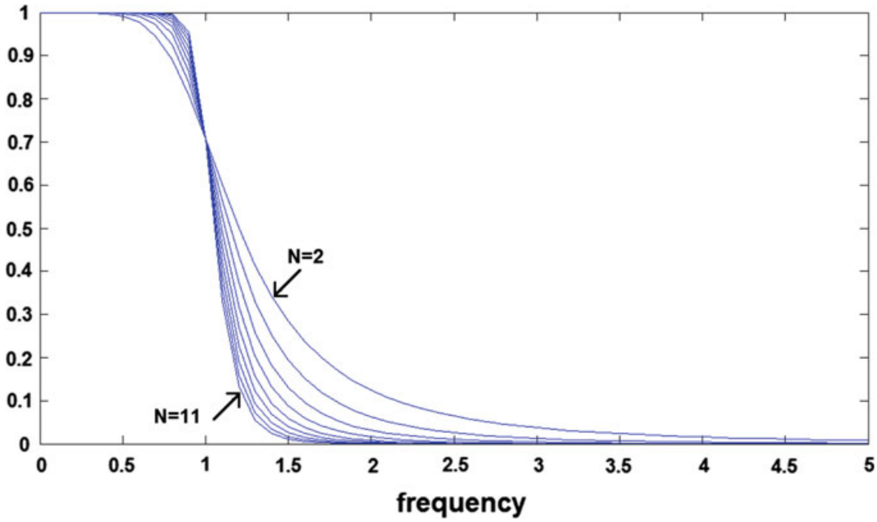

Fig. 2.4 Magnitude response plot for various orders of the Butterworth filter with $w_c = 1$ rad/sec and $A = 1$

- Given the magnitude response of the Butterworth filter at $w = 0$ (say A), magnitude of the transfer function is lesser than m at the stop band frequency (ws in rad/sec), cutoff frequency (wc in rad/sec), and sampling frequency Fs, and the order of the filter N is computed as follows.
- Obtain the prewarped frequency corresponding to ws and wc as pws and pwc as follows:

$$wcd = \frac{w_c}{Fs};\ wsd = \frac{w_s}{Fs};$$
$$pwc = \frac{2}{Ts}tan(\frac{wcd}{2});\ pws = \frac{2}{Ts}tan(\frac{wsd}{2});$$

- The order of the filter N is computed as $N = \frac{\frac{A^2}{m^2}-1}{2log(\frac{pws}{pwc})}$.
- For the typical value of N as even, the values for b_k are computed as $b_k = sin(\frac{(2k-1)\pi}{2N})$, $c_k = 1$, $B_k = A^{\frac{2}{N}}$ for $k = 1 \cdots \frac{N}{2}$.
- For the typical value of N as odd, the values for b_k are computed as $b_k = sin(\frac{(2k-1)\pi}{2N})$, $c_k = 1$, $B_k = A^{\frac{2}{N}}$ for $k = 1 \cdots \frac{N-1}{2}$ and $B_0 = 1$ and $c_0 = 1$. Thus the analog filter $H_a(s)$ is obtained (refer (1.7) and (1.8)).
- Mapping from the s-domain to z-domain ($H(S)$ to $H(z)$) is obtained by substituting $s = \frac{2}{Ts}\frac{1-z^{-1}}{1+z^{-1}}$.
- Thus the bilinear transformation-based Butterworth filter $H(z)$ is obtained.

2.2.4 Design of Digital IIR Filter Using Chebyshev Analog Filter and Impulse-Invariant Transformation

- Butterworth filter has the smooth magnitude response, but the cutoff is not usually very sharp. This is circumvented using the chebyfilter analog filter.
- The magnitude response of the Chebyshev filter is given as follows:

$$|H(jw)| = \frac{A}{[1+\varepsilon^2 C_N(\frac{w}{w_c})]^{\frac{1}{2}}}, \tag{2.10}$$

  where $C_N(x) = cos(Ncos^{-1}x)$ for $x \leq\ 1$ and $C_N(x) = cosh(Ncosh^{-1}x)$ for $x > 1$ (refer Fig. 2.5 for the typical magnitude response plot for various orders (red color for N odd and blue color for N as even) is of the Chebyshev filter with $w_c = 2$ rad/sec, $A = 1$ and $\varepsilon = 0.2$. Also, Fig. 2.6 shows the case when $\varepsilon = 2$.
- Given the ripple width R, maximum amplitude of the transfer function A, pass band cutoff frequency $w_p = w_c$ (magnitude at w_c is given as $\frac{A}{\sqrt{1+\varepsilon^2)}}$), where $(A - \frac{A}{\sqrt{1+\varepsilon^2}})$ is the ripple width R and the magnitude of the transfer function at the stop band frequency (ws in rad/sec) is lesser than m, the order of the filter N is computed as follows:

$$\varepsilon = \sqrt{\frac{R}{A-R}}; r = \frac{w_s}{w_p}; C = \sqrt{\frac{1-m^2}{m^2\varepsilon^2}}; N = \frac{acosh(C)}{acosh(r)}.$$

- For the typical value of N, the values for b_k are computed as $b_k = 2Y_N sin(\frac{(2k-1)\pi}{2N})$, $c_k = (Y_N)^2 + cos^2(\frac{(2k-1)\pi}{2N})$, where $Y_N = \frac{1}{2}([\frac{1}{\varepsilon}^2 + \frac{1}{\varepsilon}]^{\frac{1}{N}} + [\frac{1}{\varepsilon}^2 - \frac{1}{\varepsilon}]^{\frac{-1}{N}})$ and B_k is chosen by choosing the required amplitude (either A for $N = odd$ or $\frac{A}{(1+\varepsilon^2)^{\frac{1}{2}}}$ for $N = even$ at $w = 0$).
- Mapping from the s-domain to z-domain ($H(S)$ to $H(z)$) is obtained by substituting the term of the form $\frac{A_k}{(S-p_k)}$ of $H_a(s)$ with $\frac{A_k}{1-e^{-jwT_s}z^{-1}}$. This is done by representing $\frac{B_k w_c^2}{S^2+b_k w_c s+c_k w_c^2}$ as the summation of two partial fractions for every k.
- Thus the digital Butterworth impulse-invariant filter $H(z)$ is obtained.

2.2.5 Design of Digital IIR Filter Using Chebyshev Analog Filter and Bilinear Transformation

- Butterworth filter has the smooth magnitude response, but the cutoff is not usually very sharp. This is circumvented using the chebyshev analog filter.
- The magnitude response of the Chebyshev filter is given as (2.10).

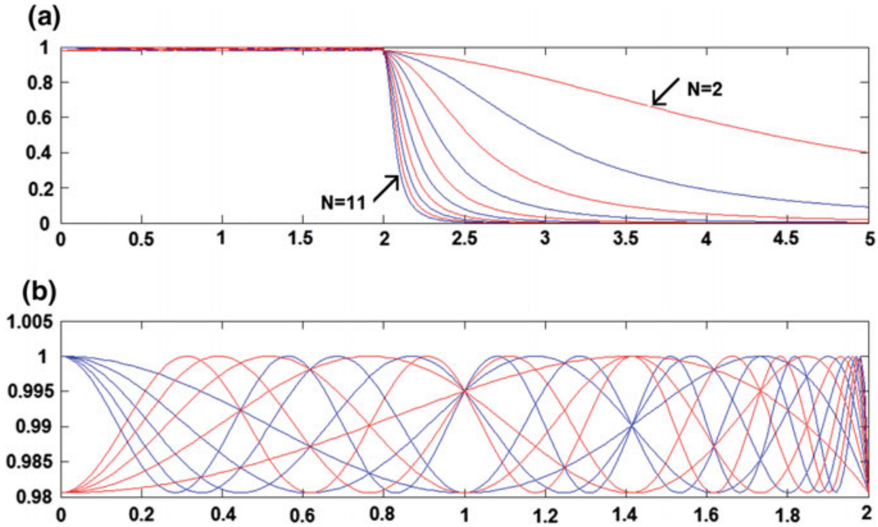

Fig. 2.5 Magnitude response plot for various orders (*red color* for N odd and *blue color* for N as even) is of the Chebyshev filter with $w_c = w_p = 2$ rad/sec (refer Sects. 2.2.4 and 2.2.5), $A = 1$ and $\varepsilon = 0.2$

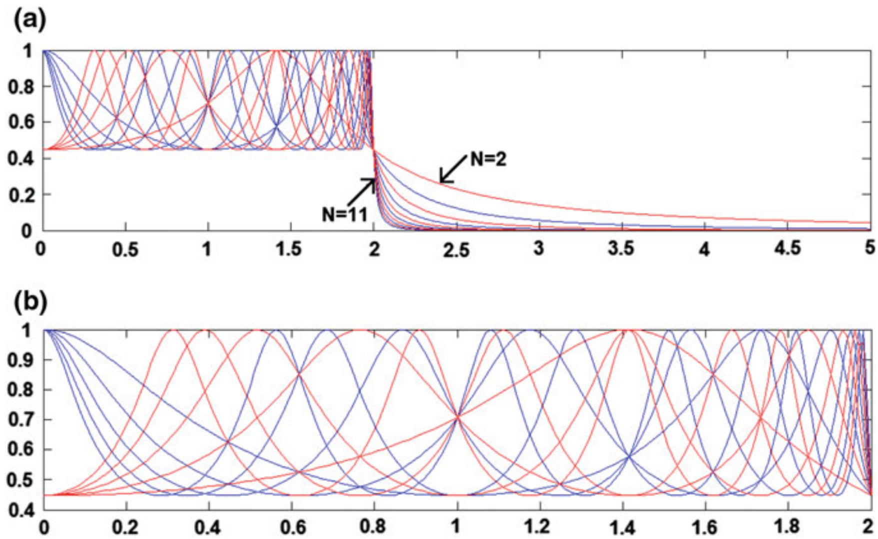

Fig. 2.6 Magnitude response plot for various orders (*red color* for N odd and *blue color* for N as even) is of the Chebyshev filter with $w_c = w_p = 2$ rad/sec, $A = 1$ and $\varepsilon = 2$

- Given the ripple width R, maximum amplitude of the transfer function A, pass band cutoff frequency $w_p = w_c$ (magnitude at w_c is given as $\frac{A}{\sqrt{1+\varepsilon^2)}}$, where $(A - \frac{A}{\sqrt{1+\varepsilon^2}})$ is the ripple width R and the magnitude of the transfer function at the stop band frequency (ws in rad/sec) is lesser than m), the order of the filter N is computed as follows.
- Obtain the prewarped frequency corresponding to ws and wc as pws and pwc as follows:

$$wcd = \frac{w_c}{Fs}; wsd = \frac{w_s}{Fs};$$
$$pwc = \frac{2}{Ts}tan(\frac{wcd}{2}); pws = \frac{2}{Ts}tan(\frac{wsd}{2});$$

$$\varepsilon = \sqrt{\frac{R}{A-R}}; r = \frac{pws}{pwp}; C = \sqrt{\frac{1-m^2}{m^2\varepsilon^2}}; N = \frac{acosh(C)}{acosh(r)}.$$

- For the typical value of N, the values for b_k are computed as $b_k = 2Y_N sin(\frac{(2k-1)\pi}{2N})$, $c_k = (Y_N)^2 + cos^2(\frac{(2k-1)\pi}{2N})$, where $Y_N = \frac{1}{2}([\frac{1}{\varepsilon}^2 + \frac{1}{\varepsilon}]^{\frac{1}{N}} + [\frac{1}{\varepsilon}^2 - \frac{1}{\varepsilon}]^{\frac{-1}{N}})$ and B_k is chosen by choosing the required amplitude (either A for $N = odd$ or $\frac{A}{(1+\varepsilon^2)^{\frac{1}{2}}}$) for $N = even$ at $w = 0$).
- Mapping from the s-domain to z-domain ($H(S)$ to $H(z)$) is obtained by substituting $s = \frac{2}{Ts}\frac{1-z^{-1}}{1+z^{-1}}$.
- Thus the bilinear transformation-based Chebyshev filter $H(z)$ is obtained.

```
%plotbuttermag.m
%Magnitude response of the Butterworth filter
function [res]=plotbuttermag(A,fc)
figure
for N=3:1:11
f=0:0.1:5;
M=A./(1+(f/fc).^(2*N)).^(1/2);
plot(f,M)
hold on
end

%plotchebymag.m
function [res]=plotchebymag(A,fc,epsilon)
figure
subplot(2,1,1)
%Magnitude response of the Chebyshev filter
for N=3:2:11
M=[];
for f=0:0.01:5;
M=[M A./(1+(epsilon^2)*CN(f/fc,N)^2)^(1/2)];
end
plot(0:0.01:5,M)
hold on
end
```

```
for N=2:2:11
M=[];
for f=0:0.01:5;
M=[M A./(1+(epsilon^2)*CN(f/fc,N)^2)^(1/2)];
end
plot(0:0.01:5,M,'r')
hold on
end
subplot(2,1,2)
for N=3:2:11
M=[];
for f=0:0.01:2;
M=[M A./(1+(epsilon^2)*CN(f/fc,N)^2)^(1/2)];
end
plot(0:0.01:2,M)
hold on
end

for N=2:2:11
M=[];
for f=0:0.01:2;
M=[M A./(1+(epsilon^2)*CN(f/fc,N)^2)^(1/2)];
end
plot(0:0.01:2,M,'r')
hold on
end
```

```
%CN.m
function [res]=CN(f,N)
switch f<1
    case 0
        res=cos(N*acos(f));
    case 1
        res=cosh(N*acosh(f));
end
```

```
%butterworthorder.m
function [N]=butterworthorder(A,wc,ws,m)
%Let the maximum frequency content is set as 10000 Hz
%A is the magnitude at w=0
%wc is the cut-off frequency at which
%the magnitude is A/sqrt(2)in rad/sec
%ws is the stop band cutoff frequency
%(in rad/sec) at which the magnitude expected is lesser than m
N=log(((A^2)/(m^2))-1)/(2*log(ws/wc));
N=ceil(N);
fc=wc/(2*pi);
f=0:1:10000;
M=A./(1+(f/fc).^(2*N)).^(1/2);
figure
plot(f,M)
%digitalbutterworth.m
function [NUM,DEN,H]=digitalbutterworth(A,wc,ws,m,Fs,option)
%option 1: Impulse-invariant technique
%option 2: Bilinear transformation technique
```

```
switch option
    case 1
[N]=butterworthorder(A,wc,ws,m);
N
Ts=1/Fs;
order=mod(N,2);
if(order==0)
    N1=N;
else
    N1=N-1;
end
b=0;
for k=1:1:(N1/2)
b(k)=2*sin((2*k-1)*pi/(2*N))
end
Ck=1;
Bk=(A)^(2/N);
B0=1;
c0=1;
%Converting s domain to z-domain
if(N~=1)
for k=1:1:length(b)
[NU,DE]=impulses2z(Bk,Ck,b(k),wc,Fs)
res1{k}=NU;
res2{k}=DE;
end
H=1;
NUM1=res1;
DEN1=res2;
for k=1:1:(N1/2)
[H1,W]=freqz(NUM1{k},DEN1{k});
H=H.*H1;
end
end
if(N==1)
    H=1;
end
if(order==1)
[H2,W]=freqz([B0*wc],[1 -exp(-c0*wc*Ts)])
H=H.*H2;
end
H=abs(H)/max(abs(H))*A;
figure
plot((W*Fs)/(2*pi),H)

if(N==1)
    NUM{1}=[B0*wc];
    DEN{1}=[1 -exp(-c0*wc*Ts)];
else
NUM=NUM2;
DEN=DEN2;
end
```

```
case 2
%Frequency prewarping
%Needs to design the digital filter with cutoff frequency
wcd=(wc/Fs);
wsd=(ws/Fs);
Ts=1/Fs;
pwc=(2/Ts)*tan(wcd/2);
pws=(2/Ts)*tan(wsd/2);
[N]=butterworthorder(A,pwc,pws,m);
N
order=mod(N,2);
if(order==0)
    N1=N;
else
    N1=N-1;
end
b=0;
for k=1:1:(N1/2)
b(k)=2*sin((2*k-1)*pi/(2*N));
end
Ck=1;
Bk=(A)^(2/N);
B0=1;
c0=1;
if(N~=1)
for k=1:1:length(b)
    [NU,DE]=bilinears2z(Bk,Ck,b(k),pwc,Fs);
    res1{k}=NU;
    res2{k}=DE;
end
NUM2=res1;
DEN2=res2;
H=1;
for k=1:1:(N1/2)
[H1,W]=freqz(NUM2{k},DEN2{k});
H=H.*H1;
end
else
H=1;
end

if(order==1)
[H2,W]=freqz([B0*pwc*Ts B0*pwc*Ts],[(2+c0*pwc*Ts) -2+c0*pwc*Ts]);
H=H.*H2;
end
H=abs(H)/max(abs(H))*A;
figure
plot((W*Fs)/(2*pi),H)

if(N==1)
    NUM{1}=[B0*pwc*Ts B0*pwc*Ts];
    DEN{1}=[(2+c0*pwc*Ts) -2+c0*pwc*Ts];
else
NUM=NUM2;
DEN=DEN2;
end

end
```

```
%impulses2z.m
function [NUM,DEN]=impulses2z(Bk,Ck,bk,wc,Fs)
Ts=1/Fs;
vector=[1 bk*wc Ck*(wc^2)];
[p]=roots(vector);
NUM=[0 (exp(p(1)*Ts)-exp(p(2)*Ts))];
NUM=NUM*Bk*(wc^2)/(p(1)-p(2));
DEN=conv([1 -1*exp(p(1)*Ts)],[1 -1*exp(p(2)*Ts)]);
```

```
%bilinears2z.m
function [NUM,DEN]=bilinears2z(Bk,Ck,bk,wc,Fs)
Ts=1/Fs;
NUM=[Bk*(wc^2)*(Ts^2) 2*Bk*(wc^2)*(Ts^2) Bk*(wc^2)*(Ts^2)];
DEN=[4-2*bk*wc*Ts+Ck*(wc^2)*(Ts^2) ...
-8+2*Ck*(wc^2)*(Ts^2) 4+2*bk*wc*Ts+Ck*(wc^2)*(Ts^2)];
```

```
%digitalchebyshev.m
function [NUM,DEN,H]=digitalchebyshev(A,R,wp,ws,m,Fs,option)
%option 1: Impulse invariant technique
%option 2: Bilinear transformation technique
switch option
    case 1
[N]=chebyshevorder(A,R,wp,m,ws)
N
Ts=1/Fs;
order=mod(N,2);
if(order==0)
    N1=N;
else
    N1=N-1;
end
epsilon=sqrt(R/(A-R));
t=(((1/epsilon^2)+1)^(1/2)+(1/epsilon));
Y= (1/2)*(t^(1/N)-t^(-1/N));
b=0;
for k=1:1:(N1/2)
b(k)=2*Y*sin((2*k-1)*pi/(2*N));
C(k)=Y^2+(cos((2*k-1)*pi/(2*N)))^(2);
end
Bk=(A)^(2/N);
B0=Bk;
C0=Y;
wc=wp;
%Converting s domain to z domain
if(N~=1)
for k=1:1:length(b)
[NU,DE]=impulses2z(Bk,C(k),b(k),wc,Fs)
res1{k}=NU;
res2{k}=DE;
end
H=1;
NUM1=res1;
DEN1=res2;
for k=1:1:(N1/2)
[H1,W]=freqz(NUM1{k},DEN1{k});
H=H.*H1;
```

```
end
else
H=1;
end


if(order==1)
[H2,W]=freqz([B0*wc],[1 -exp(-C0*wc*Ts)]);
H=H.*H2;
end
H=abs(H)/max(abs(H))*A;
figure
plot((W*Fs)/(2*pi),H)

if(N==1)
    NUM{1}=[B0*wc];
    DEN{1}=[1 -exp(-C0*wc*Ts)];
else
NUM=NUM1;
DEN=DEN1;
end



case 2
%Frequency prewarping
%Needs to design the digital filter with cutoff frequency
wpd=(wp/Fs);
wsd=(ws/Fs);
Ts=1/Fs;
pwp=(2/Ts)*tan(wpd/2);
pws=(2/Ts)*tan(wsd/2);
[N]=chebyshevorder(A,R,pwp,m,pws)
N
order=mod(N,2);
if(order==0)
    N1=N;
else
    N1=N-1;
end
epsilon=sqrt(R/(A-R));
t=(((1/epsilon^2)+1)^(1/2)+(1/epsilon));
Y= (1/2)*(t^(1/N)-t^(-1/N));
b=0;
for k=1:1:(N1/2)
b(k)=2*Y*sin((2*k-1)*pi/(2*N));
C(k)=Y^2+(cos((2*k-1)*pi/(2*N)))^(2);
end
Bk=(A)^(2/N);
B0=Bk;
C0=Y;
pwc=pwp;

if(N~=1)
for k=1:1:length(b)
    [NU,DE]=bilinears2z(Bk,C(k),b(k),pwc,Fs);
    res1{k}=NU;
    res2{k}=DE;
end
NUM2=res1;
```

```
DEN2=res2;
H=1;
for k=1:1:(N1/2)
[H1,W]=freqz(NUM2{k},DEN2{k});
H=H.*H1;
end
else
    H=1;
end


if(order==1)
[H2,W]=freqz([B0*pwc*Ts B0*pwc*Ts],[(2+C0*pwc*Ts) -2+C0*pwc*Ts]);
H=H.*H2;
end
H=abs(H)/max(abs(H))*A;
figure
plot((W*Fs)/(2*pi),H)


if(N==1)
    NUM{1}=[B0*pwc*Ts B0*pwc*Ts];
    DEN{1}=[(2+C0*pwc*Ts) -2+C0*pwc*Ts];
else
NUM=NUM2;
DEN=DEN2;
end

end
```

```
%chebyshevorder.m
function [N]=chebyshevorder(A,R,wp,m,ws)
%Let the maximum frequency content is set as 10000 Hz
%R is the ripple width
%A/sqrt(2) is the amplitude expected at wp=wc in rad/sec
%The amplitude expected at stopband cutoff frequency ws in rad/sec is lesser
%than m

epsilon=sqrt(R/(A-R));
r=ws/wp;
C=sqrt(((1/(m^2))-1)/(epsilon^2));
N=ceil(acosh(C)/acosh(r));
fc=wp/(2*pi);
M=[];
for f=0:1:10000;
M=[M A./(1+(epsilon^2)*CN(f/fc,N)^2)^(1/2)];
end
figure
plot(0:1:10000,M)
```

2.2.6 Comments on Fig. 2.7 and Fig. 2.8

1. Figure 2.7a shows the intended magnitude response of the Butterworth low-pass filter, which is obtained by plotting (2.9) for the typical values of N and wc. Figure 2.7b shows the magnitude response of the actually designed Butterworth filter. This is obtained by mapping $H_a(s)$ (refer (2.7) and (2.8)) to $H(z)$, followed by computing the magnitude response of the transfer function $H(z)$. It is seen that the intended magnitude response and the magnitude response of the designed filter are almost identical.
2. Figure 2.8a shows the intended magnitude response of the Chebyshev low-pass filter, which is obtained by plotting the (2.10) for the typical values of N, wc, and ε. Figure 2.8c shows the magnitude response of the actually designed Chebyshev filter. This is obtained by mapping $H_a(s)$ (refer (2.7) and (2.8)) to $H(z)$, followed by computing the magnitude response of the transfer function $H(z)$. It is seen that the intended magnitude response and the magnitude response of the designed filter are almost identical.
3. For the bilinear transformation, we need to get the prewarped specification to design the intended low-pass filter that has the magnitude response as shown in Fig. 2.7a (Butterworth filter) and Fig. 2.8a (Chebyshev filter). The magnitude

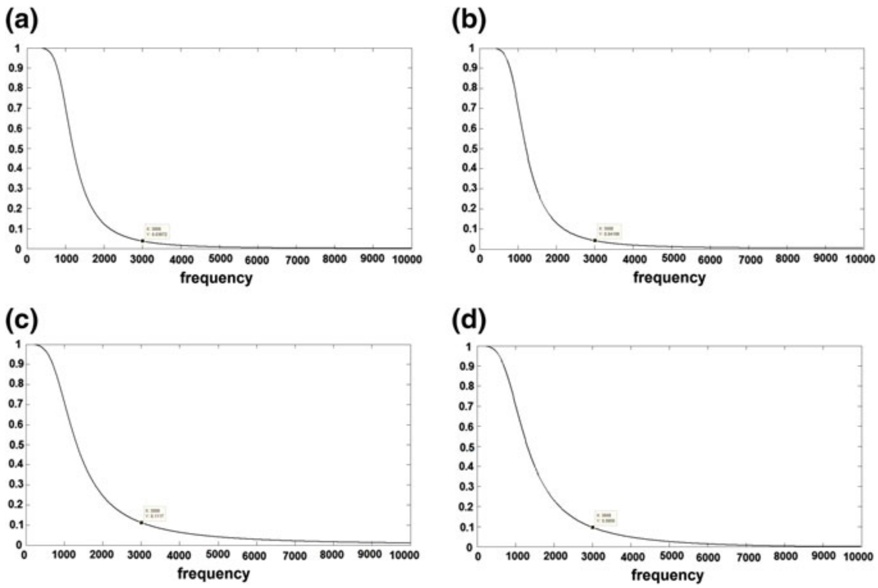

Fig. 2.7 Magnitude response of the designed Butterworth IIR low-pass filter (with magnitude response less than 0.1 at $fs = 3000$ Hz (stop band frequency) and 3dB cutoff at $fc = 500$ Hz (refer Sects. 2.2.2 and 2.2.3). The sampling frequency is $Fs = 20000$ Hz. **a** Intended low-pass filter. **b** Actually designed filter using impulse-invariant technique. **c** Specification after frequency prewarping. **d** Actual designed filter using bilinear transformation

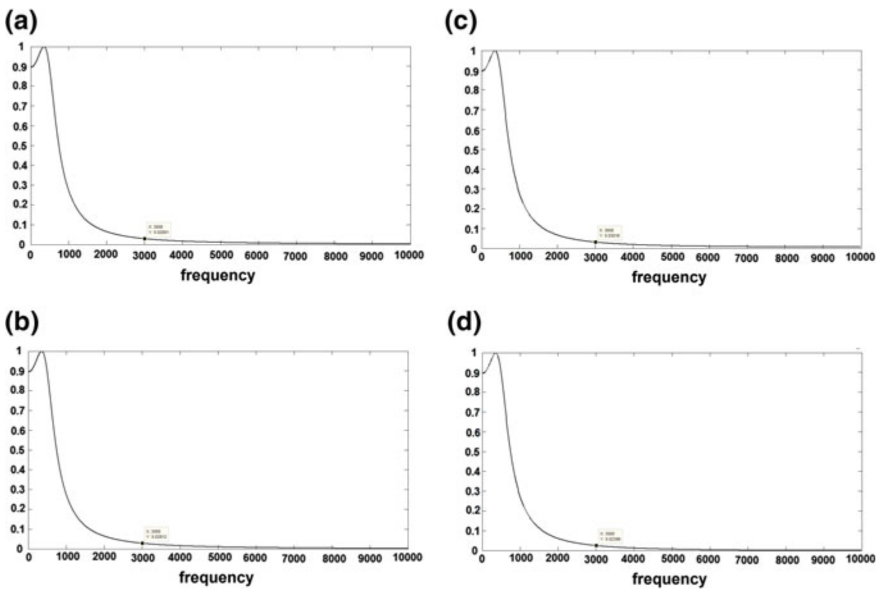

Fig. 2.8 Magnitude response of the designed Chebyshev IIR low-pass filter (with magnitude response less than 0.1 at $fs = 3000$ Hz (stop band frequency), $fc = 500$ Hz (refer Sects. 2.2.4 and 2.2.5) and $Ripplewidth(R) = 0.2$. The sampling frequency is $Fs = 20000$ Hz. **a** Intended low-pass filter. **b** Specification after frequency pre-warping. **c** Actually designed filter using impulse-invariant technique. **d** Actual designed filter using bilinear transformation

response of the IIR filter with the prewarped frequency specifications is shown in Fig. 2.7c (Butterworth filter) and Fig. 2.8b (Chebyshev filter) and the magnitude response of the actually designed IIR filter using bilinear transformation is shown in Fig. 2.7d (Butterworth filter) and Fig. 2.8d (Chebyshev filter). It is seen that amplitude of the magnitude response of the filter after transformation is lesser than the corresponding value in the prewarped specification. This helps in avoiding overlapping of spectrum.

2.2.7 Design of High-Pass, Bandpass, and Band-Reject IIR Filter

2.2.7.1 High-Pass Filter

Given the low-pass filter transfer function $H(e^{jw_d})$ with cutoff w_c radians, the high-pass filter is obtained as $H(e^{j(\pi - w_d)})$ with cutoff $\pi - w_c$. This is equivalent to replacing z with $-z$ in the z-transformation corresponding to LPF to obtain the HPF z-transform. Digital Butterworth high-pass filter using impulse invariant trans-

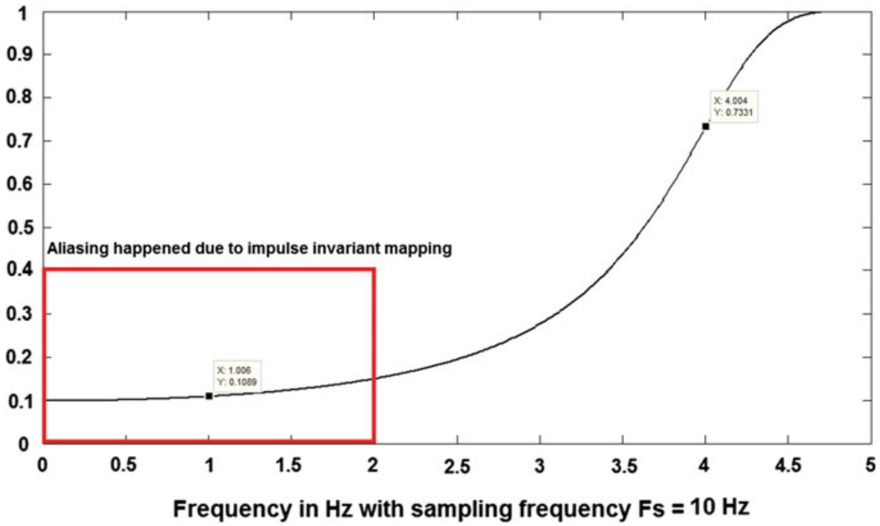

Fig. 2.9 Magnitude response of the Butterworth high-pass filter using impulse-invariant mapping

formation and bilinear transformation with pass band cutoff 8π rad/sec, stop band cutoff frequency 2π rad/sec, and sampling frequency $F_s = 10$ Hz is illustrated in Figs. 2.9 and 2.10, respectively. It is seen from Fig. 2.9 that the Aliasing occur at the lower frequencies. It is also noted that there exists nonzero amplitude at DC (0 Hz). This is the undesirable characteristics and hence impulse-invariant mapping is not usually used to design high-pass filter. This is circumvented using the bilinear transformation and is illustrated in Fig. 2.10.

```
%ButterworthHPFdemo.m
%Digital Butterworth high-pass filter using
%Impulse invariant and bilinear transformation with pass band cutoff
%2*pi*4 rad/sec, stop band cutoff frequency 2*pi*1 rad/sec
%magnitude at the stop band lesser than 0.1 and the sampling frequency 10 Hz
[NUM,DEN,H]=digitalbutterworthHPF(1,2*pi*4,2*pi*1,0.1,10,1)
[NUM,DEN,H]=digitalbutterworthHPF(1,2*pi*4,2*pi*1,0.1,10,1)
```

```
%digitalbutterworthHPF.m
function [NUM,DEN,H]=digitalbutterworthHPF(A,wc,ws,m,Fs,option)
wc=(pi-(wc/Fs))*Fs
ws=(pi-(ws/Fs))*Fs
%option 1: Impulse invariant technique
%option 2: Bilinear transformation technique
switch option
    case 1
[N]=butterworthorder(A,wc,ws,m);
N
Ts=1/Fs;
order=mod(N,2);
```

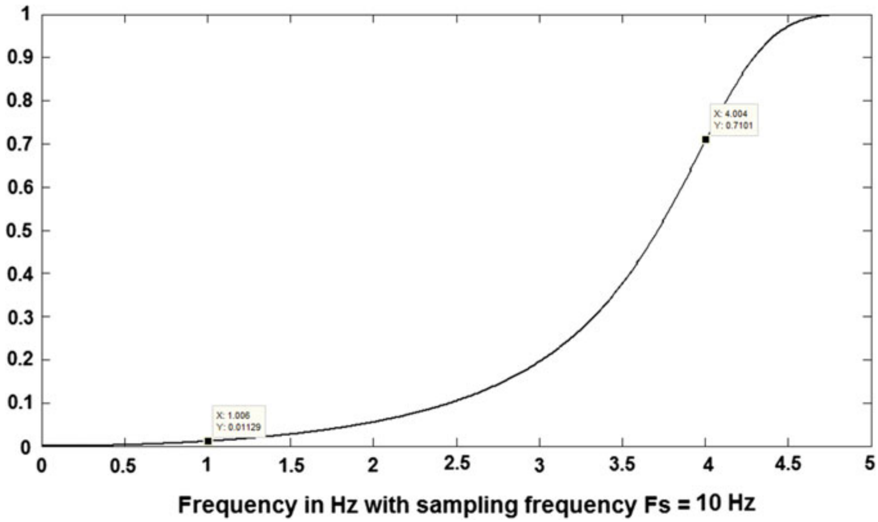

Fig. 2.10 Magnitude response of the Butterworth high-pass filter using bilinear transformation mapping

```
if(order==0)
    N1=N;
else
    N1=N-1;
end
b=0;
for k=1:1:(N1/2)
b(k)=2*sin((2*k-1)*pi/(2*N))
end
Ck=1;
Bk=(A)^(2/N);
B0=1;
c0=1;
%Converting s domain to z domain
if(N~=1)
for k=1:1:length(b)
[NU,DE]=impulses2z(Bk,Ck,b(k),wc,Fs)
L1=length(NU)
if(mod(L1,2)==1)
L=(L1+1)/2;
s1=[ones(1,L/2);zeros(1,L/2)]*2-1
s1=reshape(s1,1,size(s1,1)*size(s1,2))
s1=[s1 1];
else
L=L1/2;
s1=[ones(1,L);zeros(1,L)]*2-1
s1=reshape(s1,1,size(s1,1)*size(s1,2))
end
res1{k}=NU.*s1;

L2=length(DE);
if(mod(L2,2)==1)
```

```
L=(L2+1)/2;
s2=[ones(1,L/2);zeros(1,L/2)]*2-1;
s2=reshape(s2,1,size(s2,1)*size(s2,2));
s2=[s2 1];
else
L=L2/2;
s2=[ones(1,L);zeros(1,L)]*2-1;
s2=reshape(s2,1,size(s2,1)*size(s2,2));
end
res2{k}=DE.*s2
end
H=1;
NUM1=res1;
DEN1=res2;
for k=1:1:(N1/2)
[H1,W]=freqz(NUM1{k},DEN1{k})
H=H.*H1;
end
else
    H=1;
end

if(order==1)
[H2,W]=freqz([B0*wc],[1 exp(-c0*wc*Ts)]);
H=H.*H2;
end
H=abs(H)/max(abs(H))*A;
figure
plot((W*Fs)/(2*pi),H)

if(N==1)
NUM{1}=[B0*wc];
DEN{1}=[1 exp(-c0*wc*Ts)];
else
NUM=NUM1;
DEN=DEN1;
end


case 2
%Frequency prewarping
%Needs to design the digital filter with cutoff frequency
wcd=(wc/Fs);
wsd=(ws/Fs);
Ts=1/Fs;
pwc=(2/Ts)*tan(wcd/2);
pws=(2/Ts)*tan(wsd/2);
[N]=butterworthorder(A,pwc,pws,m);
N
order=mod(N,2);
if(order==0)
    N1=N;
else
    N1=N-1;
end
b=0;
for k=1:1:(N1/2)
b(k)=2*sin((2*k-1)*pi/(2*N));
end
Ck=1;
```

```
Bk=(A)^(2/N);
B0=1;
c0=1;
if(N~=1)
for k=1:1:length(b)
[NU,DE]=bilinears2z(Bk,Ck,b(k),pwc,Fs)
L1=length(NU);
if(mod(L1,2)==1)
L=(L1+1)/2;
s1=[ones(1,L/2);zeros(1,L/2)]*2-1;
s1=reshape(s1,1,size(s1,1)*size(s1,2));
s1=[s1 1];
else
L=L1/2;
s1=[ones(1,L/2);zeros(1,L/2)]*2-1;
s1=reshape(s1,1,size(s1,1)*size(s1,2));
end
res1{k}=NU.*s1;
L2=length(DE);
if(mod(L2,2)==1)
L=(L2+1)/2;
s2=[ones(1,L/2);zeros(1,L/2)]*2-1;
s2=reshape(s2,1,size(s2,1)*size(s2,2));
s2=[s2 1];
else
L=L2/2;
s2=[ones(1,L);zeros(1,L)]*2-1;
s2=reshape(s2,1,size(s2,1)*size(s2,2));
end
res2{k}=DE.*s2;
end
NUM2=res1;
DEN2=res2;
H=1;
for k=1:1:(N1/2)
[H1,W]=freqz(NUM2{k},DEN2{k});
H=H.*H1;
end
else
H=1;
end

if(order==1)
[H2,W]=freqz([B0*pwc*Ts -B0*pwc*Ts],[(2+c0*pwc*Ts) 2-c0*pwc*Ts]);
H=H.*H2;
end
H=abs(H)/max(abs(H))*A;
figure
plot((W*Fs)/(2*pi),H)

if(N==1)
NUM{1}=[B0*pwc*Ts -B0*pwc*Ts]
DEN{1}=[(2+c0*pwc*Ts) 2-c0*pwc*Ts]
else
NUM=NUM2;
DEN=DEN2;
end
end
```

```
%chebyshevHPFdemo.m
%Digital Chebyshev high-pass filter using
%Impulse invariant and Bilinear transformation with pass band cutoff
%2*pi*4 rad/sec, stop band cutoff frequency 2*pi*1 rad/sec, Ripple width 0.5
%magnitude at the stop band lesser than 0.1 and the sampling frequency 10 Hz
[NUM,DEN,H]=digitalchebyshevHPF(1,0.5,2*pi*4,2*pi*1,0.1,10,1)
[NUM,DEN,H]=digitalchebyshevHPF(1,0.5,2*pi*4,2*pi*1,0.1,10,2)
```

```
%digitalchebyshevHPF.m
function [NUM,DEN,H]=digitalchebyshevHPF(A,R,wp,ws,m,Fs,option)
wp=(pi-(wp/Fs))*Fs
ws=(pi-(ws/Fs))*Fs
%option 1: Impulse invariant technique
%option 2: Bilinear transformation technique
switch option
    case 1
[N]=chebyshevorder(A,R,wp,m,ws)
N
Ts=1/Fs;
order=mod(N,2);
if(order==0)
    N1=N;
else
    N1=N-1;
end
epsilon=sqrt(R/(A-R));
t=(((1/epsilon^2)+1)^(1/2)+(1/epsilon));
Y= (1/2)*(t^(1/N)-t^(-1/N));
b=0;
for k=1:1:(N1/2)
b(k)=2*Y*sin((2*k-1)*pi/(2*N));
C(k)=Y^2+(cos((2*k-1)*pi/(2*N)))^(2);
end
Bk=(A)^(2/N);
B0=Bk;
C0=Y;
wc=wp;
%Converting s domain to z domain
if(N~=1)
for k=1:1:length(b)
[NU,DE]=impulses2z(Bk,C(k),b(k),wc,Fs)
L1=length(NU)
if(mod(L1,2)==1)
L=(L1+1)/2;
s1=[ones(1,L/2);zeros(1,L/2)]*2-1
s1=reshape(s1,1,size(s1,1)*size(s1,2))
s1=[s1 1];
else
L=L1/2;
s1=[ones(1,L);zeros(1,L)]*2-1
s1=reshape(s1,1,size(s1,1)*size(s1,2))
end
res1{k}=NU.*s1;

L2=length(DE);
if(mod(L2,2)==1)
L=(L2+1)/2;
s2=[ones(1,L/2);zeros(1,L/2)]*2-1;
```

```
s2=reshape(s2,1,size(s2,1)*size(s2,2));
s2=[s2 1];
else
L=L2/2;
s2=[ones(1,L);zeros(1,L)]*2-1;
s2=reshape(s2,1,size(s2,1)*size(s2,2));
end
res2{k}=DE.*s2
end
H=1;
NUM1=res1;
DEN1=res2;
for k=1:1:(N1/2)
[H1,W]=freqz(NUM1{k},DEN1{k});
H=H.*H1;
end
else
H=1;
end
if(order==1)
[H2,W]=freqz([B0*wc],[1 exp(-C0*wc*Ts)]);
H=H.*H2;
end
H=abs(H)/max(abs(H))*A;
figure
plot((W*Fs)/(2*pi),H)

if(N==1)
NUM{1}=[B0*wc];
DEN{1}=[1 exp(-C0*wc*Ts)];
else
NUM=NUM1;
DEN=DEN1;
end



case 2
%Frequency prewarping
%Needs to design the digital filter with cutoff frequency
wpd=(wp/Fs);
wsd=(ws/Fs);
Ts=1/Fs;
pwp=(2/Ts)*tan(wpd/2);
pws=(2/Ts)*tan(wsd/2);
[N]=chebyshevorder(A,R,pwp,m,pws)
N
order=mod(N,2);
if(order==0)
    N1=N;
else
    N1=N-1;
end
epsilon=sqrt(R/(A-R));
t=(((1/epsilon^2)+1)^(1/2)+(1/epsilon));
Y= (1/2)*(t^(1/N)-t^(-1/N));
b=0;
for k=1:1:(N1/2)
b(k)=2*Y*sin((2*k-1)*pi/(2*N));
C(k)=Y^2+(cos((2*k-1)*pi/(2*N)))^(2);
```

```
end
Bk=(A)^(2/N);
B0=Bk;
C0=Y;
pwc=pwp;
if(N~=1)
for k=1:1:length(b)
    [NU,DE]=bilinears2z(Bk,C(k),b(k),pwc,Fs);
    L1=length(NU)
if(mod(L1,2)==1)
L=(L1+1)/2;
s1=[ones(1,L/2);zeros(1,L/2)]*2-1
s1=reshape(s1,1,size(s1,1)*size(s1,2))
s1=[s1 1];
else
L=L1/2;
s1=[ones(1,L);zeros(1,L)]*2-1
s1=reshape(s1,1,size(s1,1)*size(s1,2))
end
res1{k}=NU.*s1;

L2=length(DE);
if(mod(L2,2)==1)
L=(L2+1)/2;
s2=[ones(1,L/2);zeros(1,L/2)]*2-1;
s2=reshape(s2,1,size(s2,1)*size(s2,2));
s2=[s2 1];
else
L=L2/2;
s2=[ones(1,L);zeros(1,L)]*2-1;
s2=reshape(s2,1,size(s2,1)*size(s2,2));
end
res2{k}=DE.*s2;
end
NUM2=res1;
DEN2=res2;
H=1;
for k=1:1:(N1/2)
[H1,W]=freqz(NUM2{k},DEN2{k});
H=H.*H1;
end
else
H=1;
end


if(order==1)
[H2,W]=freqz([B0*pwc*Ts -B0*pwc*Ts],[(2+C0*pwc*Ts) 2-C0*pwc*Ts]);
H=H.*H2;
end
H=abs(H)/max(abs(H))*A;
figure
plot((W*Fs)/(2*pi),H)

if(N==1)
NUM{1}=[B0*pwc*Ts -B0*pwc*Ts ];
```

```
DEN{1}=[(2+C0*pwc*Ts) 2-C0*pwc*Ts ];
else
NUM=NUM2;
DEN=DEN2;
end

end
```

2.2.7.2 Bandpass Filter

Bandpass filter is obtained as the cascade of low-pass filter with cutoff frequency wc_2 and high-pass filter with cutoff frequency wc_1 (Figs. 2.11 and 2.12). The bandpass filter with $wc_1 = 2\pi$ rad/sec and $wc_2 = 8\pi$ rad/sec is illustrated in Fig. 2.13a–c (Butterworth filter) and Fig. 2.13d–f (Chebyshev filter) using bilinear transformation technique. It is constructed using the cascade connection of low-pass filter (with cutoff frequency $wc_2 = 8\pi$ rad/sec and stop band cutoff frequency $ws_2 = 2\pi 0.1\frac{F_s}{2}$ rad/sec), followed by the high-pass filter (with cutoff frequency $wc_1 = 2\pi$ rad/sec and $ws_1 = 2\pi 0.9\frac{F_s}{2}$ rad/sec). Impulse-invariant (lead to overlapping) is not usually chosen to design other than low-pass filter. Hence, illustration of bandpass filter using bilinear transformation is demonstrated.

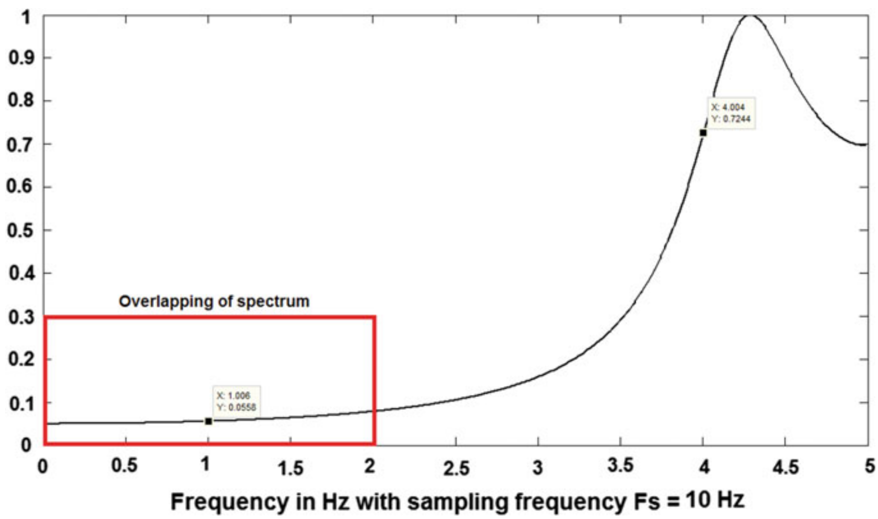

Fig. 2.11 Magnitude response of the Chebyshev high-pass filter using impulse-invariant mapping. It is seen that the magnitude is nonzero at $f = 0$ Hz. This is due to overlapping of spectrum

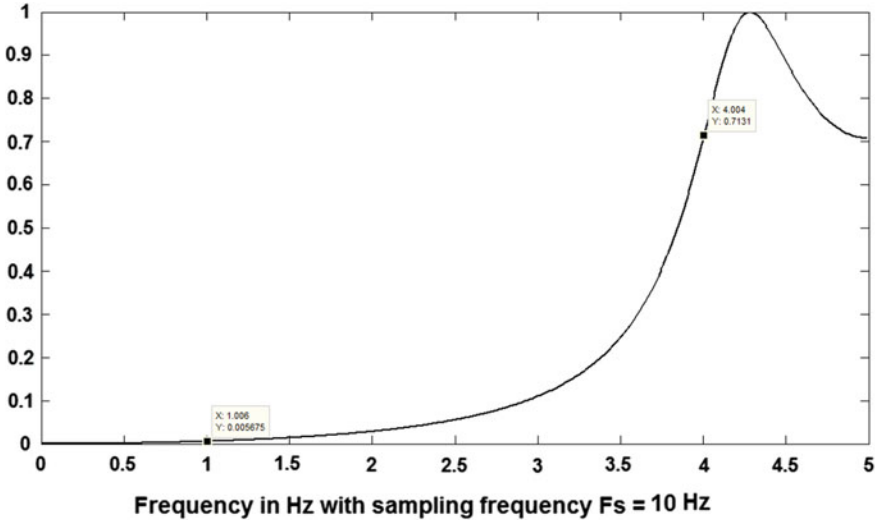

Fig. 2.12 Magnitude response of the Chebyshev high-pass filter using bilinear transformation mapping

```
%IIRBPFDEMO.m
A=1;
wc1=2*pi*1;
wc2=2*pi*4;
m=0.001;
Fs=10;
Ripple=0.5;

%Using Butterworth filter and impulse-invariant transformation
[NUM,DEN,H]=digitalBPF(1,Ripple,wc1,wc2,m,Fs,1,2);
figure;
plot(linspace(0,Fs/2,length(H)),abs(H));

%Using Chebyshevfilter and bilinear transformation
[NUM,DEN,H]=digitalBPF(1,Ripple,wc1,wc2,m,Fs,2,2);
figure
plot(linspace(0,Fs/2,length(H)),abs(H));
```

```
function [NUM,DEN,H]=digitalBPF(A,R,wc1,wc2,m,Fs,option1,option2)
%R is the ripple width used in case of Chebyshev filter
%A is the maximum amplitude of the filter
%H is the normalized magnitude response of the designed filter
%wc1 and wc2 are the cutoff frequencies in rad/sec
%option1:1->Butterworth 2->Chebyshev filter
%option2: 1->Impulse invariant 2->Bilinear
Fmax=Fs/2;
```

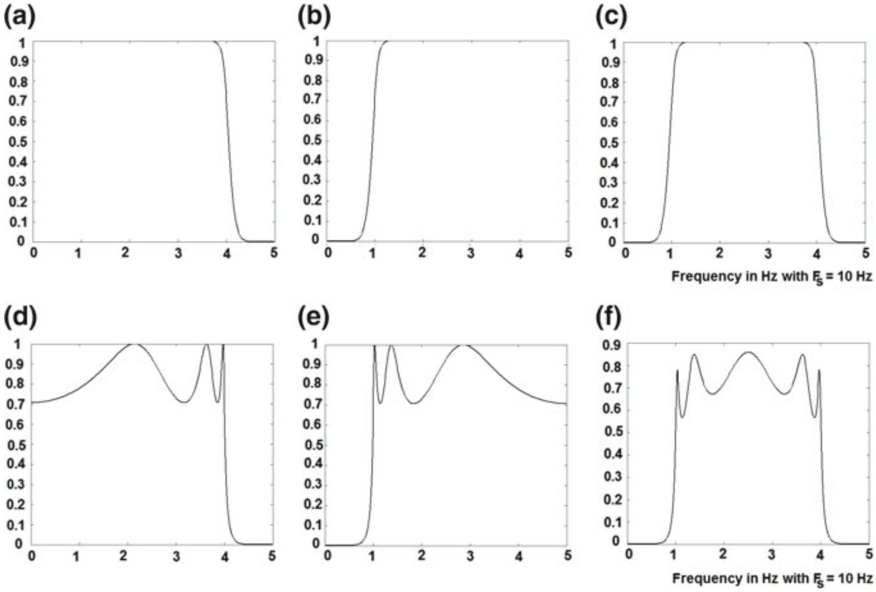

Fig. 2.13 Bandpass filter using bilinear transformation. **a** Butterworth low-pass filter. **b** Butterworth high-pass filter. **c** Corresponding Butterworth bandpass filter as the cascade of low-pass and high-pass filter. **d** Chebyshev low-pass filter. **e** Chebyshev high-pass filter. **f** Corresponding Chebyshev bandpass filter as the cascade of low-pass and high-pass filter

```
ws1=2*pi*0.1*(Fmax);
ws2=2*pi*0.9*(Fmax);
switch option1
    case 1
        switch option2
            case 1
[N1,D1,H1]=digitalbutterworth(A,wc2,ws2,m,Fs,1);
[N2,D2,H2]=digitalbutterworthHPF(A,wc1,ws1,m,Fs,1);
            case 2
[N1,D1,H1]=digitalbutterworth(A,wc2,ws2,m,Fs,2);
[N2,D2,H2]=digitalbutterworthHPF(A,wc1,ws1,m,Fs,2);
        end
    case 2
        switch option2
            case 1
[N1,D1,H1]=digitalchebyshev(A,R,wc2,ws2,m,Fs,1);
[N2,D2,H2]=digitalchebyshevHPF(A,R,wc1,ws1,m,Fs,1);
            case 2
[N1,D1,H1]=digitalchebyshev(A,R,wc2,ws2,m,Fs,2);
[N2,D2,H2]=digitalchebyshevHPF(A,R,wc1,ws1,m,Fs,2);
        end
end

temp1=1;
temp2=1;
temp3=1;
```

```
for i=1:1:length(N1)
temp1=conv(temp1,N1{i})
temp2=conv(temp2,D1{i})
end
for i=1:1:length(N2)
temp1=conv(temp1,N2{i})
temp2=conv(temp2,D2{i})
end
NUM=temp1;
DEN=temp2;
H=H1.*H2;
```

2.2.7.3 Band-reject Filter

Band-reject filter is obtained as the parallel connection of low-pass filter with cutoff frequency wc_1 and the high-pass filter with cutoff frequency wc_2. The band-reject filter with $wc_1 = 2\pi$ rad/sec and $wc_2 = 8\pi$ rad/sec is illustrated in Fig. 2.14a–c (Butterworth filter) and Fig. 2.14d–f (Chebyshev filter) using bilinear transformation technique. It is constructed using the parallel connection of low-pass filter (with cutoff frequency $wc_1 = 2\pi$ rad/sec and stop band cutoff frequency $ws_1 = 2\pi 0.9\frac{F_s}{2}$), followed by the high-pass filter (with cutoff frequency $wc_2 = 2\pi$ rad/sec and $ws_2 = 2\pi 0.1\frac{F_s}{2}$). Impulse-invariant (lead to overlapping) is not usually chosen to design other than low-pass filter. Hence, realization using the bilinear transformation is demonstrated.

```
%IIRBRFdemo.m
A=1;
wc1=2*pi*1;
wc2=2*pi*4;
m=0.001;
Fs=10;
Ripple=0.5;

%Using Butterworth filter and impulse-invariant transformation
[NUM,DEN,H]=digitalBRF(1,Ripple,wc1,wc2,m,Fs,1,2);
figure;
plot(linspace(0,Fs/2,length(H{1})),(1/2)*(abs(H{1})+abs(H{2})));

%Using Chebyshev filter and bilinear transformation
[NUM,DEN,H]=digitalBRF(1,Ripple,wc1,wc2,m,Fs,2,2);
figure
plot(linspace(0,Fs/2,length(H{1})),(1/2)*(abs(H{1})+abs(H{2})));


%digitalBRF.m
function [NUM,DEN,H]=digitalBRF(A,R,wc1,wc2,m,Fs,option1,option2)
%option1:1->Butterworth 2->Chebyshev filter
%option2: 1->Impulse invariant 2->Bilinear
%R is the ripple width used in case of Chebyshev filter
Fmax=Fs/2;
ws1=2*pi*0.9*(Fmax);
ws2=2*pi*0.1*(Fmax);
switch option1
    case 1
```

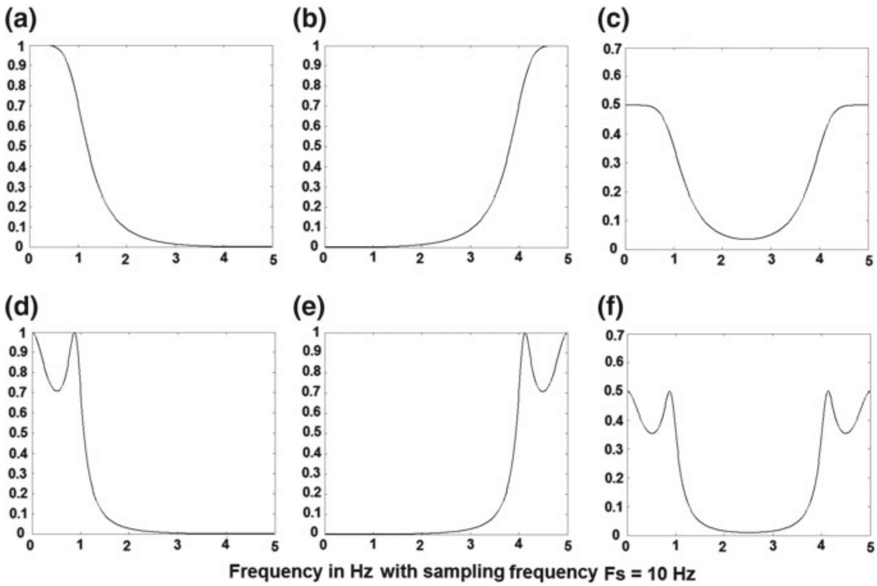

Fig. 2.14 Band-reject filter using bilinear transformation. **a** Butterworth low-pass filter. **b** Butterworth high-pass filter. **c** Corresponding band-reject filter as the parallel summation of low-pass and high-pass filter. **d** Chebyshev low-pass filter. **e** Chebyshev high-pass filter. **f** Corresponding Chebyshev band-reject filter as the parallel summation of low-pass and high-pass filter

```
        switch option2
            case 1
[N1,D1,H1]=digitalbutterworth(A,wc1,ws1,m,Fs,1);
figure
[N2,D2,H2]=digitalbutterworthHPF(A,wc2,ws2,m,Fs,1);
            case 2
[N1,D1,H1]=digitalbutterworth(A,wc1,ws1,m,Fs,2);
[N2,D2,H2]=digitalbutterworthHPF(A,wc2,ws2,m,Fs,2);
        end
    case 2
        switch option2
            case 1
[N1,D1,H1]=digitalchebyshev(A,R,wc1,ws1,m,Fs,1);
[N2,D2,H2]=digitalchebyshevHPF(A,R,wc2,ws2,m,Fs,1);
            case 2
[N1,D1,H1]=digitalchebyshev(A,R,wc1,ws1,m,Fs,2);
[N2,D2,H2]=digitalchebyshevHPF(A,R,wc2,ws2,m,Fs,2);
        end
end

NUM=N1;
DEN=D1;
H{1}=H1;
H{2}=H2;
```

2.3 Realization

Let the transfer function of the typical IIR filter is given as follows:

$$H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots a_p z^{-p}}{b_0 + b_1 z^{-1} + b_2 z^{-3} + \cdots b_q z^{-q}}. \tag{2.11}$$

Realization of the IIR filter is the method of obtaining the output sequence $y(n)$ corresponding to the input sequence $x(n)$ to the linear IIR filter $h(n)$. This is done as follows.

2.3.1 Direct Form 1

Let $X(z)$, $Y(z)$ be the z-transformation of the sequence $x(n)$ and $y(n)$, respectively:

$$\frac{Y(z)}{X(z)} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \cdots a_p z^{-p}}{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + \cdots + b_q z^{-q}}. \tag{2.12}$$

Taking inverse z-transformation, we get the following difference equations:

$$\begin{aligned} y(n) = \ & \frac{a_0}{b_0} x(n) + \frac{a_1}{b_0} x(n-1) + \frac{a_2}{b_0} x(n-2) + \cdots + \frac{a_p}{b_0} x(n-p) \\ & - \frac{b_1}{b_0} y(n-1) - \frac{b_2}{b_0} y(n-2) - \cdots - \frac{b_q}{b_0} y(n-q). \end{aligned}$$

2.3.2 Direct Form 2

Let $\frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \frac{W(z)}{X(z)}$, $\frac{Y(z)}{W(z)} = a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots a_p z^{-p}$ and $\frac{W(z)}{X(z)} = \frac{1}{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots b_q z^{-q}}$. Taking inverse z-transformation, we get the following difference equations:

$$\begin{aligned} w(n) &= -\frac{1}{b_0} x(n) - \frac{b_1}{b_0} w(n-1) - \frac{b_2}{b_0} w(n-2) \\ y(n) &= a_0 w(n) + a_1 w(n-1) + a_2 w(n-2). \end{aligned}$$

We see that to realize IIR filter using Direct form I, we need number of $(p+q)$ number of taps. But to realize using Direct form II, we need only $max(p,q)$ number of taps at the cost of time required for the computation.

2.3.3 Illustration

Consider the input signal $x(t) = \sum_{k=1}^{k=3} A_k sin(2\pi f_k t)$ is sampled using the sampling frequency Fs to obtain the discrete sequence $x(n) = \sum_{k=1}^{k=3} sin(2\pi f_k n T s)$. The digital impulse-invariant Butterworth IIR filter is designed to filter f_3 as given below.

1. Let $A_1 = 1$, $A_2 = 1$ and $A_3 = 1$, $f_1 = 10$, $f_2 = 15$ and $f_3 = 200$.
2. The specification is obtained as follows: Butterworth low-pass filter is designed with $A = 1$, $wc = 2 * pi * 30$, $ws = 2 * pi * 100$, $Fs = 500$ and the amplitude is lesser than 0.1 at ws.
3. The transfer function of the filter is obtained as

$$H(z) = \frac{53.7905Z^{-1}}{1 - 1.4779Z^{-1} + 0.5868Z^{-2}}. \tag{2.13}$$

4. Direct form 1: For the input sequence $x(n)$, the corresponding output sequence is obtained as follows: $y(n) = 53.79x(n-1) - 1.477y(n-1) - 0.5868y(n-2)$.
5. Direct form 2: For the input sequence $x(n)$, the corresponding output sequence is obtained as follows: $w(n) = -x(n) + 1.477w(n-1) - 0.5868w(n-2)$, $y(n) = 53.79w(n-1)$.
6. The number of taps needed for realization of the filter is 3 for Direct form I (DF1) and 2 for Direct form II (DF2). The elapsed time required for DF1 and DF2 realization is given as 0.005619 and 0.009127 s, respectively.
7. Figure 2.15 illustrates the realization of IIR filter using Direct form I and are identical with that of the magnitude response realized using Direct form II. Figure 2.16 illustrates the magnitude response of IIR filter corresponding to the transfer function (2.13).

```
%realizeiir.m
A1=1;
A2=1;
A3=1;
f1=10;
f2=15;
f3=200;
A=1;
wc=2*pi*30;
ws=2*pi*100;
Fs=500;
Ts=1/Fs;
m=0.1;
n=0:1:1000;
S=A1*sin(2*pi*f1*n*Ts)+A2*sin(2*pi*f2*n*Ts)+A3*sin(2*pi*f3*n*Ts);
[NUM,DEN,H]=digitalbutterworth(A,wc,ws,m,Fs,1);
NUM{1}
DEN{1}
temp1=1;
temp2=1;
for k=1:1:length(NUM)
temp1=conv(temp1,NUM{k});
```

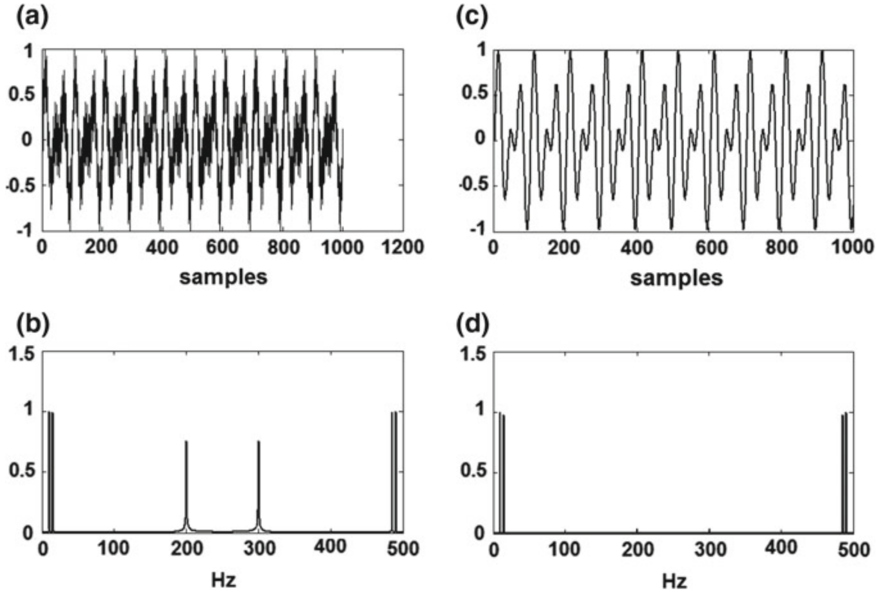

Fig. 2.15 Demonstration on the Direct form I realization of IIR filter using **a** input signal, **b** Filtered signal, **c** spectrum of the input signal, **d** spectrum of the filtered signal

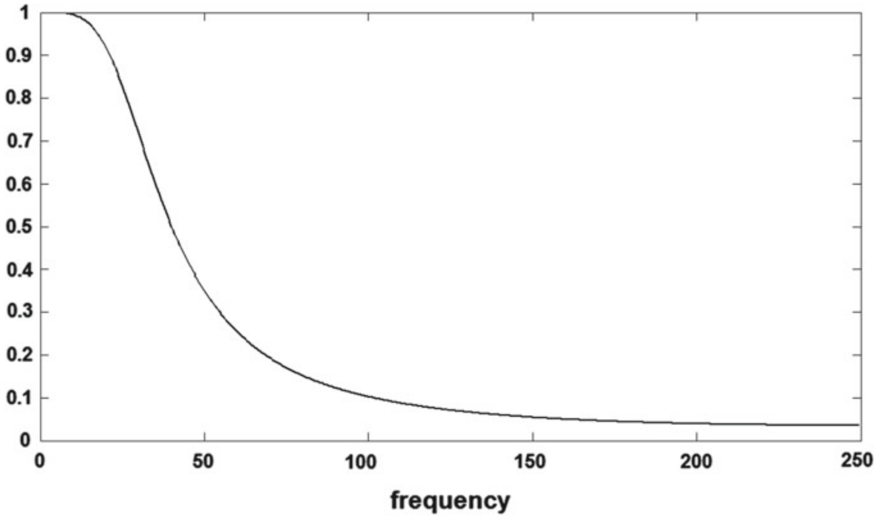

Fig. 2.16 Magnitude response of the IIR filter used to filter the input signal (refer Fig. 2.15)

```
end
for k=1:1:length(DEN)
temp2=conv(temp2,DEN{k});
end
temp1=temp1+eps;
[H,W]=freqz(temp1,temp2);
figure
plot(Fs*W/(2*pi),abs(H)/max(abs(H)))
%Direct form I realization
y=zeros(1,length(temp2)+1);
tic
for n=length(y)+1:1:1000
temp=0;
for r=0:1:length(temp1)-1
temp=temp+temp1(r+1)*S(n-r);
end
for s=1:1:length(temp2)-1
temp=temp-1*temp2(s+1)*y(n-s);
end
temp=temp/temp2(1);
y=[y temp];
end
toc
S=S/max(S);
y=y/max(y);
FRS=abs(fft(S))/max(abs(fft(S)));
FRy=abs(fft(y))/max(abs(fft(y)));
figure
subplot(2,2,1)
plot(S)
subplot(2,2,2)
plot(y)
subplot(2,2,3)
plot(linspace(0,Fs,length(S)),FRS)
subplot(2,2,4)
plot(linspace(0,Fs,length(y)),FRy)

%Directform II realization
M=max(length(temp1),length(temp2));
y=zeros(1,M+1);
w=zeros(1,M+1);
tic
for n=length(w):1:1000
temp=0;
temp=S(n);
for r=1:1:length(temp2)-1
temp=temp-temp2(r+1)*w(n-r);
end
w(n)=temp/temp2(1);
temp=0;
for s=0:1:length(temp1)-1
temp=temp+temp1(s+1)*w(n-s);
end

y=[y temp];
end
toc
S=S/max(S);
y=y/max(y);
```

```
FRS=abs(fft(S))/max(abs(fft(S)));
FRy=abs(fft(y))/max(abs(fft(y)));
figure
subplot(2,2,1)
plot(S)
subplot(2,2,2)
plot(y)
subplot(2,2,3)
plot(linspace(0,Fs,length(S)),FRS)
subplot(2,2,4)
plot(linspace(0,Fs,length(y)),FRy)
```

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