

Chapter 2

Possibilistic Framework for Multi-Objective Optimization Under Uncertainty

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Abstract Optimization under uncertainty is an important line of research having today many successful real applications in different areas. Despite its importance, few works on multi-objective optimization under uncertainty exist today. In our study, we address combinatorial multi-objective problem under uncertainty using the possibilistic framework. To this end, we firstly propose new Pareto relations for ranking the generated uncertain solutions in both mono-objective and multi-objective cases. Secondly, we suggest an extension of two well-known Pareto-base evolutionary algorithms namely, SPEA2 and NSGAII. Finally, the extended algorithms are applied to solve a multi-objective Vehicle Routing Problem (VRP) with uncertain demands.

Keywords Multi-objective optimization • Uncertainty • Possibility theory • Evolutionary algorithms • Vehicle routing problem

2.1 Introduction

Most real-world decision problems are multi-objective in nature as they require the simultaneous optimization of multiple and usually conflicting objectives. These multi-objective problems are a very important and widely discussed research topic. Yet, despite the massive number of existing resolution methods and techniques for multi-objective optimization, there still many open questions in this area. In fact, there is no consideration of uncertainty in the classical multi-objective concepts and techniques, which makes their application to real-life optimization problems impossible.

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Moreover, uncertainty characterizes almost all practical applications, in which the big amount of data provides certainly some unavoidable imperfections. This imperfection might result from using unreliable information sources caused by inputting data incorrectly, faulty reading instruments or bad analysis of some training data. It may also be the result of poor decision-maker opinions due to any lack of its background knowledge or even due to the difficulty of giving a perfect qualification for some costly situations. The classical way to deal with uncertainty is the probabilistic reasoning, originated from the middle of the seventeenth century [19]. However, probability theory was considered for a long time as a very good quantitative tool for uncertainty treatment, but as good as it is, this theory is only appropriate when all numerical data are available, which is not always the case. Indeed, there are some situations such as the case of total ignorance, which are not well handled and which can make the probabilistic reasoning unsound [26]. Therefore, a panoply of non-classical theories of uncertainty have recently emerged such as fuzzy sets theory [33], possibility theory [34] and evidence theory [25]. Among the aforementioned theories of uncertainty, our interest will focus on possibility theory which offers a natural and simple model to handle uncertain data and presents an appropriate framework for experts to express their partial beliefs numerically or qualitatively. Nevertheless, while the field of optimization under uncertainty has gained considerable attention during several years in the mono-objective context, only few studies have been focused on treating uncertain optimization problems within a multi-objective setting. This chapter addresses the multi-objective optimization problems under uncertainty in the possibilistic setting [23].

The remainder of the chapter is organized as follows. Section 2.2 recalls the main concepts of deterministic multi-objective optimization. Section 2.3 gives an overview of existing approaches for multi-objective optimization under uncertainty. Section 2.4 presents in detail our proposed possibilistic framework after briefly recalling the basics of possibility theory. Finally, Sect. 2.5 describes an illustrative example on a multi-objective vehicle routing problem with uncertain demands and summarizes the obtained results.

2.2 Background on Deterministic Multi-Objective Optimization

Deterministic multi-objective optimization is the process of optimizing systematically and simultaneously two or more conflicting objectives subject to certain constraints. In contrast to mono-objective optimization, a multi-objective optimization problem does not restrict to find a unique global solution but it aims to find the most preferred exact solutions among the best ones.

Formally, a basic multi-objective optimization problem (MOP), defined in the sense of minimization of all the objectives, consists of solving a mathematical program of the form:

$$MOP = \begin{cases} \text{Min } F(x) = (f_1(x), f_2(x), \dots, f_n(x)) \\ \text{s.t. } x \in S \end{cases} \quad (2.1)$$

where n ($n \geq 2$) is the number of objectives and $x = \{x_1, \dots, x_k\}$ is the set of decision variables from the decision space S , which represents the set of feasible solutions associated with equality and inequality constraints. $F(x)$ is the vector of independent objectives to be minimized. This vector F can be defined as a cost function in the objective space by assigning an objective vector \vec{y} which represents the quality of the solution (or fitness).

$$F : X \rightarrow Y \subseteq R^n, \quad F(x) = \vec{y} = \begin{pmatrix} y_1 \\ \dots \\ y_n \end{pmatrix} \quad (2.2)$$

In order to identify better solutions of a given MOP, other concepts of optimality should be applied such as *Pareto dominance*, *Pareto optimality*, *Pareto optimal set* and *Pareto front*. Without loss of generality, we assume that the sense of minimization of all the objectives is considered in the following concepts definition:

An objective vector $\mathbf{x} = (x_1, \dots, x_n)$ is said to *Pareto dominate* another objective vector $\mathbf{y} = (y_1, \dots, y_n)$ (denoted by $\mathbf{x} \prec_p \mathbf{y}$) if and only if no component of \mathbf{y} is smaller than the corresponding component of \mathbf{x} and at least one component of \mathbf{x} is strictly smaller:

$$\forall i \in 1, \dots, n : x_i \leq y_i \wedge \exists i \in 1, \dots, n : x_i < y_i. \quad (2.3)$$

For a minimization $MOP(F, S)$, a solution $x^* \in X$ is *Pareto optimal* (also known as efficient, non-dominated or non-inferior) if for every $x \in X$, $F(x)$ does not dominate $F(x^*)$, that is, $F(x) \not\prec_p F(x^*)$.

A *Pareto optimal set* P^* is defined as:

$$P^* = \{x \in X / \exists x' \in X, F(x') \not\prec_p F(x)\}. \quad (2.4)$$

The image of this *Pareto optimal set* P^* in the objective space is called *Pareto front* PF^* defined as:

$$PF^* = \{F(x), x \in P^*\}. \quad (2.5)$$

Yet, finding the true *Pareto front* of a general MOP is NP-hard. Thus, the main goal of multi-objective optimization is to identify a good approximation of the *Pareto front*, from which the decision maker can select an optimal solution based on the current situation. The approximated front should satisfy two properties: (1) convergence or closeness to the exact Pareto front and (2) uniform diversity of the obtained solutions around the Pareto front. Figure 2.1 illustrates an example of approximated front having a very good spread of solutions (uniform diversity) but a bad convergence, since the solutions are far from the true Pareto front.

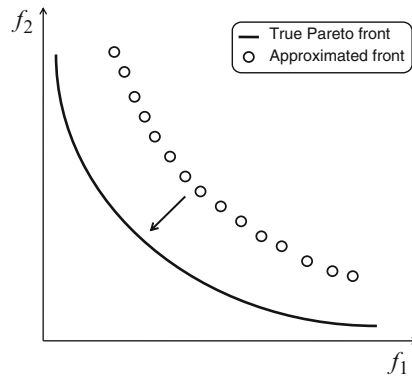


Fig. 2.1 Example of Pareto front with uniform diversity and bad convergence

There are several deterministic optimization methods to deal with multi-objective combinatorial problems, such as the metaheuristics, which mark a great revolution in the field of optimization. A review of various metaheuristics can be found in [29]. Among the well-know metaheuristics, evolutionary algorithms seem particularly suitable for both theoretical and practical MOPs, since they have the ability to search partially ordered spaces for several alternative trade-offs [6, 5, 7]. Some of the most popular multi-objective evolutionary algorithms (MOEAs) are: Multi-Objective Genetic Algorithm (MOGA) [11], Niche-Pareto Genetic Algorithm (NPGA) [14], Pareto-Archived Evolutionary Strategy (PAES) [18], Strength Pareto Evolutionary Algorithms (SPEA, SPEA2) [35, 36] and Non-dominated Sorting Genetic Algorithms (NSGA, NSGAI) [8, 9].

Such algorithms are based on three main components namely, Fitness assignment, Diversity preserving and Elitism.

Fitness Assignment

Fitness Assignment allows to guide the search algorithm toward Pareto optimal solutions for a better convergence. The fitness assignment procedure assigns to each objective vector, a scalar-valued fitness that measures the quality of solution. According to the fitness assignment strategy, four different categories can be identified:

- Pareto-based assignment: based on the concept of dominance and Pareto optimality to guide the search process. The objective vectors are scalarized using the dominance relation.
- Scalar-based assignment: based on the MOP transformation into a mono-objective problem by using for example aggregation methods and weighted metrics.
- Criterion-based assignment: based on the separate handling of various non commensurable objectives by performing a sequential search according to a given preference order of objectives or by handling the objectives in parallel.
- Indicator-based assignment: based on the use of performance quality indicators to drive the search toward the Pareto front.

Diversity Preserving

Diversity Preserving used to generate a diverse set of Pareto solutions. According to the strategy of density estimation, three categories can be distinguished:

- Distance-based density assessment: based on the distance between individuals in the feature space. Examples of techniques are, Niche sharing, Clustering, K th nearest neighbor and Crowding.
- Grid-based density assessment: based on the way in which a number of individuals residing within predetermined cells are located. Histogram method is an example.
- Distribution-based density assessment: based on the probability density of individuals using for example probability density estimation functions.

Elitism

Elitism consists in archiving the best solutions found (e.g, Pareto optimal solutions) in order to prevent the loss of good solutions during the search process. Archiving process can be done using an archive (elite population) or an external population and its strategy of update usually relies on size, convergence and diversity criteria. Depending on the manner in which the archiving process is performed, MOEAs can be classified into two categories, namely non-elitist and elitist MOEAs. Moreover, almost all MOEAs follow the same basic steps in the search process [12], as outlined in the following pseudo code:

Generic MOEA Framework
Initialize random population P While (Stopping condition is not satisfied) Fitness evaluation of solutions in P; Environmental selection of “good” solutions; Diversity preserving of candidate solutions; Update and store elite solutions into an external population or archive; Mating selection to create the mating pool for variation; Variation by applying crossover and mutation operators; End While

An MOEA begins its search with a population of solutions usually generated at random. Thereafter, an iterative optimization process takes place by the use of six search operators: evaluation of the population individuals, environmental selection to choose better solutions based on their fitness, diversity preservation of candidate solutions, updating and archiving the solutions into an external population or archive, mating selection operator in which solutions are picked from the updated population to fill an intermediate mating pool and finally variation operator to generate new solutions. The process stops when one or more pre-specified stopping conditions are met.

All the above concepts and techniques of deterministic multi-objective optimization are widely used and applied successfully to several combinatorial decision problems in many interesting areas, but their application to real-life decision making situations often faces some difficulties. Yet, most of real-world optimization problems are naturally subject to various types of uncertainties caused by many sources such as missing information, forecasting, data approximation or noise in measurements. These uncertainties are very difficult to avoid in practical applications and so should be taken into account within the optimization process. Therefore, a variety of methodologies and approaches for handling optimization problems under uncertainty have been proposed in the last years. Unfortunately, almost all of them have been devoted to solve such problems in the mono-objective context, while only few studies have been performed in the multi-objective setting. A review of some existing approaches for uncertain multi-objective optimization will be summarized in the next section.

2.3 Existing Approaches for Uncertain Multi-Objective Optimization

Uncertain multi-objective optimization has gained more and more attention in recent years [17], since it closely reflects the reality of many real-world problems. Such problems, known as multi-objective problems under uncertainty, are naturally characterized by the necessity of optimizing simultaneously several objectives subject to a set of constraints and while considering that some input data are ill-known and without knowing what their full effects will be. In these problems, the set of objectives and/or constraints to be satisfied can be affected by the uncertainty of input data or uncontrollable problem parameters. Hence, the aim of optimization in this case will be to find solutions of a multi-objective problem that are not only feasible and optimal but also their objectives and/or constraints are allowed to have some acceptable (or minimal) uncertainties. These uncertainties can take different forms in terms of distribution, bounds, and central tendency.

Yet, considering the uncertainty in the objective functions seems to be very applicable but highly critical, since the propagation of input uncertainties to the objectives may have a major impact on the whole optimization process and consequently on the problem solutions. In most of the existing approaches for dealing with multi-objective problems under uncertainty, the objective functions to be optimized are transformed into different forms in order to simplify their resolution by eliminating one of the two basic characteristics of such problems: multi-objectivity and uncertainty propagation. In fact, some of these approaches have been often limited to simply reduce the problem to mono-objective context by considering the set of objectives as if there's only one, using for example an aggregation function (a weighted sum) of all the objectives [13] or preferring only one objective to be optimized (based on a preference indicator) and fixing the remaining objectives as constraints [24]. The considered single objective is then optimized using appropriate mono-objective methods for uncertainty treatment.

Some other approaches have been focused on treating the problem as multi-objective but with ignorance of uncertainty propagation to the objective functions by converting them into deterministic functions using statistical properties. For example, in [30], expectation values are used to approximate the observed interval-valued objectives and so the goal became to optimize the expected values of these objectives. In [2], the average value per objective is firstly computed and then a ranking method based on the average values of objectives is proposed. Similarly, [9] suggested to consider the mean value for each objective vectors and then to apply classical deterministic multi-objective optimizers. Nevertheless, the uncertainty of objective values must not be ignored during the optimization process, because if the input data or parameters are highly uncertain, how can the optimizer simply state that the uncertainty of outputs is completely certain? It may be feasible only for simplicity or other practical reasons as long as the algorithm performance will not be affected.

To this end, some distinct approaches have been suggested to handle the problem as-is without erasing any of its multi-objective or uncertain characteristics by introducing a particular multi-objective optimizer for this purpose. Indeed, [21, 22, 1] proposed to display uncertainty in objective functions through intervals of belief functions and then introduced an extensions of Pareto dominance for ranking the generated interval-valued objectives. Hughes [15, 16] suggested to express uncertainty in the objectives via special types of probability distributions and then independently proposed a stochastic extension of Pareto dominance. Our interest in this chapter will focus on handling multi-objective problems under uncertainty in the possibilistic setting while considering the uncertainty propagation to the set of objectives to be optimized.

2.4 Proposed Possibilistic Framework for Multi-Objective Problems Under Uncertainty

This section provides firstly a brief background on possibility theory and then presents in detail the proposed possibilistic framework for solving multi-objective problems with uncertain data. The framework is composed of three main stages: Adaptation of possibilistic setting, New Pareto optimality and Extension of some optimization algorithms to our uncertain context.

2.4.1 Basics on Possibility Theory

Possibility theory, issued from Fuzzy Sets theory, was introduced by Zadeh [34] and further developed by Dubois and Prade [10]. This theory offers a flexible tool for representing uncertain information such as expressed by humans. Its basic building block is the notion of possibility distribution, denoted by π and defined as the following:

Let $V = \{X_1, \dots, X_n\}$ be a set of state variables whose values are ill-known. We denote by x_i any instance of X_i and by D_{X_i} the domain associated with X_i . $\Omega = D_{X_1} \times \dots \times D_{X_n}$ denotes the universe of discourse, which is the cartesian product of all variable domains V . Vectors $\omega \in \Omega$ are often called realizations or simply “states” (of the world). The agent’s knowledge about the value of the x_i ’s can be encoded by a possibility distribution π that corresponding to a mapping from the universe of discourse Ω to the scale $[0, 1]$, i.e. $\pi : \Omega \rightarrow [0, 1]$; $\pi(\omega) = 1$ means that the realization ω is totally possible and $\pi(\omega) = 0$ means that ω is an impossible state. It is generally assumed that there exist at least one state ω which is totally possible— π is said then to be *normalized*. Extreme cases of knowledge are presented by:

- *complete knowledge* i.e. $\exists \omega_0 \in \Omega, \pi(\omega_0) = 1$ and $\forall \omega \neq \omega_0, \pi(\omega) = 0$.
- *total ignorance* i.e. $\forall \omega \in \Omega, \pi(\omega) = 1$ (all values in Ω are possible).

From π , one can describe the uncertainty about the occurrence of an event $A \subseteq \Omega$ via two dual measures: the possibility $\Pi(A)$ and the necessity $N(A)$ expressed by:

$$\Pi(A) = \sup_{\omega \in A} \pi(\omega). \quad (2.6)$$

$$N(A) = 1 - \Pi(\neg A) = 1 - \sup_{\omega \notin A} \pi(\omega) \quad (2.7)$$

Measure $\Pi(A)$ corresponds to the possibility degree (i.e. the plausibility) of A and it evaluates to what extent A is consistent (i.e. not contradictory) with the knowledge represented by π . Yet, the expression “*it is possible that A is true*” does not entail anything about the possibility nor the impossibility of A . Thus, the description of uncertainty about the occurrence of A needs its dual measure $N(A)$ which corresponds to the extent to which A is impossible and it evaluates at which level A is certainly implied by the π (the certainty degree of A). Main properties of these two dual measures are summarized in Table 2.1.

Table 2.1 Possibility measure Π and necessity measure N

$\Pi(A) = 1$ and $\Pi(\bar{A}) = 0$	$N(A) = 1$ and $N(\bar{A}) = 0$	A is certainly true
$\Pi(A) = 1$ and $\Pi(\bar{A}) \in]0, 1[$	$N(A) \in]0, 1[$ and $N(\bar{A}) = 0$	A is somewhat certain
$\Pi(A) = 1$ and $\Pi(\bar{A}) = 1$	$N(A) = 0$ and $N(\bar{A}) = 0$	Total ignorance

The particularity of the possibilistic scale is that it can be interpreted in two manners: in an ordinal manner, i.e. when the possibility degrees reflect only an ordering between the possible values and in a numerical manner, i.e. when the handled values make sense in the ranking scale.

Technically, a possibility distribution is a normal fuzzy set (at least one membership grade equals 1). Indeed, all fuzzy numbers can be interpreted as specific possibility distributions. More precisely, given a variable X whose values are restricted by a fuzzy set F characterized by its membership function μ_F , so that π_X is taken as equal to the membership function $\mu_F(x)$. Thus, the possibility and necessity measures will be expressed in terms of supremum degrees of the μ_F , i.e.

$\Pi(X) = \sup_{x \in X} \mu_F(x)$ and $N(X) = 1 - \sup_{x \notin X} \mu_F(x)$. In this work, we are interested in a particular form of possibility distributions, namely those represented by triangular fuzzy numbers and commonly known as triangular possibility distributions. A triangular possibility distribution π_X is defined by a triplet $[\underline{x}, \hat{x}, \bar{x}]$, as shown in Fig. 2.2, where $[\underline{x}, \bar{x}]$ is the interval of possible values called its bounded support and \hat{x} denotes its kernel value (the most plausible value).

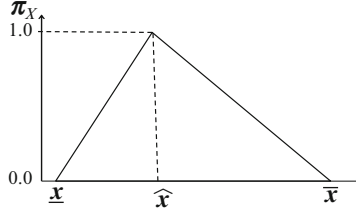


Fig. 2.2 Triangular possibility distribution

In the remaining, we use $X = [\underline{x}, \hat{x}, \bar{x}] \subseteq \mathbb{R}$ to denote the triangular fuzzy number X , meaning that X is represented by a triangular possibility distribution π_X . This representation is characterized by a membership function μ_X which assigns a value within $[0, 1]$ to each element in $x \in X$. Its mathematical definition is given by:

$$\mu_X(x) = \begin{cases} \frac{x - \underline{x}}{\hat{x} - \underline{x}}, & \underline{x} \leq x \leq \hat{x} \\ 1, & x = \hat{x} \\ \frac{\bar{x} - x}{\bar{x} - \hat{x}}, & \hat{x} \leq x \leq \bar{x} \\ 0, & \text{otherwise.} \end{cases} \quad (2.8)$$

However, in practical use of triangular fuzzy numbers, a ranking procedure needs to be applied for decision-making. In other words, one triangular fuzzy number needs to be evaluated and compared with the others in order to make a choice among them. Indeed, all possible topological relations between two triangular fuzzy numbers $A = [\underline{a}, \hat{a}, \bar{a}]$ and $B = [\underline{b}, \hat{b}, \bar{b}]$ may be covered by only four different situations, which are: Fuzzy disjoint, Fuzzy weak overlapping, Fuzzy overlapping and Fuzzy inclusion [20]. These situations, illustrated in Fig. 2.3 should be taken into account for ranking triangular fuzzy numbers.

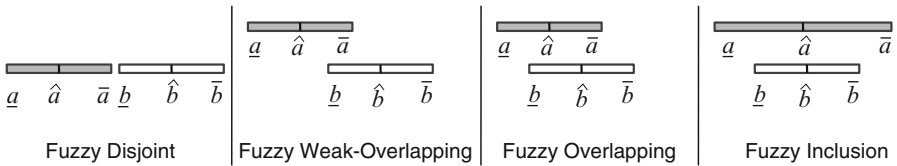


Fig. 2.3 Possible topological situations for two TFNs

2.4.2 Adaptation of Possibilistic Setting

In the following, we choose to express the uncertain data of multi-objective problems under uncertainty using triangular possibility distributions (i.e. triangular fuzzy numbers) as defined in the previous subsection. Then, as a multi-objective optimization problem under uncertainty involves the simultaneous satisfaction of several objectives respecting a set of constraints and while considering some input data uncertainties, we assume that the observed objectives and some constraints (especially those depends on uncertain variables) are affected by the used form of these uncertainties.

Thus, as in our case, uncertainty is represented by a triangular form, the uncertain constraints in such a problem may be disrupted by this fuzzy form and so will be fuzzy constraints. Yet, the satisfaction of such constraints cannot be directly predicted since it is difficult to estimate directly that a fuzzy constraint is fully satisfied or fully violated. At this level, we propose firstly to use the two measures of possibility theory Π and N in order to express the satisfaction of a given fuzzy constraint, as follows:

Let $X = [\underline{x}, \hat{x}, \bar{x}] \subseteq \mathbb{R}$ be a triangular fuzzy variable, let x be any instance of X , let v be a given fixed value and let $C = (X \leq v)$ be a fuzzy constraint that depends only on the value of X and whose membership function is $\mu(x)$, then we have the measures $\Pi(X \leq v)$ and $N(X \leq v) = 1 - \Pi(X > v)$ are equal to:

$$\Pi(\tilde{X} \leq v) = \sup \mu_{x \leq v}(x) = \begin{cases} 1 & \text{if } v > \hat{x} \\ \frac{v - \underline{x}}{\hat{x} - \underline{x}} & \text{if } \underline{x} \leq v \leq \hat{x} \\ 0 & \text{if } v < \underline{x}. \end{cases} \quad (2.9)$$

$$N(\tilde{X} \leq v) = 1 - \sup \mu_{x > v}(x) = \begin{cases} 1 & \text{if } v > \bar{x} \\ \frac{v - \hat{x}}{\bar{x} - \hat{x}} & \text{if } \hat{x} \leq v \leq \bar{x} \\ 0 & \text{if } v < \hat{x}. \end{cases} \quad (2.10)$$

These formulas will be used to express the degrees that a solution satisfies the fuzzy constraint.

Example 1 As an example of constraint satisfaction expressed by the possibility and necessity measures, we have $Q = [\underline{q}, \hat{q}, \bar{q}] = [20, 45, 97]$ is a triangular fuzzy quantity of objects, $M = 50$ is the maximum size of a package and $C = (Q \leq M)$ is the fuzzy constraint which imposes that the total quantity of objects must be less than or equal to the package size. In this case, $\Pi(Q \leq M) = 1$ because $M = 50 > \hat{q} = 45$ and $N(Q \leq M) = \frac{M - \hat{q}}{\bar{q} - \hat{q}} = \frac{50 - 45}{97 - 45} = 0.096$ because $\hat{q} = 45 \leq M = 50 \leq \bar{q} = 97$.

Note that, a constraint may fail even though its possibility achieves 1 and holds even though its necessity is 0. In addition, an often used definition says that the possibility measure Π gives always the best case and shows the most optimist attitude, while the necessity N gives the worst case and shows the most pessimist attitude. Then, as presented above, Π and N are related to each others by a dual relationship. Therefore, a combination of these two measures allows the expression of both optimistic

and pessimistic attitude of the decision maker. From these remarks, we can conclude that it is more efficient at this step to use the linear combination of possibility and necessity measures proposed by Brito et al. [4], rather than treating each measure separately. This linear combination is defined as the following:

Given a constraint A , its weight denoted by $W(A)$ which corresponds to the combination of the weighted possibility and necessity, is expressed by:

$$W(A) = \lambda \Pi(A) + (1 - \lambda) N(A) \geq \alpha. \quad (2.11)$$

where the parameter $\lambda \in [0, 1]$, measures the degree of optimism or confidence of the decision maker such that:

$$\lambda = \begin{cases} 1 & \text{Total optimistic case} \\ 0 & \text{Total pessimistic case} \\ 0.5 & \text{Neither optimistic nor pessimistic.} \end{cases} \quad (2.12)$$

and $\alpha \in [0, 1]$ is a given threshold of satisfaction fixed by the decision maker. This formula indicates that the weight measure $W(A)$ must be higher than a given threshold α . The higher it is, the greater the constraint will be satisfied.

Secondly, knowing that propagating the uncertainty of multi-objective problem's data through the resolution model leads often to uncertain formulation of objective functions and as in our case the uncertain data are represented by triangular fuzzy numbers, the objective functions will be consequently disrupted by this fuzzy form. Let us assume that, a multi-objective triangular-valued function can be mathematically defined as:

$$F : X \rightarrow Y \subseteq (R \times R \times R)^n, \\ F(x) = \vec{y} = \begin{pmatrix} y_1 = [\underline{y}_1, \hat{y}_1, \overline{y}_1] \\ \vdots \\ y_n = [\underline{y}_n, \hat{y}_n, \overline{y}_n] \end{pmatrix} \quad (2.13)$$

Clearly, in this case, the classical multi-objective techniques cannot be applied since they are only meant for deterministic case. Therefore, a need for special optimization methods techniques to handle the generated triangular-valued functions is evident. To this end, we first introduce a new Pareto dominance over triangular fuzzy numbers, in both mono-objective and multi-objective cases.

2.4.3 New Pareto Optimality over Triangular Fuzzy Numbers

In this section, we first present new mono-objective dominance relations between two TFNs. Then, based on these mono-objective dominance, we define a new Pareto dominance between vectors of TFNs, for multi-objective case. Note that, the minimization sense is considered in all our definitions.

2.4.3.1 Mono-Objective Dominance Relations

In the mono-objective case, three dominance relations over triangular fuzzy numbers are defined: Total dominance (\prec_t), Partial strong-dominance (\prec_s) and Partial weak-dominance (\prec_w).

Definition 1 Total Dominance

Let $y = [\underline{y}, \hat{y}, \bar{y}] \subseteq \mathbb{R}$ and $y' = [\underline{y'}, \hat{y'}, \bar{y'}] \subseteq \mathbb{R}$ be two triangular fuzzy numbers. y dominates y' totally or certainly (denoted by $y \prec_t y'$) if: $\bar{y} < \underline{y'}$.

This dominance relation represents the fuzzy disjoint situation between two triangular fuzzy numbers and it imposes that the upper bound of y is strictly inferior than the lower bound of y' as shown by case (1) in Fig. 2.4.

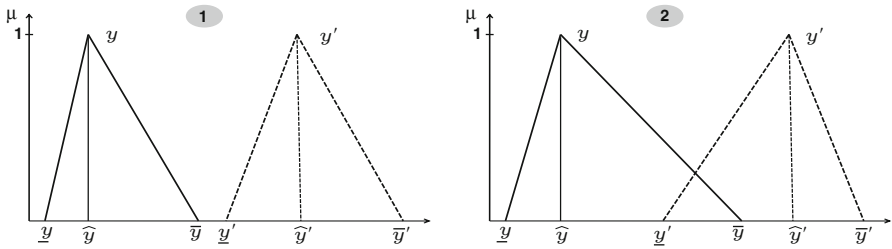


Fig. 2.4 Total dominance and partial strong-dominance

Definition 2 Partial Strong-Dominance

Let $y = [\underline{y}, \hat{y}, \bar{y}] \subseteq \mathbb{R}$ and $y' = [\underline{y'}, \hat{y'}, \bar{y'}] \subseteq \mathbb{R}$ be two triangular fuzzy numbers. y strong dominates y' partially or uncertainly (denoted by $y \prec_s y'$) if:

$$(\bar{y} \geq \underline{y'}) \wedge (\hat{y} \leq \underline{y'}) \wedge (\bar{y} \leq \bar{y'}).$$

This dominance relation appears when there is a fuzzy weak-overlapping between both triangles and it imposes that firstly there is at most one intersection between them and secondly this intersection should not exceed the interval of their kernel values $[\hat{y}, \hat{y'}]$, as shown by case (2) in Fig. 2.4.

Definition 3 Partial Weak-Dominance

Let $y = [\underline{y}, \hat{y}, \bar{y}] \subseteq \mathbb{R}$ and $y' = [\underline{y'}, \hat{y'}, \bar{y'}] \subseteq \mathbb{R}$ be two triangular fuzzy numbers. y weak dominates y' partially or uncertainly (denoted by $y \prec_w y'$) if:

1. Fuzzy overlapping

$$[(\underline{y} < \underline{y'}) \wedge (\bar{y} < \bar{y'})] \wedge [((\hat{y} \leq \underline{y'}) \wedge (\bar{y} > \bar{y'})) \vee ((\hat{y} > \underline{y'}) \wedge (\bar{y} \leq \bar{y'})) \vee ((\hat{y} > \underline{y'}) \wedge (\bar{y} > \bar{y'}))].$$

2. Fuzzy Inclusion

$$(\underline{y} < \underline{y'}) \wedge (\bar{y} \geq \bar{y'}).$$

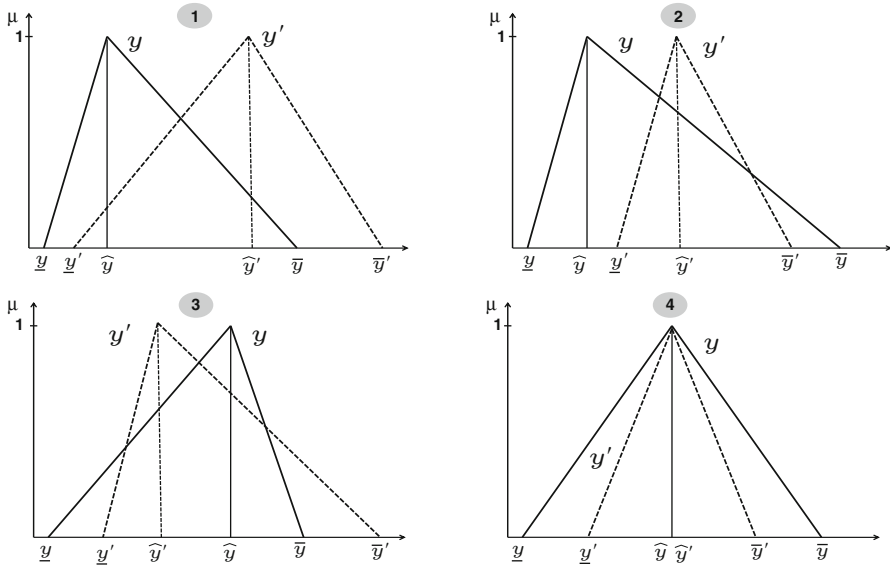


Fig. 2.5 Partial weak-dominance

In this dominance relation, the two situations of fuzzy overlapping and inclusion may occur. Figure 2.5 presents four examples of possible cases, where in (1) and (3) y and y' are overlapped, while, in (2) and (4) y' is included in y .

Yet, the partial weak-dominance relation cannot discriminate all possible cases and leads often to some incomparable situations as for cases (3) and (4) in Fig. 2.5. These incomparable situations can be distinguished according to the kernel value positions in fuzzy triangles. Thus, we propose to consider the kernel values configuration as condition to identify the cases of incomparability, as follows:

$$\hat{y} - \hat{y}' = \begin{cases} < 0, & y \prec_w y' \\ \geq 0, & y \text{ and } y' \text{ can be incomparable.} \end{cases}$$

Subsequently, to handle the identified incomparable situations (with kernel condition $\hat{y} - \hat{y}' \geq 0$), we introduce another comparison criterion, which consists in comparing the discard between both fuzzy triangles as follows:

$$y \prec_w y' \Leftrightarrow (\underline{y}' - \underline{y}) \leq (\bar{y}' - \bar{y})$$

Similarly, it is obvious that: $y' \prec_w y \Leftrightarrow (\underline{y}' - \underline{y}) > (\bar{y}' - \bar{y})$.

It is easy to check that in the mono-objective case, we obtain a total pre-order between two triangular fuzzy numbers, contrarily to the multi-objective case, where the situation is more complex and it is common to have some cases of indifference.

2.4.3.2 Pareto Dominance Relations

In the multi-objective case, we propose to use the mono-objective dominance relations, defined previously, in order to rank separately the triangular fuzzy solutions of each objective function. Then, depending on the types of mono-objective dominance

founded for all the objectives, we define the Pareto dominance between the vectors of triangular fuzzy solutions. In this context, two Pareto dominance relations: *Strong Pareto dominance* (\prec_{SP}) and *Weak Pareto dominance* (\prec_{WP}) are introduced.

Definition 4 *Strong Pareto Dominance.*

Let \vec{y} and \vec{y}' be two vectors of triangular fuzzy numbers. \vec{y} strong Pareto dominates \vec{y}' (denoted by $\vec{y} \prec_{SP} \vec{y}'$) if:

- (a) $\forall i \in 1, \dots, n : y_i \prec_t y'_i \vee y_i \prec_s y'_i$
- (b) $\exists i \in 1, \dots, n : y_i \prec_t y'_i \wedge \forall j \neq i : y_j \prec_s y'_j$
- (c) $\exists i \in 1, \dots, n : (y_i \prec_t y'_i \vee y_j \prec_s y'_j) \wedge \forall j \neq i : y_j \prec_w y'_j$.

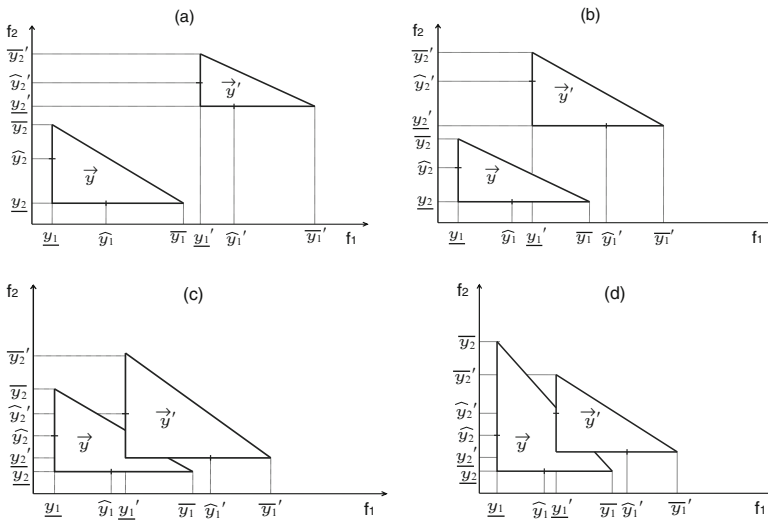


Fig. 2.6 Strong Pareto dominance

The strong Pareto dominance holds if either y_i total dominates or partial strong dominates y'_i in all the objectives (Fig. 2.6a: $y_1 \prec_t y'_1$ and $y_2 \prec_t y'_2$), either y_i total dominates y'_i in one objective and partial strong dominates it in another (Fig. 2.6b: $y_1 \prec_s y'_1$ and $y_2 \prec_t y'_2$), or at least y_i total or partial strong dominates y'_i in one objective and weak dominates it in another (Fig. 2.6c, d: $y_1 \prec_s y'_1$ and $y_2 \prec_w y'_2$).

Definition 5 *Weak Pareto dominance*

Let \vec{y} and \vec{y}' be two vectors of triangular fuzzy numbers. \vec{y} weak Pareto dominates \vec{y}' (denoted by $\vec{y} \prec_{WP} \vec{y}'$) if: $\forall i \in 1, \dots, n : y_i \prec_w y'_i$.

The weak Pareto dominance holds if y_i weak dominates y'_i in all the objectives (Fig. 2.7a). Yet, a case of indifference (defined below) can occur if there is a weak dominance with inclusion type in all the objectives (Fig. 2.7b).

Definition 6 *Case of Indifference*

Two vectors of triangular fuzzy numbers are indifferent or incomparable (denoted by $\vec{y} \parallel \vec{y}'$) if: $\forall i \in 1, \dots, n : y_i \subseteq y'_i$.

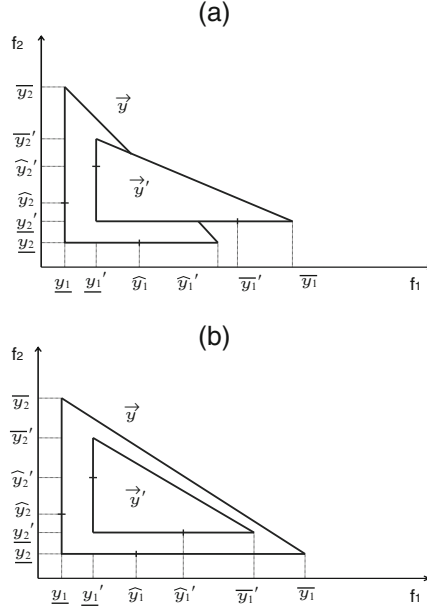


Fig. 2.7 (a) Weak Pareto dominance and (b) Case of indifference

The proposed Pareto dominance in bi-dimensional objective space can easily be generalized for ranking more than two objectives. Note that, if the considered triangular objectives are non-independent, the estimation in a bi-dimensional space can have different distributions (non-triangular) like linear shapes. Finally, the issue now is how integrate this dominance in the research process of multi-objective optimization algorithms.

2.4.4 Extended Optimization Algorithm

In the following, we present an extension of two well-known Pareto-based multi-objective evolutionary algorithms: SPEA2 [36] and NSGAII [9], in order to enable them handling a multi-objective problem with triangular-valued objectives. Both algorithms have proved to be very powerful tools for multi-objective optimization. Due to their population-based nature, they are able to generate multiple optimal solutions in a single run with respect to the good convergence and diversification of obtained solutions. We call our two extended algorithms respectively, ESPEA2—Strength Pareto Evolutionary Algorithm2 and ENSGAII—Non-dominated Sorting Genetic Algorithm II.

2.4.4.1 ESPEA2

SPEA2 is an improved version of the Strength Pareto Evolutionary Algorithm SPEA initially proposed by Zitzler and Thiele [35]. This evolutionary algorithm uses mainly three techniques: a dominance based approach as fitness assignment strategy, a nearest neighbor technique that allows a good diversity preservation and an archive with fixed size that guarantees the elitist storage of optimal solutions. To extend such techniques to triangular fuzzy context, we propose firstly to replace the classical dominance approach by the new Pareto dominance approach proposed for ranking triangular-valued objectives. Secondly, an adaptation of the nearest neighbor technique is introduced. Indeed, in SPEA2, this technique is based on *Euclidean distance* to estimate the density in its neighborhood and it consists in calculating for each solution (objective vector) the distance to its k-nearest neighbor and then adding the reciprocal value to the fitness vector. Yet, as in our case the solutions are triangular objective vectors and knowing that the *Euclidean distance* should be applied only between two exact vectors, we propose to use the expected value as a defuzzification method [32] in order to approximate the considered triangular vectors, such that for each triangular fuzzy number $y_i = [\underline{y}_i, \hat{y}_i, \bar{y}_i]$, the expected value is defined by:

$$E(y_i) = (\underline{y}_i + 2 \times \hat{y}_i + \bar{y}_i) / 4 \quad (2.14)$$

Then, the Euclidean distance between two triangular vectors $\vec{y} = (y_1, \dots, y_n)$ and $\vec{y}' = (y'_1, \dots, y'_n)$ can be applied as follows:

$$D(\vec{y}, \vec{y}') = D(E(\vec{y}), E(\vec{y}')) = \sqrt{\sum_{i=1..n} (E(y_i) - E(y'_i))^2} \quad (2.15)$$

Finally, we adapt the SPEA2 archive to triangular space in order to enable it keeping the obtained triangular solutions. These extensions are integrated into the research process of SPEA2 by modifying the following steps:

- Evaluation: Rank individuals using the new Pareto dominance \prec_{TP} .
- Environmental selection:
 1. Copy all non-dominated individuals having fitness values lower than one in the triangular archive A with fixed size N.
 2. if A is too large ($\text{size}(A) > N$) then, reduce A by means of truncation operator based on Nearest neighbor method to keep only the non-dominated individuals with good spread.
 3. else if A is too small ($\text{size}(A) < N$) then, fill A with the best dominated individuals.
 4. otherwise ($\text{size}(A) = N$), the environmental selection is completed.
- Mating selection: Perform binary tournament selection with replacement on the archive A in order to fill the mating pool.

2.4.4.2 ENSGAI

NSGAI is an extension of an elitism PMOEA called Non-dominated Sorting Genetic Algorithm NSGA, originally proposed by Deb and Srinivas [8]. Unlike the SPEA2 algorithm, NSGAI uses a crowded-comparison operator as diversity preservation technique in order to maintain a uniformly spread front by front. In addition, it does not use an explicit archive for the elitism operation, it only consider the population as a repository to store both elitist and non-elitist solutions. To extend NSGAI to triangular context, we propose at the first step to use the new Pareto dominance between triangular-valued objectives in order to ensure the fitness assignment procedure, in which a dominance depth strategy is applied. At the second stage, we provide an adaptation of the crowded-comparison operator. Indeed, this operator uses the *Crowding Distance* that serves to get a density estimation of individuals surrounding a particular individual in the population. More precisely, the total *Crowding Distance* CD of an individual is the sum of its individual objectives' distances, that in turn are the differences between the individual and its closest neighbors. For the i th objective function y_i , this distance is expressed by:

$$CD(i) = \sum_{i=1..n} (f_{y_i}(i+1) - f_{y_i}(i-1)) / (f_{y_i}^{max} - f_{y_i}^{min}) \quad (2.16)$$

Where f_{y_i} is the fitness value of its neighbors $(i-1)$ and $(i+1)$, $f_{y_i}^{max}$ and $f_{y_i}^{min}$ are respectively the maximum and minimum value of y_i .

However, as in our case, the objective functions are represented by triangular fuzzy values, we propose also to approximate these triangular numbers by calculating their expected values (Eq. (2.14)) before applying the Crowding distance. Finally, it is necessary to adapt both Evaluation and Selection steps in NSGAI, like in SPEA2 algorithm. The distinctive features of NSGAI lie in using the crowding comparison procedure as truncation operator to reduce the population in the environmental selection step and also in considering it as a second selection criteria when two solutions have the same rank in the tournament selection step.

2.5 Application on a Multi-Objective Vehicle Routing Problem

The Vehicle Routing Problem (VRP) is an important combinatorial optimization problem, widely used in a large number of real-life applications [31]. The classical VRP consists in finding optimal routes used by a set of identical vehicles, stationed at a central depot, to serve a given set of customers geographically distributed and with known demands. Through the years, many variants and models derived from the basic VRP have been discussed and examined in the literature. In this work, we are interested in a well-known variant of VRP, the so-called Multi-objective VRP with Time Windows and Uncertain Demands (MO-VRPTW-UD). This variant is

based firstly on the principle of classical VRP, where all the data are deterministic, excepting the customer demands which are uncertain, meaning that the actual demand is only known when the vehicle arrives at the customer location. Several researchers have tried to solve this problem and proposed to deal with the uncertainty of demands using different ways such as probability distributions, dempster belief functions and possibility distributions [1, 13, 28]. In our case, the uncertainty of demands is represented via triangular fuzzy numbers (defined previously) and the objectives to be optimized are respectively, the minimization of the total traveled distance and the total tardiness time.

Formally, a MO-VRPTW-UD may be defined as follows:

Let $G(N,A)$ be a weighted directed graph with an arc set A and a node set $N_i = \{N_0, \dots, N_n\}$ where the node N_0 is the central depot and the other nodes $N_i \neq N_0$ represent the customers. For each customer is associated an uncertain demand dm_i . Only one vehicle k with a limited capacity Q , is allowed to visit each customer. A feasible vehicle route R is represented by the set of served customers, starting and ending at the central depot: $R_k = (N_0, N_1, \dots, N_n, N_0)$. X_{ij}^k denotes the decision variable which is equal to 1 if the vehicle k travels directly from node N_i to node N_j and to 0 otherwise. d_{ij} denotes the symmetric traveled distance between two nodes (N_i, N_j) . This distance is proportional to the corresponding travel time t_{ij} . Figure 2.8 illustrates an example of MO-VRPTW-UD, with a central depot, three vehicles ($V1, V2, V3$) having a maximum capacity $Q = 10$ and a set of eight customers represented by nodes. Each customer $i = 1 \dots 8$ has an uncertain demand expressed in our case by a triangular fuzzy number $dm = [\underline{dm}_i, \widehat{dm}, \overline{dm}]$ (Ex: the fuzzy demand of the customer 1 is $dm_1 = [2, 7, 11]$).

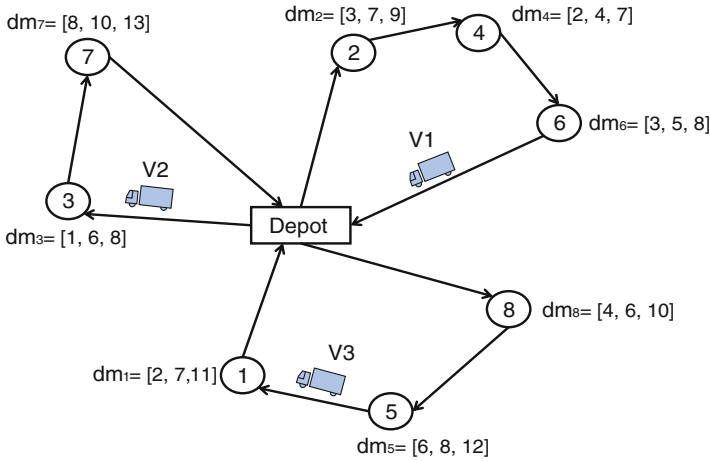


Fig. 2.8 Example of Mo-VRPTW-UD

The main constraints of this problem are: Vehicle capacity constraint, Distance constraint and Time windows constraint.

(1) Vehicle capacity constraint

This constraint imposes that the sum of customer demands in each route must not exceed the limited capacity of associated vehicle. It may be defined as: $\sum_{i=1}^n dm_{N_i} \leq Q$. Yet, as in our case the customers demands are fuzzy values $\widetilde{dm} = [\underline{dm}, \widehat{dm}, \overline{dm}]$, we cannot directly verify if the capacity constraint is satisfied or not and so clearly the constraint satisfaction changes to fuzzy. For example, consider the customer 7 with fuzzy demand $dm_7 = [8, 10, 13]$ shown in Fig. 2.8, we cannot check if dm_7 is lower, equal or higher than $Q = 10$ in order to estimate the transportation costs in terms of time spent and traveled distance. Thus, to handle the satisfaction of this constraint, we propose to use firstly the two measures Π and N of fuzzy constraint satisfaction defined previously. For this example, we obtain $\Pi(dm \leq Q) = 1$ and $N(dm \leq Q) = 0$. Then, by applying the linear combination given by Eq. (2.11) (with for example $\lambda = 0.5$ and $\alpha = 0.2$), we can conclude that the satisfaction of the fuzzy capacity constraint is possible ($W(dm \leq Q) = 0.5 > 0.2$).

(2) Distance constraint

This constraint imposes that each vehicle with a limited capacity Q must deliver goods to the customers according to their uncertain demands dm , with the minimum transportation costs in term of traveled distance. In other words, if the capacity constraint of a vehicle is not satisfied, the delivery fails and causes wasted costs. Therefore, to calculate the traveled distance on a route R defined a priori, three different situations may be found:

- *The demand of a customer is lower than the vehicle capacity ($\sum_{i=1}^f dm_{N_i} < Q$):* In this case, the vehicle will serve the current customer f and then move to the next one $(f + 1)$.
- *The demand of a customer is equal to the vehicle capacity ($\sum_{i=1}^f dm_{N_i} = Q$):* In this case, the priori optimization strategy is used. In fact, the vehicle leaves the depot to serve the first customer f with its total capacity. As it becomes empty, this vehicle will return to the depot to load and serve the next customer $(f + 1)$. Thus, the traveled distance will be: $D(R) = d_{N_0N_1} + \sum_{i=1}^{f-1} d_{N_iN_{i+1}} + d_{N_fN_0} + d_{N_0N_{f+1}} + \sum_{i=f+1}^{n-1} d_{N_iN_{i+1}} + d_{N_nN_0}$.
- *The demand of a customer is higher than the vehicle capacity ($\sum_{i=1}^f dm_{N_i} > Q$):* In this case, the vehicle will serve the customer f with its total capacity ($Q - \sum_{i=1}^{f-1} d_{N_i}$), go to the depot to load, return back to the same customer f to deliver the remaining quantity and then move to the next customer $(f + 1)$. Thus, the traveled distance will be: $D(R) = d_{N_0N_1} + \sum_{i=1}^{n-1} d_{N_iN_{i+1}} + d_{N_fN_0} + d_{N_0N_f} + d_{N_nN_0}$.

Yet as in our case the demands are represented by a triplet of fuzzy values, we propose to calculate separately the distance for each value of the triangular fuzzy demand based on the three situations presented above. Consequently, the traveled distance will be calculated three times and so obtained as triangular number $D = [\underline{D}, \widehat{D}, \overline{D}]$.

(3) Time windows constraint

This constraint imposes that each customer will be served within its time window that represents the interval of time planned for receiving the vehicle service. This means that, if the vehicle arrives too soon, it should wait until the arrival time of its time window to serve the customer, while if it arrives too late (after the fixed departure time), wasted cost in term of tardiness time appears. The time windows constraint uses the following notations:

- The central depot has a time window $[0, l_0]$, meaning that each vehicle that leaves the depot at a time 0 goes back to the depot before the time l_0 .
- Each customer i will be served within his time window $[e_i, l_i]$ by exactly one vehicle, where the lower bound e_i represents the earliest arrival time and the upper bound l_i represents the latest arrival time for the visit of vehicles.
- A waiting time W_i means that the vehicles must arrive before the lower bound of the window e_i .
- A_i, B_i refers respectively to the arrival and the departure times to the customer i .
- Each customer imposes a service time S_i^k that corresponds to the goods loading/unloading time used by the vehicle.
- t_{ij} refers to the travel time from customer i to j .

Firstly, the time needed to serve two consecutive customers i and j is defined as follows:

$$x_{ij}^k(S_i^k + t_{ij} + S_j^k) \leq 0 \quad \text{with} \quad A_i + S_i^k \leq B_i.$$

Besides, a vehicle must arrive at a customer i between the time window $[e_i, l_i]$, but if it arrives before the lower bound e_i , it must wait a while W_i . This waiting time is calculated as follows:

$$W_i = \begin{cases} 0 & \text{if } A_i \geq e_i \\ e_i - A_i & \text{otherwise.} \end{cases}$$

where, the arrival time at customer i is equal to: $A_i = B_{i-1} + t_{i,i-1}$ and the departure time is equal to: $B_i = A_i + W_i + S_i$. While, if the vehicle arrives at a customer i after the upper bound of its time window l_i , a tardiness time must be calculated as follows:

$$T_i = \begin{cases} 0 & \text{if } A_i \leq l_i \\ A_i - l_i & \text{otherwise.} \end{cases}$$

In the case of routes failure, wasted costs in term of tardiness time will appear. Yet, knowing that the travel time depends mainly on the traveled distance and as in our case the obtained distance is a triangular value, the time spent to serve customers will be disrupted by this triangular form and consequently the tardiness time will be

also obtained as triangular fuzzy number $T = [\underline{T}, \hat{T}, \overline{T}]$. Finally, all these constraints combined with the constraints of classical VRP model the MO-VRPTW-UD problem (Fig. 2.8).

To solve the MO-VRPTW-UD problem, the two extended SPEA2 and NSGAII algorithms based on our new Pareto optimality are applied. These algorithms are implemented with the version 1.3-beta of ParadisEO under Linux, especially with the ParadisEO-MOEO module dedicated to multi-objective optimization [3]. Subsequently, to validate the proposed algorithms, we choose to test our VRP application using the Solomon's benchmark, which is considered as a basic reference for the evaluation of several VRP resolution methods [27]. More precisely, six different Solomon's instances are used in our experimentation, namely, C101, C201, R101, R201, RC101 and RC201. Yet, in these instances, all the input values are exact and so the uncertainty of customer demands is not taken into account. At this level, we propose to generate for each instance the triangular fuzzy version of crisp demands in the following manner. Firstly, the kernel value (\widehat{dm}) for each triangular fuzzy demand dm is kept the same as the current crisp demand dm_i of the instance. Then, the lower (\underline{dm}) and upper (\overline{dm}) bounds of this triangular fuzzy demand are uniformly sampled at random in the intervals $[50\%dm, 95\%dm]$ and $[105\%dm, 150\%dm]$, respectively. This fuzzy generation manner ensures the quality and reliability of generated fuzzy numbers. Finally, each of the six sampled fuzzy instances is tested on the both algorithms executed 30 times. Since 30 runs have been performed on each algorithm SPEA2 and NSGAII, we obtained for each instance, 30 sets of optimal solutions that represent the Pareto fronts of our problem. Each solution shows the lowest traveled distance and tardiness time, which are represented by triangular numbers. Examples of two Pareto fronts obtained for one execution of the instance C101 using each algorithm are shown in Figs. 2.10 and 2.11, where the illustrated fronts are composed by a set of triangles, such that each triangle represents one Pareto optimal solution. For instance, the bold triangular (in Fig. 2.10) represents an optimal solution with minimal distance (the green side) equal to [2413, 2515, 2623] and tardiness time (the red side) equal to [284, 312, 295, 280, 315, 322]. Note that, both algorithms converge to optimal fronts approximation in a very short run-time (Approx. 0.91 min for SPEA2 and 2.30 min for NSGAII). However, we cannot compare results with the obtained results of other proposed approaches for solving MO-VRPTW-UD because of incompatibilities between the objectives to be optimized.

To assess the performance of our both algorithms, we propose to use two well-known unary quality indicators:

(i) *Hypervolume Indicator* (I_H) [38], considered one of the few indicators that measures the approximation quality in terms of convergence and diversity simultaneously. This intuitive quality indicator needs the specification of a reference point Z^{Max} that denotes an upper bound over all the objectives and a reference set Z_N^* of non-dominated solutions. In our case, the quality of a given output set A in comparison to Z_N^* is measured using the Hypervolume difference metric I_H^- . As shown in Fig. 2.9, this indicator computes the difference between these two sets by measuring the portion of the objective space weakly dominated by Z_N^* and not by A .

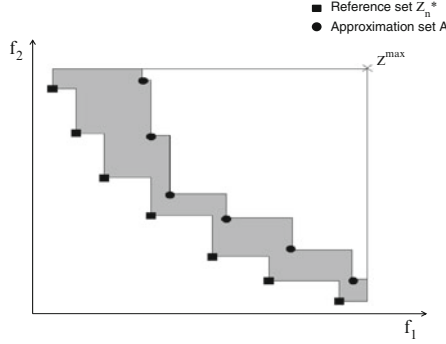


Fig. 2.9 Hypervolume difference indicator

(ii) *Epsilon Indicator* (I_ϵ) [37], dedicated to the measure of approximations quality in term of convergence. More explicitly, this indicator is used to compare non-dominated approximations and not the solutions. In our case, we use the additive ϵ -indicator ($I_{\epsilon+}$) which is a distance based indicator that gives the minimum factor by which an approximation A has to be translated in the criterion space to weakly dominate the reference set Z_N^* . This indicator can be defined as follows:

$$I_{\epsilon+}^1(A) = I_{\epsilon+}(A, Z_N^*) \quad (2.17)$$

where

$$I_{\epsilon+}(A, B) = \min\{\forall z \in B, \exists z' \in A : z'_i - \epsilon \leq z_i, \forall 1 \leq i \leq n\} \quad (2.18)$$

However, these two indicators are only meant to evaluate the quality of deterministic Pareto front approximations. Thus, to enable them evaluating our uncertain approximations (i.e., Triangular fuzzy solutions), we propose to consider the expected values of the triangular solutions (Function) as the sample of values to be used for the qualification of our both algorithms. In other words, the both indicators are simply applied on the samples of expected values computed for each instance. Therefore, as in our case 30 runs per algorithm have been performed, we obtain 30 Hypervolume differences and 30 epsilon measures for each tested sample. Once all these values are computed, we need to use statistical analysis to be able to compare our two algorithms. To this end, we choose to use Wilcoxon statistical test described in [37].

Table 2.2 gives a comparison of SPEA2 and NSGAII algorithms for the six tested instances. This comparison based on the results of I_H^- and $I_{\epsilon+}$ indicators, shows that the SPEA2 algorithm is significantly better than the NSGAII algorithm on all the instances, excepting the instances R201 and RC201, where for I_H^- there is no significant difference between the approximations of both algorithms.

Table 2.2 Algorithms comparison using Wilcoxon test with a P -value=0.5%

Instances	Algorithms	I_H^-		$I_{\varepsilon+}$	
		SPEA2	NSGAII	SPEA2	NSGAII
C101	SPEA2	-	\prec	-	\prec
	NSGAII	\succ	-	\succ	-
C201	SPEA2	-	\prec	-	\prec
	NSGAII	\succ	-	\succ	-
R101	SPEA2	-	\prec	-	\prec
	NSGAII	\succ	-	\succ	-
R201	SPEA2	-	\equiv	-	\prec
	NSGAII	\equiv	-	\succ	-
RC101	SPEA2	-	\prec	-	\prec
	NSGAII	\succ	-	\succ	-
RC201	SPEA2	-	\equiv	-	\prec
	NSGAII	\equiv	-	\succ	-

According to the metric under consideration (I_H^- or $I_{\varepsilon+}$), either the algorithm located at a specific row is significantly better (\prec) than the algorithm located at a specific column, either it is worse (\succ) or there is no significant difference between both (\equiv)

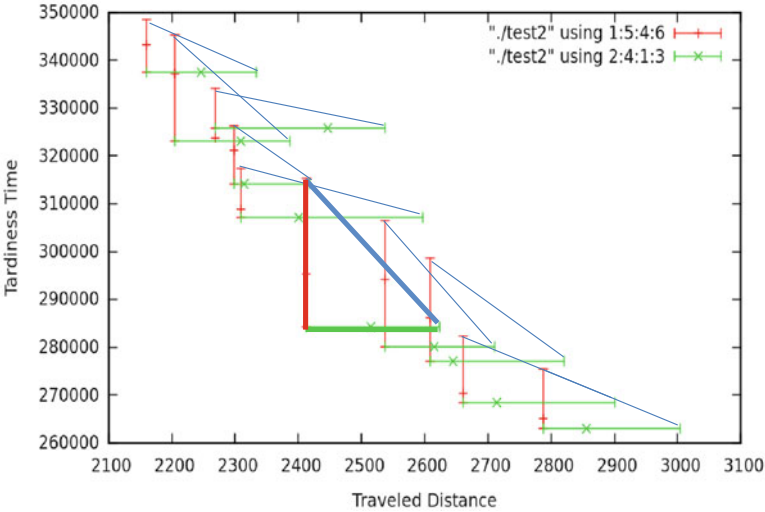


Fig. 2.10 Pareto front (C101-SPEA2)

2.6 Conclusion

This chapter addresses the multi-objective problems with fuzzy data, in particular, with triangular-valued objective functions. To solve such problems, we have proposed an extension of two multi-objective evolutionary algorithms: SPEA2 and NSGAII by integrating a new triangular Pareto dominance. The implemented algorithms have been applied on a multi-objective vehicle routing problem with uncertain demands and then experimentally validated on the Solomon’s benchmark.

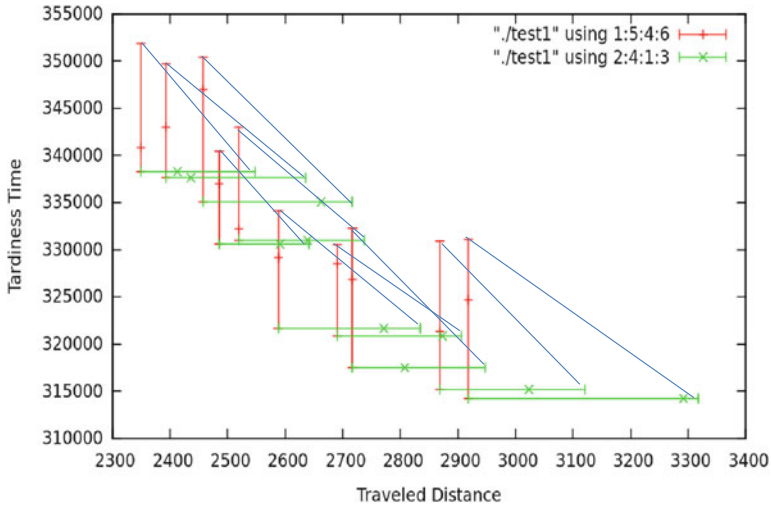


Fig. 2.11 Pareto front (C101-NSGAI)

Subsequently, we have obtained an encouraging results. As a future work, we intend to refine the algorithmic features by introducing for example a new fuzzy distance for the density estimation techniques and to extend the proposed Pareto dominance for ranking other fuzzy shapes like trapezoidal fuzzy numbers. Another perspective will be the extension of multi-objective performance metrics to uncertain context (i.e, fuzzy context).

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