Chapter 2
Selection of Static Supply Portfolio

2.1 Introduction

In a customer-driven supply chain manufacturers should be prepared to produce different products to meet different customer needs. Each product is typically composed of many common and non-common (custom) parts that can be sourced from different suppliers with different supply capacity. An important issue is how to best allocate the orders for parts among various part suppliers to fulfill all customer orders for products and to achieve a high customer service level at a low cost and, in addition, to mitigate the impact of supply chain disruption risks. Supply chain management, in particular, deals with selection of supply portfolio, i.e., selection of suppliers and order quantity allocation under uncertain quality of supplied materials and reliability of on-time delivery. The decision maker needs to decide from which supplier to purchase parts required to meet customer demand. The decision is based on price, quality and reliability criteria that may conflict each other. For example, the supplier offering the lowest price may not have the best quality or the supplier with the best quality may not deliver on time. In stochastic supply settings, supplier selection allows the producer do decide whether it should cooperate with a low cost, yet risky suppliers over more expensive but possibly more reliable suppliers. A common risk-neutral objective of minimizing expected cost or maximizing expected service level is therefore influenced by uncertainty and risk. As a result, new non-risk-neutral objectives of minimizing and maximizing the number of outcomes that could occur above an acceptable cost level and below an acceptable service level, respectively, are observed in practice. Furthermore, to reduce the fixed ordering costs of creating contracts and maintaining relationships with suppliers, the number of suppliers and the total number of orders should be minimized. On the other hand, however, the selection of more suppliers may divert the risk of unreliable supplies. In global supply chains a multi-regional suppliers base is a frequent solution, where suppliers from different geographic regions are selected. Then, in addition to independent local disruptions (i.e., equipment breakdown, fires, etc.) that are uniquely associated with a

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particular supplier, the supplies of parts are also subject to regional disruptions (e.g., floods, hurricanes, earthquakes, economic crisis, etc.) simultaneously of all suppliers in the same region.

This chapter deals with selection of a static supply portfolio under disruption risks, i.e., for determining a single-period supply portfolio. The proposed portfolio approach allows the two popular in financial engineering percentile measures of risk, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) to be applied for managing the risk of supply disruptions. For a finite number of scenarios, CVaR allows the evaluation of worst-case costs (or worst-case service level) and shaping of the resulting cost (service level) distribution through optimal supplier selection and order allocation decisions, i.e., the selection of optimal supply portfolio. Since common parts can be efficiently managed by material requirement planning methods, the focus in this chapter is on supplies of custom parts that can be critical in a make-to-order manufacturing.

The following SMIP models are presented in this chapter:

SP_E(c) for risk-neutral selection of supply portfolio to minimize expected cost;
SP_E(sl) for risk-neutral selection of supply portfolio to maximize expected service level;
SP_CV(c) for risk-averse selection of supply portfolio to minimize CVaR of cost;
SP_CV(sl) for risk-averse selection of supply portfolio to maximize CVaR of service level;
SP_ECV(c) for mean-risk selection of supply portfolio to optimize trade-off between expected cost and CVaR of cost;
SP_ECV(sl) for mean-risk selection of supply portfolio to optimize trade-off between expected service level and CVaR of service level.

In the computational experiments described in Sect. 2.6, single-region and multi-region sourcing subject to local and regional disruption risks are illustrated with numerical examples.

In the next chapter the portfolio approach will be to enhanced for a multi-period supplier selection and order allocation in the presence of supply chain disruption and delay risks, where in the scenario analysis the low probability and high impact supply disruptions are combined with the high probability and low impact supply delays. Unlike for a single-period problem considered in this chapter, in a multi-period setting the decision maker needs to decide from which supplier and when to purchase custom parts required for each customer order to meet customer requested due dates at a low cost and a high customer service level and to mitigate the impact of disruption and delay risks.

2.2 Problem Description

In the supply chain under consideration various types of products are assembled by a single producer to satisfy customer orders, using custom parts purchased from multiple suppliers (for notation used, see Table 2.1). Each supplier can provide the producer with custom parts for all customer orders. However, the suppliers have different limited capacity and, in addition, differ in price and quality of offered parts and in reliability of delivery of parts. Let $I = \{1, \ldots, \overline{I}\}$ be the set of $\overline{I}$ suppliers and $J = \{1, \ldots, \overline{J}\}$ the set of $\overline{J}$ customer orders for the products, known ahead of time.

Table 2.1 Notation: static supply portfolio

	Indices
i	= supplier, $i \in I$
j	= customer order, $j \in J$
r	= geographic region, $r \in R$
s	= disruption scenario, $s \in S$
	Input Parameters
c_i	= capacity of supplier i
d_j	= demand for parts required for customer order j
D	= $\sum_{j \in J} d_j$ - total demand for parts
e_i	= cost of ordering parts from supplier i
h_j	= per unit penalty cost of unfulfilled customer order j
o_{ij}	= unit price of parts for customer order j purchased from supplier i
p_i	= local disruption probability for supplier i
p^r	= regional disruption probability for all suppliers in region r
α	= confidence level
ρ_i	= expected defect rate of supplier i

Each order $j \in J$ is described by the quantity d_j of required custom parts. Denote by c_i the capacity of supplier $i \in I$, by e_i the cost of ordering parts from supplier $i \in I$, and by o_{ij} the unit purchasing price of parts for customer order $j \in J$ from supplier $i \in I$.

The ordered parts are dispatched to the producer after the completion time of their manufacturing. For each supplier, however, the quality of delivered part may vary randomly. When the suppliers are selected the risk of defective parts can be considered using past observations. Since quality of parts vary among different suppliers, a different average defect rate can be associated with each supply portfolio. Let ρ_i be the expected defect rate of supplier i.

The suppliers are assumed to be located in $\overline{R}$ disjoint geographic regions. Denote by $I^r \subseteq I$ the subset of suppliers in region $r \in R = \{1, \ldots, \overline{R}\}$, where $\bigcup_{r \in R} I^r = I$. The supplies of parts are subject to random local disruptions that are uniquely associated with a particular supplier, which may arise from equipment breakdowns, local labor strike, fires, etc. Denote by p_i the local disruption probability for supplier i, i.e., the parts ordered from supplier i are delivered without disruptions with probability $(1 - p_i)$, or not at all with probability p_i. In addition to independent local disruptions of each supplier individually, the supplies of parts are also subject to regional correlated disruption of all suppliers in the same region simultaneously, with probability p^r for region $r \in R$.

Denote by π_i the disruption probability of every supplier $i \in I^r, r \in R$

$$\pi_i = p^r + (1 - p^r)p_i; \; i \in I^r, r \in R. \tag{2.1}$$

Let $S = \{1, \dots, \overline{S}\}$ be the index set of $\overline{S} = 2^{\overline{I}}$ disruption scenarios, where each scenario $s \in S$ defines a subset $I_s \subset I$ of non-disrupted suppliers. The supplies from every supplier, $i \in I \setminus I_s$, can be independently disrupted either by a local or by a regional disaster event. The probability P_s of each disruption scenario $s \in S$ is a product over all regions $r \in R$ of probabilities P_s^r of realizing disruption scenario s for suppliers in I^r,

$$P_s = \prod_{r \in R} P_s^r, \tag{2.2}$$

where P_s^r is (cf. Sect. 1.3)

$$P_s^r = \begin{cases} (1 - p^r) \prod_{i \in I^r \bigcap I_s} (1 - p_i) \prod_{i \in I^r \setminus I_s} p_i & \text{if } I^r \bigcap I_s \neq \emptyset \\ p^r + (1 - p^r) \prod_{i \in I^r} p_i & \text{if } I^r \bigcap I_s = \emptyset. \end{cases} \tag{2.3}$$

The producer does not need to pay for ordered and defective or undelivered parts. However, the producer may be charged with a much higher cost of unfulfilled customer orders for products, caused by the shortage of parts, undelivered due to supply disruptions. Let h_j be the per unit penalty cost of unfulfilled customer order j.

The decision maker needs to decide from which suppliers to purchase custom parts required for each customer order to achieve a minimum cost of ordering, purchasing and shortages and to mitigate the impact of disruption risks by minimizing the potential worst-case cost or maximizing the potential worst-case service level.

2.3 Models for Risk-Neutral Decision-Making

In this section two SMIP models **SP_E(c)** and **SP_E(sl)** are proposed for risk-neutral selection of a static supply portfolio, i.e., for determining a single-period supply portfolio to minimize expected cost and maximize expected service level, respectively. The static supply portfolio is defined below, (for definition of problem variables, see Table 2.2).

$$(V_1, \dots, V_{\overline{I}}),$$

where

$$\sum_{i \in I} V_i = 1$$

and $0 \leq V_i \leq 1$ is the fraction of the total demand for parts ordered from supplier i, and V_i is determined by the custom parts allocation variables v_{ij}

Table 2.2 Variables: static supply portfolio

		First stage variables
u_i	=	1, if an order for parts is placed on supplier i; otherwise $u_i = 0$ (supplier selection)
v_{ij}	=	the fraction of demand for parts required for customer order j ordered from supplier i (allocation of demand for custom parts)
		Auxiliary variables
V_i	=	the fraction of total demand for parts allocated to supplier i (supply portfolio: allocation of total demand for parts)
VaR^c		Cost-at-Risk, the targeted cost such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is below VaR^c
VaR^{sl}		Service-at-Risk, the targeted service level such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is above VaR^{sl}
$\mathscr{C}_s$	$\geq$	0, the tail cost for scenario s, i.e., the amount by which costs in scenario s exceed VaR^c
$\mathscr{S}_s$	$\geq$	0, the tail service level for scenario s, i.e., the amount by which VaR^{sl} exceeds service level in scenario s

$$V_i = \sum_{j \in J} d_j v_{ij}/D; \;\; i \in I. \tag{2.4}$$

When deciding on a static supply portfolio it is assumed that the orders for all parts are simultaneously placed on selected suppliers (e.g., at time 0), and each supplier delivers all the ordered parts at the earliest possible delivery date. Therefore, the allocation of orders for parts among the suppliers is not combined with the allocation of orders among the planning periods. Nevertheless, the static portfolio should be checked against the risk of supply disruptions across all potential disruption scenarios.

Notice that Table 2.2 does not explicitly define the second stage variables for the SMIP problem considered. The seconds stage variables are simply demand allocation variables for realized disruption scenarios s, $\tilde{v}_{ij}^s$; $i \in I, j \in J, s \in S$, defined as follows

$$\tilde{v}_{ij}^s = \begin{cases} v_{ij} & \text{if } \; i \in I_s, j \in J, s \in S \\ 0 & \text{if } \; i \notin I_s, j \in J, s \in S. \end{cases}$$

In view of the above definition, an explicit introduction of the second stage variables $\tilde{v}_{ij}^s$ into the SMIP model formulations is not required.

In a risk-neutral operating conditions the overall quality of the supply portfolio can be measured by the expected cost per part, E^c, (2.5), or expected service level E^{sl}, (2.6).

$$E^c = \sum_{i\in I} e_i u_i/D + \sum_{i\in I}\sum_{j\in J} o_{ij} d_j v_{ij}/D$$
$$+\sum_{s\in S}\sum_{i\notin I_s}\sum_{j\in J} P_s(h_j - o_{ij}) d_j v_{ij}/D \tag{2.5}$$
$$E^{sl} = \sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J} P_s d_j v_{ij}/D \tag{2.6}$$

The expected cost E^c includes, cost of ordering,
$\sum_{i\in I} e_i u_i/D$,
cost of purchasing non defective parts,
$\sum_{i\in I}\sum_{j\in J} o_{ij} d_j v_{ij}/D$,
and cost of shortage of parts due to supply disruptions (cost of unfulfilled customer orders less cost of non delivered parts),
$\sum_{s\in S}\sum_{i\notin I_s}\sum_{j\in J} P_s(h_j - o_{ij}) d_j v_{ij}/D$.
The purchase orders for parts are assumed to be inflated by the reject rates ρ_i of defective parts, i.e., are equal to $(1+\rho_i)d_j v_{ij}$ for all $i \in I, j \in J$. However, since the producer does not need to pay for ordered and defective parts in the amount of $\rho_i d_j v_{ij}$, the corresponding purchasing cost per part for delivered parts is simply given by $\sum_{i\in I}\sum_{j\in J} o_{ij} d_j v_{ij}/D - \sum_{s\in S}\sum_{i\notin I_s}\sum_{j\in J} P_s o_{ij} d_j v_{ij}/D$.

The expected cost per part, E^c, (2.5), can also be written as follows

$$E^c = \sum_{i\in I} e_i u_i/D + \sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J} P_s o_{ij} d_j v_{ij}/D$$
$$+\sum_{s\in S}\sum_{i\notin I_s}\sum_{j\in J} P_s h_j d_j v_{ij}/D, \tag{2.7}$$

where $\sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J} P_s o_{ij} d_j v_{ij}/D$ is the expected purchasing cost per part for delivered parts.

The expected service level, E^{sl}, (2.6), is a surrogate measure of expected customer demand fulfillment rate and represents the expected fraction of fulfilled demand for required parts.

The SMIP models **SP_E(c)** and **SP_E(sl)** are formulated below. The supply portfolio will be optimized by minimizing expected cost per part, E^c, (2.5) or by maximizing expected service level, E^{sl}, (2.6).

SP_E(c): *Selection of risk-neutral* **S***upply* **P***ortfolio to minimize expected cost*

Minimize (2.5)

subject to

1. Supply portfolio selection constraints:

- the total demand for parts required for each customer order must be fully allocated among suppliers,

- for each selected supplier the total quantity of ordered parts cannot exceed the supplier capacity,
- parts cannot be ordered from non-selected suppliers,
- at least one customer order should be assigned to each selected supplier,

$$\sum_{i \in I} v_{ij} = 1; \; j \in J \tag{2.8}$$

$$\sum_{j \in J} (1 + \rho_i) d_j v_{ij} \le c_i u_i; \; i \in I \tag{2.9}$$

$$v_{ij} \le u_i; \; i \in I, j \in J \tag{2.10}$$

$$\sum_{j \in J} v_{ij} \ge u_i; \; i \in I \tag{2.11}$$

2. Non-negativity and integrality conditions

$$u_i \in \{0, 1\}; \; i \in I \tag{2.12}$$

$$v_{ij} \in [0, 1]; \; i \in I, j \in J. \tag{2.13}$$

Notice that if $h_j = h \forall j \in J$, i.e., per unit penalty cost of unfulfilled customer order is identical for all orders j, then E^c, (2.7), can be expressed by the following simplified formula

$$E^c = \sum_{i \in I} e_i u_i / D + \sum_{s \in S} \sum_{i \in I_s} \sum_{j \in J} P_s o_{ij} d_j v_{ij} / D + h(1 - E^{sl}), \tag{2.14}$$

where E^{sl} is the expected service level (2.6).

SP_E(sl): *Selection of risk-neutral supply portfolio to maximize expected service level*

Maximize (2.6)
subject to (2.8)–(2.13).

If total available capacity of all suppliers is less than total demand for required parts, i.e., $\sum_{i \in I} c_i/(1 + \rho_i) \le \sum_{j \in J} d_j$, then the demand allocation equality constraints (2.8) should be replaced by inequalities

$$\sum_{i \in I} v_{ij} \le 1; \; j \in J; \tag{2.15}$$

otherwise no feasible solution exists.

A simple upper bound on the expected service level, E^{sl}, (2.6), is derived below.

Proposition 2.1

$$E^{sl} \leq \min\{1, \sum_{r\in R}\sum_{i\in I^r}(1-p^r)(1-p_i)c_i/(1+\rho_i)D\}. \tag{2.16}$$

Proof The supply portfolio selection constraints (2.9) imply that

$$\begin{aligned} \sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J} P_s d_j v_{ij}/D \leq \\ \sum_{r\in R}\sum_{i\in I^r}\sum_{j\in J}(1-p^r)(1-p_i)d_j v_{ij}/D \leq \\ \sum_{r\in R}\sum_{i\in I^r}(1-p^r)(1-p_i)c_i u_i/(1+\rho_i)D \leq \\ \sum_{r\in R}\sum_{i\in I^r}(1-p^r)(1-p_i)c_i/(1+\rho_i)D, \end{aligned}$$

where $(1-p^r)(1-p_i) = 1-\pi_i$, (2.1), is non-disruption probability of supplier $i \in I^r$.

Since E^{sl} cannot be greater than 1, its upper bound is 1, if $\sum_{r\in R}\sum_{i\in I^r}(1-p^r)(1-p_i)c_i/(1+\rho_i)D > 1$.

In the proposed models the parts required for each customer order are assumed to be partially provided by one or more suppliers and the customer order allocation variable v_{ij} represents the fraction of all parts required for order j provided by supplier i. In some practical cases, all custom parts of the same type that are required for a customer order are purchased from a single supplier. Then, the corresponding continuous allocation variable v_{ij} should be redefined as a binary assignment variable denoting whether or not all parts required for order j are provided by supplier i. If all v_{ij} are defined to be binary variables, then **SP_E(c)** and **SP_E(sl)** become pure stochastic binary programs.

2.4 Models for Risk-Averse Decision-Making

In the risk-averse selection of supply portfolio under disruption risks, the confidence level α is fixed by the decision maker to control the risk of losses due to supply disruptions. We assume that the decision maker is willing to accept only portfolios for which the total probability of scenarios with costs greater than VaR^c or with service level lower than VaR^{sl} is not greater than $1-\alpha$. Furthermore, a risk aversive decision maker wants to minimize the expected worst-case costs exceeding VaR^c or to maximize the expected worst-case service level below VaR^{sl}.

Define by $\mathscr{C}_s$ the tail cost for scenario s, where tail cost is defined as the amount by which costs in scenario s exceed VaR^c. In a similar way, define by $\mathscr{S}_s$ the tail service level for scenario s, where tail service level is defined as the nonnegative amount by which VaR^{sl} exceeds service level in scenario s.

The portfolio will be optimized by calculating VaR^c and minimizing CVaR^c simultaneously or by calculating VaR^{sl} and maximizing CVaR^{sl}, respectively. By measuring CVaR^c or CVaR^{sl}, the magnitude of the tail costs or the tail service level is considered to achieve a more accurate estimate of the risks of minimizing cost or maximizing service level, respectively. When using CVaR^c to minimize worst-case costs and CVaR^{sl} to maximize worst-case service level, CVaR^c is always not less than VaR^c and CVaR^{sl} is always not greater than VaR^{sl}, respectively.

In the proposed model CVaR is represented by an auxiliary function (2.17) and (2.20) introduced by Rockafellar and Uryasev (2000). The SMIP models **SP_CV(c)** and **SP_CV(sl)** for selection of risk-averse supply portfolio to reduce the risk of high costs and the risk of low service level, respectively, is formulated below.

SP_CV(c): *Selection of risk-averse supply portfolio to minimize CVaR of cost*

Minimize

$$CVaR^c = VaR^c + (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{C}_s \tag{2.17}$$

subject to

1. Supply portfolio selection constraints: (2.8)–(2.11)

2. Risk constraints:

- the tail cost for scenario s is defined as the nonnegative amount by which cost per part in scenario s exceeds VaR^c,

$$\mathscr{C}_s \geq \sum_{i \in I} e_i u_i / D + \sum_{i \in I} \sum_{j \in J} o_{ij} d_j v_{ij} / D + \sum_{i \notin I_s} \sum_{j \in J} (h_j - o_{ij}) d_j v_{ij} / D - VaR^c; \;\; s \in S \tag{2.18}$$

3. Non-negativity and integrality conditions: (2.12), (2.13) and

$$\mathscr{C}_s \geq 0; \;\; s \in S. \tag{2.19}$$

SP_CV(sl): *Selection of risk-averse supply portfolio to maximize CVaR of service level*

Maximize

$$CVaR^{sl} = VaR^{sl} - (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{S}_s \tag{2.20}$$

subject to

1. Supply portfolio selection constraints: (2.8)–(2.11)

2. Risk constraints:

- the tail service level for scenario s is defined as the nonnegative amount by which VaR^{sl} exceeds service level in scenario s,

$$\mathscr{S}_s \geq VaR^{sl} - \sum_{i \in I_s} \sum_{j \in J} d_j v_{ij}/D; \;\; s \in S \tag{2.21}$$

3. Non-negativity and integrality conditions: (2.12), (2.13) and

$$\mathscr{S}_s \geq 0; \;\; s \in S. \tag{2.22}$$

Note that as $\mathscr{C}_s$ and $\mathscr{S}_s$ are constrained of being positive, the model **SP_CV(c)** tries to decrease VaR^c and the model **SP_CV(sl)** tries to increase VaR^{sl}, respectively. Hence they positively impact the objective functions. However, large reduction in VaR^c and large increase in VaR^{sl} may result in more scenarios with positive tail costs and with positive tail service levels, respectively.

If for some customer order j all required parts must be supplied by a single supplier, then the corresponding nonnegative allocation variable v_{ij} should be redefined as a binary assignment variable denoting whether or not all parts required for order j are provided by supplier i, similarly as for the risk-neutral models **SP_E(c)** and **SP_E(sl)**.

2.5 Models for Mean-Risk Decision-Making

In the single objective approach the supply portfolio is selected by minimizing either the expected cost per part, E^c, (2.5), the expected service level, E^{sl}, (2.6), the expected worst-case cost per part, $CVaR^c$, (2.17) or the expected worst-case service level, $CVaR^{sl}$, (2.20). In this section the two cost functions and the two service level functions are considered simultaneously, and a bi-objective selection of supply portfolio is presented aimed at minimizing both objective functions to balance expected costs or expected service level with the risk tolerance. This trade-off model is known as the mean-risk model (e.g., Ogryczak and Ruszczynski 2002), formulated as the optimization of a composite objective consisting of the expected cost (service level) and the CVaR as a risk measure.

The nondominated solution set of the bi-objective supply portfolio can be found by the parameterization on λ the weighted-sum programs **SP_ECV(c)** and **SP_ECV(sl)** presented below. The mean-risk program **SP_ECV(c)** is based on model **SP_CV(c)** with the addition of objective (2.5) of model **SP_E(c)**. Similarly, the mean-risk program **SP_ECV(sl)** is based on model **SP_CV(sl)** with the addition of objective (2.6) of model **SP_E(sl)**.

SP_ECV(c): *Selection of mean-risk supply portfolio to minimize weighted sum of expected cost and CVaR of cost*

Minimize

$$\lambda E^c + (1-\lambda)CVaR^c \tag{2.23}$$

where $0 \leq \lambda \leq 1$,
subject to (2.5), (2.8)–(2.13), (2.17)–(2.19).

SP_ECV(sl): *Selection of mean-risk supply portfolio to maximize weighted sum of expected service level and CVaR of service level*

Maximize

$$\lambda E^{sl} + (1-\lambda)CVaR^{sl} \tag{2.24}$$

where $0 \leq \lambda \leq 1$,
subject to (2.6), (2.8)–(2.13), (2.20)–(2.22).

Steuer (1996) proved that for mixed integer programs, there may be portions of the nondominated set (nearby weakly nondominated solution) that the above approach is unable to compute, even if the complete parameterization on λ is attempted.

2.6 Computational Examples

In this section some computational examples are presented to illustrate possible applications of the proposed SMIP approach for selection of static supply portfolio under disruption risks. First, a single-region sourcing case will be illustrated, where all suppliers are located in a single geographic region, and then examples of multi-region sourcing with subsets of suppliers in different geographic regions, each subject to different regional disruption risks. For the single-region sourcing, minimization of cost is considered only, whereas for the multi-region sourcing, both minimization of cost and maximization of service level are considered.

2.6.1 Single-Region Sourcing

In this subsection all suppliers are assumed to be located in the same geographic region, and hence the regional disruption can be called a global disruption. Denote by, p^*, the global disruption probability for the entire region. Now, the probability, P_s (2.3), of each disruption scenario s, can be calculated using the following formula

$$P_s = \begin{cases} (1-p^*)\hat{P}_s & \text{if } \ I_s \neq \emptyset \\ p^* + (1-p^*)\prod_{i\in I} p_i & \text{if } \ I_s = \emptyset, \end{cases} \tag{2.25}$$

where $\hat{P}_s$ is the probability of disruption scenario s in the presence of independent local disruptive events only

$$\hat{P}_s = \prod_{i\in I_s}(1-p_i) \cdot \prod_{i\notin I_s} p_i. \tag{2.26}$$

If the probability of regional disruption $p^* = 0$, then the probability P_s reduces to $\hat{P}_s$ for independent local disruptive events.

The following parameters have been used for the example problems:

- $\overline{I}$, the number of suppliers, was equal to 7, 10 or 14 and the corresponding number $\overline{S} = 2^{\overline{I}}$ of disruption scenarios, was equal to 128, 1024 or 16384, respectively;
- $\overline{J}$, the number of customer orders, was equal to 50;
- d_j, the numbers of required parts for each customer order, were integers uniformly distributed over [100, 500], i.e., generated from a U[100;500] distribution;
- c_i, the capacity of each supplier i, was equal to $\lceil 2\sum_{j\in J} d_j/\overline{I}\rceil$ ($\lceil\cdot\rceil$ denotes the smallest integer not less than $\cdot$), i.e., the total capacity of all suppliers was equal to the double total demand for parts;
- e_i, the cost of ordering parts from supplier i, was equal to 500 for each supplier i;
- h_j, the per unit shortage cost for customer order j, was equal to 100 for all customer orders j;
- ρ_i, the expected defect rate of each supplier i, was exponentially distributed, ranging from 0.0003 to 0.03;
- o_{ij}, the unit price of parts required for each customer order j purchased from each supplier i, was uniformly distributed over [10,15] (i.e., drawn from U[10;15]) and reduced by the factor $(1-\rho_i)$ to get a lower price for parts from the suppliers with a higher defect rate;
- p_i, the local disruption probability was uniformly distributed over [0,0.06], i.e., the disruption probabilities were drawn independently from U[0;0.06];
- p^*, the global disruption probability was equal to 0.01;
- α, the confidence level, was equal to 0.50, 0.75, 0.90, 0.95 or 0.99.

For the example problems, the total demand for parts is $D = \sum_{j\in J} d_j = 14750$ parts. Solution results for the risk-neutral model **SP_E(c)** are shown in Table 2.3, and for the risk-averse model **SP_CV(c)** with different confidence levels, in Table 2.4. The size of the mixed integer programs for different number $\bar{I}$ of suppliers is represented by the total number of variables, Var., number of binary variables, Bin., number of constraints, Cons, and number of nonzero coefficients in the constraint matrix, Nonz. Table 2.4 also presents the probability $1 - F(VaR^c)$ of outcomes with worst-case cost above VaR^c. Note that the number of variables and constraints in the mixed integer program **SP_CV** grows exponentially in the number $\bar{I}$ of suppliers. The table demonstrate that the number of selected suppliers increases with the confidence level α, which indicates that the impact of disruption risks is mitigated by diversification of the supply portfolio. Note that VaR^c becomes smaller than expected cost when $\alpha = 0.50$ and $\alpha = 0.75$.

Table 2.3 Risk-neutral solutions for model **SP_E(c)**: single-region sourcing

No. of Suppliers	Expected Cost	No. of Selected Suppliers
7	12.38	4
Var. = 364, Bin. = 7, Cons. = 72, Nonz. = 1428 [a]		
10	13.18	5
Var. = 520, Bin. = 10, Cons. = 81, Nonz. = 2040 [a]		
14	12.03	7
Var. = 728, Bin. = 14, Cons. = 93, Nonz. = 2856 [a]		

[a] Var. = no. of variables, Bin. = no. of binary variables, Cons. = no. of constraints, Nonz. = no. of nonzero coefficients

The optimal risk-neutral supply portfolio for model **SP_E(c)** and 10 suppliers is shown in Fig. 2.1. In addition, the figure presents for each supplier i the expected defect rate ρ_i, the average unit price $\sum_{j\in J} o_{ij}/\bar{J}$, and disruption probabilities, π_i, (2.1). For the optimal supply portfolio the total demand was equally allocated among five suppliers with the lowest disruption probabilities.

In the computational experiments the confidence level α is set at five levels of 0.5, 0.75, 0.90, 0.95, and 0.99, which means that focus is on minimizing the highest 50%, 25%, 10%, 5%, and 1% of all scenario outcomes, i.e., costs per part.

Figure 2.2 shows the probability mass functions and the cumulative distribution functions for the optimal risk-averse portfolios with different confidence levels for 10 suppliers. Figure 2.2 indicates that the mass function of cost per part is concentrated in a few points and the resulting cumulative distribution is a discontinues step function with jumps at those points. Such results are typical for the scenario-based optimization under uncertainty, where the probability measure is concentrated in finitely many points. The resulting discontinuity (vertical jumps) of the distribution function leads to probability intervals of confidence level α with the same VaR. The discrete distributions of cost per part for the optimal supply portfolios with four different confidence levels and the corresponding probabilities concentrated at each level of cost are presented also in Table 2.5. The table shows that the probabilities are concentrated at 6, 10, 10, 10 points, respectively for the confidence level $\alpha =$ 0.5, 0.9, 0.989, 0.99. In the examples, a large probability atom is concentrated at the highest cost. As a consequence, a slight increase of the confidence level from

Table 2.4 Risk-averse solutions: single-region sourcing

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SP_CV(c)**: 7 suppliers					
Var. = 493, Bin. = 7, Cons. = 200, Nonz. = 47380 [(a)]					
CVaR^c	14.15	17.69	28.21	39.01	100.14
VaR^c	10.61	10.61	13.90	21.19	100.14
E^c	12.38	12.38	12.59	12.94	14.24
$1 - F(VaR^c)$	0.053	0.053	0.039	0.018	0
No. of suppliers selected	4	4	7	7	4
Model **SP_CV(c)**: 10 suppliers					
Var. = 1545, Bin. = 10, Cons. = 1115, Nonz. = 526338 [(a)]					
CVaR^c	16.02	21.69	31.67	42.05	100.17
VaR^c	10.35	10.35	20.29	23.19	100.17
E^c	13.18	13.18	14.06	13.96	15.89
$1 - F(VaR^c)$	0.113	0.113	0.041	0.032	0
No. of suppliers selected	5	5	9	10	5
Model **SP_CV(c)**: 14 suppliers					
Var. = 17113, Bin. = 14, Cons. = 16477, Nonz. = 11733800 [(a)]					
CVaR^c	13.60	16.70	25.59	35.94	100.24
VaR^c	10.51	10.51	13.75	16.89	100.24
E^c	12.05	12.05	12.50	12.43	15.50
$1 - F(VaR^c)$	0.115	0.097	0.058	0.029	0
No. of suppliers selected	7	7	13	11	7

[(a)] Var. = no. of variables, Bin. = no. of binary variables, Cons. = no. of constraints, Nonz. = no. of nonzero coefficients

$\alpha = 0.989$ to $\alpha = 0.99$ results in a significant change in VaR from 37.33 to 100.17, while only a slight increase of CVaR^c from 94.81 to 100.17 is observed. Moreover, the optimal portfolio has been changed significantly; for $\alpha = 0.989$ the total demand has been equally allocated among all ten suppliers ($V_i = 0.1$ forall i), whereas for $\alpha = 0.99$ among five suppliers only ($V_i = 0.2$ for $i = 1, 2, 6, 7, 8$). This degree of instability of the optimal supply portfolio due to the discontinuity in the distribution function may be distressing in practice, when a slightly higher confidence level is required. Despite the limited change in CVaR^c, the above results demonstrate that the well known misbehaviour in the dependence of VaR^c and optimal supply portfolio on the confidence level can as well be encountered when CVaR^c is applied as a risk measure.

The computational results indicate that the smaller is the number of concentration points and the greater are probability atoms concentrated at those points, the greater can be the positive difference $F(VaR^c) - \alpha$, i.e., the smaller than $1 - \alpha$ can be the probability of outcomes with cost higher than VaR^c. For example (see, Table 2.4), for $\bar{I} = 10$ and $\alpha = 0.5$, $VaR^c = 10.35$ and $F(VaR^c) = 0.88666 > 0.5$, which indicates a high concentration of probability measure at point 10.35 for the optimal supply portfolio. Actually, the probability that cost per part is 10.35 is 0.88666 (see, Table 2.5), which indicates that $VaR^c = 10.35$ is the lowest cost that may occur and

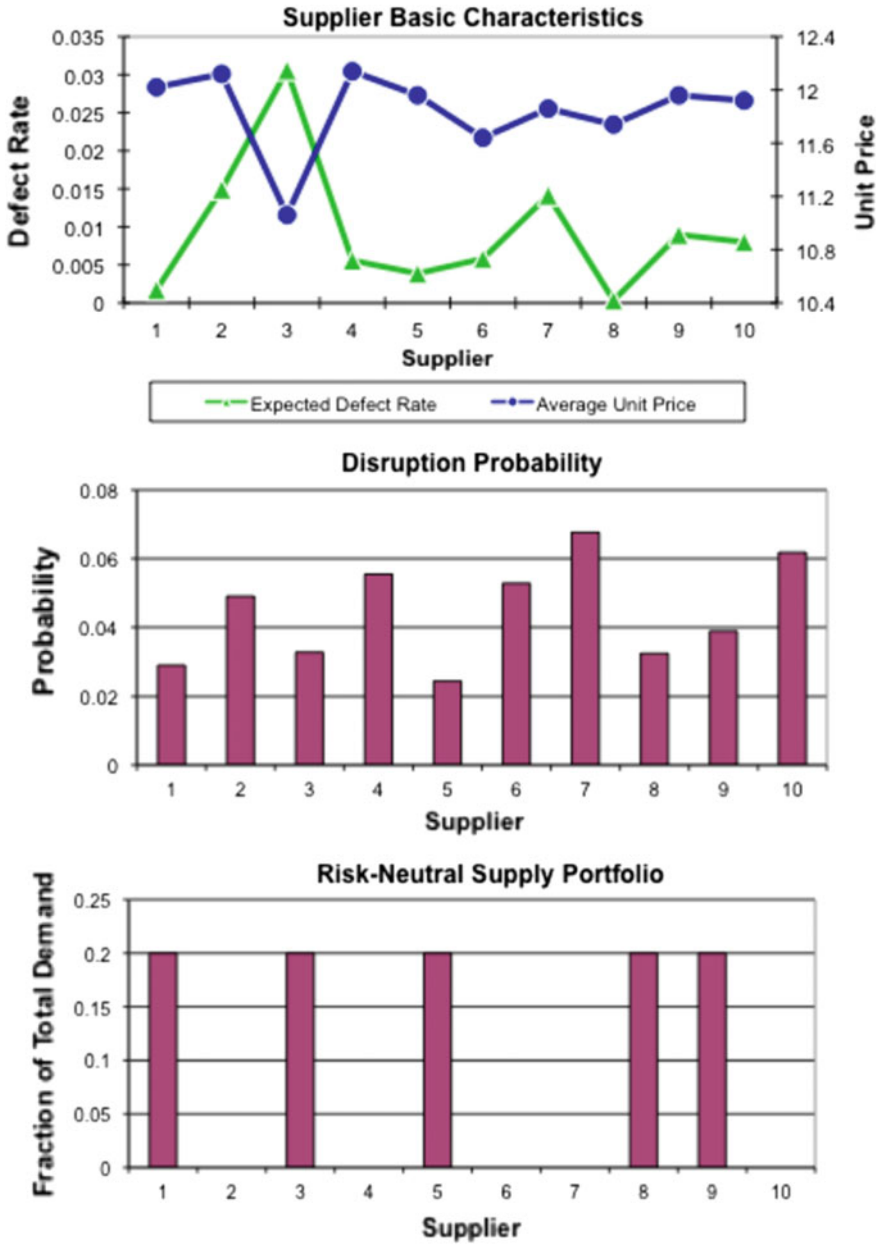

Fig. 2.1 Risk-neutral supply portfolio for model **SP_E(c)**: 10 suppliers

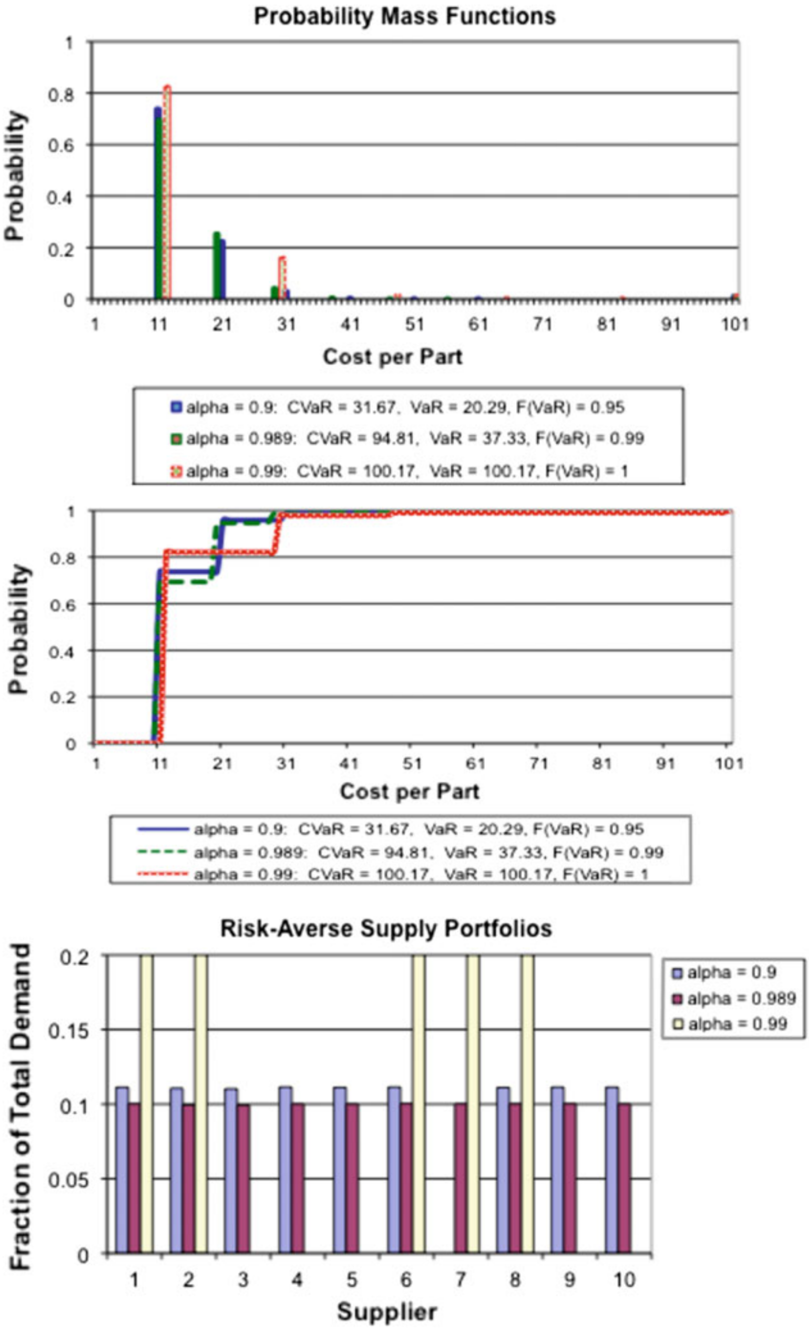

Fig. 2.2 Risk-averse supply portfolios and cost distributions for model **SP_CV(c)**: 10 suppliers

Table 2.5 Probability of cost per part for optimal risk-averse supply portfolios: 10 suppliers

Cost interval	$\alpha = 0.5$	$\alpha = 0.9$	$\alpha = 0.989$	$\alpha = 0.99$
[10, 11)	0.886661618	0.736768317	0.693881575	0
[11, 12)	0	0	0	0.821176973
[19, 20)	0	0	0.251829646	0
[20, 21)	0	0.221857039	0	0
[28, 29)	0.098887038	0	0.040311352	0
[29, 30)	0	0	0	0.156904596
[30, 31)	0	0.029090556	0	0
[37, 38)	0	0	0.003744839	0
[40, 41)	0	0.002178297	0	0
[46, 47)	0.004355651	0	0.000223405	0
[47, 48)	0	0	0	0.011507829
[50, 51)	0	0.000102579	0	0
[55, 56)	0	0	8.94e-06	0
[60, 61)	0	3.15e-06	0	0
[64, 65)	9.47e-05	0	2.43e-07	0.0004038
[70, 71)	0	6.30e-08	0	0
[73, 74)	0	0	4.41e-09	0
[80, 81)	0	7.91e-10	0	0
[82, 83)	1.01e-06	0	5.14e-11	6.76e-06
[90, 91)	0	5.66e-12	0	0
[91, 92)	0	0	3.46e-13	0
[100, 101)	0.010000004	0.01	0.01	0.010000043

that for the confidence level $\alpha = 0.5$, less than 11.33% of the cost outcomes are above VaR^c.

Moreover, if the highest cost probability is greater than $1 - \alpha$, then CVaR^c and VaR^c are identical and both equal to the highest cost. In the example for ten suppliers and $\alpha = 0.99$, the highest cost per part is 100.17 and the probability concentrated at 100.17 is $0.01000004 > 1 - \alpha$, then $VaR^c = 100.17$ is the highest cost per part that may occur and hence $CVaR^c = VaR^c = 100.17$ (see, Table 2.5 and Fig. 2.2). Similar results $CVaR^c = VaR^c = 100.14$ and $CVaR^c = VaR^c = 100.24$ have been obtained for $\alpha = 0.99$, respectively for seven and 14 suppliers (see, Table 2.4), which indicates that the corresponding probabilities of the highest cost per part are greater than $1 - \alpha = 0.01$.

If the probability measure is concentrated at the highest cost and is greater than $1 - \alpha$, so that CVaR^c and VaR^c are identical with the highest cost, then for a higher confidence level α, a smaller number of suppliers are selected, which indicates that diversification of the supply portfolio is not necessary any more. For instance, the optimal risk-averse supply portfolio selected for $\alpha = 0.99$ consists of five suppliers only, the same number as that for a much lower α (cf. Table 2.4, Fig. 2.2).

In the computational experiments the local disruption probabilities p_i were assumed to be very low and were drawn from U[0;0.06]. To study the effect of the increasing range of disruption probabilities, the probabilities have been drawn also from U[0;0.25], U[0;0.5] or U[0;1]. The effect of varying distribution of local disruption probabilities is illustrated in Fig. 2.3, where the optimal risk-averse supply portfolios are presented. Figure 2.3 indicates that for a greater range of disruption probabilities, the suppliers with the highest disruption probabilities are not selected. For example, for U[0;1] and $\alpha = 0.75$, suppliers $i = 4, 6, 7, 10$ with the four highest disruption rates were not selected, whereas for $\alpha = 0.985$, suppliers $i = 7, 10$ with the two highest disruption rates were not selected. Similar results are observed for the other distributions of disruption probability.

Table 2.6 Solutions results for model **SP_CV(c)** with binary assignment variables $v_{ij} \in \{0, 1\}$: 10 suppliers

Confidence level α	0.50	0.75	0.90	0.95	0.99
Var. = 1545, Bin. = 510, Cons. = 1105, Nonz. = 526328 [a]					
CVaR^c	16.08	21.80	31.75	42.08	100.20
VaR^c	10.36	10.40	20.16	23.21	100.20
E^c	13.22	13.25	14.07	13.94	15.74
$1 - F(VaR^c)$	0.155	0.155	0.094	0.053	0
No. of suppliers selected	6	6	9	10	6

[a] Var. = no. of variables, Bin. = no. of binary variables, Cons. = no. of constraints, Nonz. = no. of nonzero coefficients

Finally, Table 2.6 presents solution results for model **SP_CV(c)** applied to optimization of a single sourcing, with binary assignment variables v_{ij}, i.e., when for each customer order, the required parts must be provided by a single supplier only. Comparison of the results shown in Table 2.6 with the corresponding results presented in Table 2.4 for continuous allocation variables v_{ij}, indicates that in the former case both the expected cost per part and CVaR^c were slightly higher and, in addition, for a low α the number of selected suppliers was greater. Such results were expected, since model **SP_CV(c)** with the continuous allocation variables is a partial LP relaxation of that model with the binary assignment variables.

For the mean-risk approach, the subsets of nondominated solutions were computed by parameterization on $\lambda \in \{0.01, 0.10, 0.25, 0.50, 0.75, 0.90, 0.99\}$ the weighted-sum program **SP_ECV(c)**. The subset of nondominated solutions found for the selected seven levels of trade-off parameter λ is: $(E^c, CVaR^c) = (13.18, 36.32)$, (13.33, 33.97), (13.49, 32.65), (13.84, 31.84), (13.86, 31.82), (14.06, 31.67). The trade-off between the expected cost and the expected worst-case cost is clearly shown in Fig. 2.4, where the convex efficient front for the mean-risk model **SP_ECV(c)** with $\alpha = 0.9$ is presented. The results emphasize the effect of varying cost/risk preference of the decision maker.

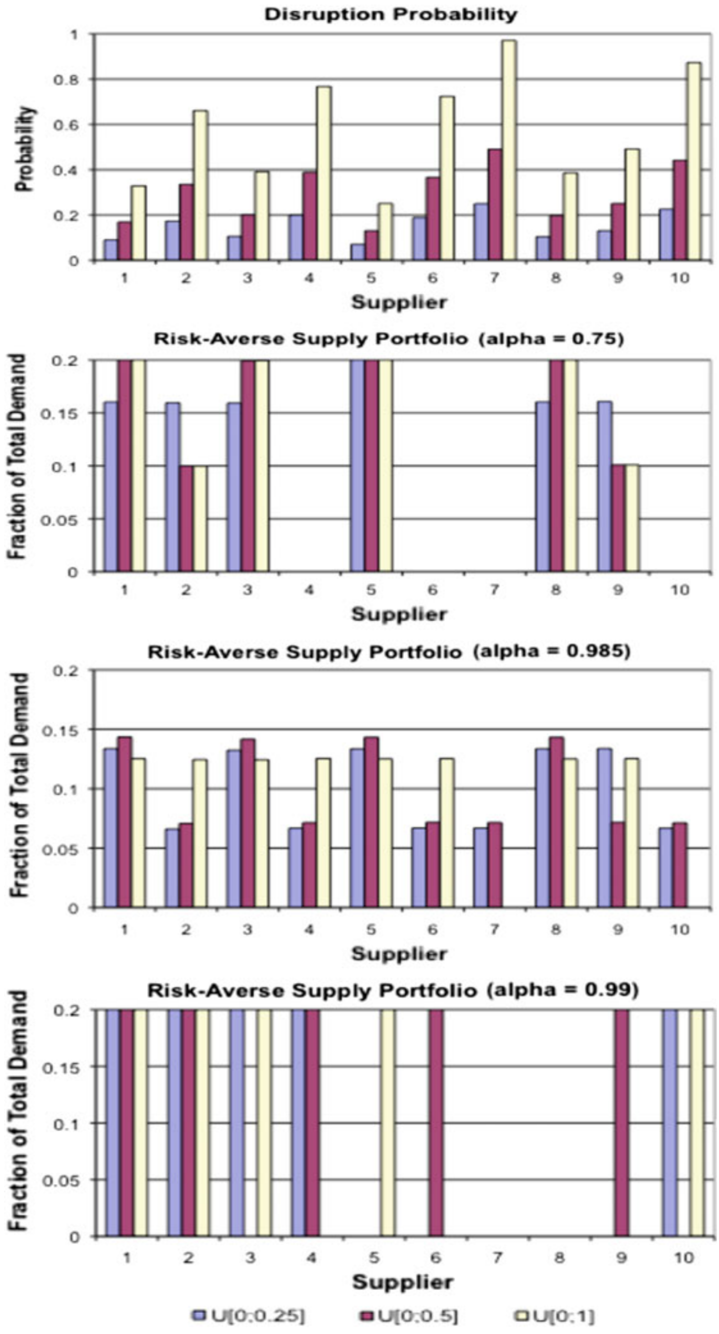

Fig. 2.3 Risk-averse supply portfolios for different local disruption probabilities for model **SP_CV(c)**: 10 suppliers

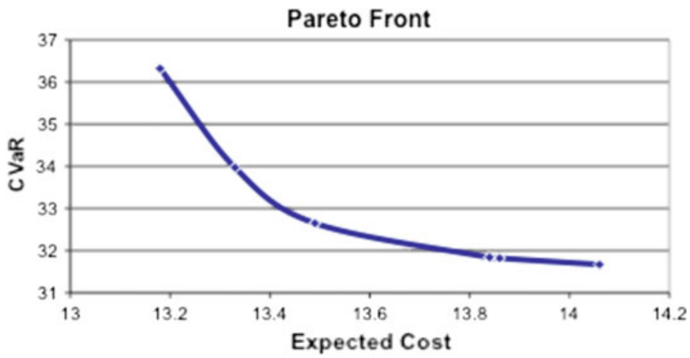

Fig. 2.4 Pareto front for mean-risk model **SP_ECV(c)**: 10 suppliers, $\alpha = 0.9$

Note that solutions to single objective models **SP_E(c)** and **SP_CV(c)** are equivalent to the nondominated solutions of the weighted-sum program **SP_ECV(c)** for $\lambda = 1$ and $\lambda = 0$, respectively.

The computational experiments were performed using the AMPL programming language and the CPLEX solver. The solver was capable of finding proven optimal solutions within CPU seconds for all examples.

2.6.2 Multi-region Sourcing

In this subsection the suppliers are assumed to be located in multiple geographic regions subject to different regional disruption risks. The following parameters used for the example problems are different from those in Sect. 2.6.1:

- $\overline{I}$, the number of suppliers, was equal to 10 and the corresponding number $\overline{S} = 2^{\overline{I}}$ of disruption scenarios, was equal to 1024;
- $\overline{R}$, the number of geographic regions, was equal to 3, and the subsets of suppliers were $I^1 = \{1, 2, 3\}$, $I^2 = \{4, 5, 6\}$ and $I^3 = \{7, 8, 9, 10\}$, respectively;
- $\overline{J}$, the number of customer orders, was equal to 25;
- d_j, the numbers of required parts for each customer order, were integers uniformly distributed over [1000, 15000] for all customer orders j. and the resulting total demand for parts was $D = 132500$;
- e_i, the cost of ordering parts, were integers in $\{5000, 6000, \ldots, 10000\}$, $\{10000, 11000, \ldots, 15000\}$ and $\{15000, 16000, \ldots, 30000\}$, respectively for suppliers $i \in I^1$, $i \in I^2$ and $i \in I^3$;
- h_j, the per unit shortage cost for each customer order j, was integer uniformly distributed over $[\max_{i \in I}(o_{ij}), 4\max_{i \in I}(o_{ij})]$, i.e., ranging from one to four times of maximum purchasing cost of required parts;

- o_{ij}, the unit price of parts for customer order j purchased from supplier i, was uniformly distributed over [13,15], [11,13] and [9,11], respectively for suppliers $i \in I^1$, $i \in I^2$ and $i \in I^3$;
- p_i, the local disruption probability was uniformly distributed over [0.005,0.01], [0.01,0.05] and [0.05;0.10], respectively for suppliers $i \in I^1$, $i \in I^2$ and $i \in I^3$, i.e., the disruption probabilities were drawn independently from U[0.005;0.01], U[0.01,0.05] and U[0.05;0.10], respectively;
- p^r, the regional disruption probability was 0.001, 0.005 and 0.01, respectively for region $r = 1$, $r = 2$ and $r = 3$;

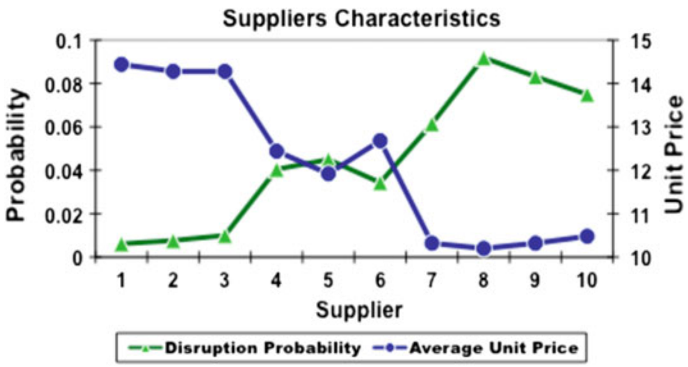

Fig. 2.5 Suppliers

Basic characteristics of suppliers: average unit price $\sum_{j \in J} o_{ij}/\bar{J}$, and disruption probability, π_i, (2.1), are presented in Fig. 2.5. Figure 2.5 indicates that the most reliable and most expensive are suppliers $i = 1, 2, 3$ in region $r = 1$, while suppliers $i = 7, 8, 9, 10$ in region $r = 3$ are most competitive and most unreliable. In particular, supplier $i = 8$ is the cheapest and most unreliable among all suppliers.

The solution results for the risk-averse models **SP_CV(c)** and **SP_CV(sl)** with different confidence levels are shown in Table 2.7. For both models the number of selected suppliers increases with the confidence level. While for model **SP_CV(c)** the cheapest, yet most unreliable supplier $i = 8$ was never selected, for model **SP_CV(sl)** all 10 suppliers are selected for $\alpha = 0.9,\ 0.95$ and $\alpha = 0.99$.

Figure 2.6 shows the optimal risk-averse supply portfolios for models **SP_CV(c)** and **SP_CV(sl)** and the three confidence levels, $\alpha = 0.75,\ 0.9,\ 0.99$. For both models and $\alpha = 0.75$ the most unreliable suppliers $i = 7, 8, 9, 10$ in region $r = 3$ are not selected and for all confidence levels most demand for parts is allocated among the three most reliable, yet most expensive suppliers, $i = 1, 2, 3$, in region $r = 1$, in particular for $\alpha = 0.75, 0.9$. Similar properties of the risk-averse supply portfolios were observed in case of single-region sourcing (see, Fig. 2.3).

Table 2.7 Risk-averse solutions: multi-region sourcing

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SP_CV(c)**					
Var. = 1285, Bin. = 10, Cons. = 1319, Nonz. = 269558 [(a)]					
CVaR^c	15.35	17.15	20.49	23.90	30.43
VaR^c	13.29	13.68	16.44	17.19	26.21
$1 - F(VaR^c)$	0.149	0.125	0.055	0.036	0.005
E^c	14.32	14.55	14.59	14.51	14.61
E^{sl} [(b)]	97.51	97.98	97.30	97.56	96.15
Suppliers Selected(% of total demand)	1(20)	1(20)	1(20)	1(20)	1(19)
	2(19)	2(19)	2(20)	2(19)	2(13)
	3(20)	3(20)	3(20)	3(20)	3(12)
		4(14)	4(8)	4(8)	4(6)
	5(8)	5(7)	5(6)	5(9)	5(6)
	6(13)	6(20)	6(6)	6(8)	6(8)
	7(20)		7(12)	7(16)	7(9)
			9(8)		9(12)
					10(15)
Model **SP_CV(sl)**					
Var. = 1285, Bin. = 10, Cons. = 1319, Nonz. = 131318 [(a)]					
CVaR^{sl}%	96.01	92.02	85.29	80.12	69.82
VaR^{sl}%	100	100	91.87	87.50	77.77
$1 - F(VaR^{sl})$	0.125	0.125	0.064	0.033	0.007
E^{sl} [(b)]	98.00	98.00	97.29	97.43	97.04
E^c	15.56	15.48	15.86	15.85	15.61
Suppliers Selected(% of total demand)	1(20)	1(20)	1(20)	1(20)	1(20)
	2(19)	2(19)	2(20)	2(20)	2(18)
	3(20)	3(20)	3(20)	3(20)	3(18)
	4(20)	4(20)	4(8)	4(8)	4(10)
	5(1)	5(1)	5(8)	5(9)	5(4)
	6(20)	6(20)	6(8)	6(9)	6(9)
			7(4)	7(3)	7(9)
			8(4)	8(3)	8(4)
			9(4)	9(3)	9(4)
			10(4)	10(3)	10(4)

[(a)] Var. = no. of variables, Bin. = no. of binary variables, Cons. = no. of constraints, Nonz. = no. of nonzero coefficients
[(b)] $(\sum_{s \in S} \sum_{i \in I_s} \sum_{j \in J} P_s d_j v_{ij} / D)100\%$

The computational experiments indicate that

- *probability of disruption a supply is a key determinant in the decision of allocation of demand among the suppliers. In a risk averse model, an order for delivery of* *parts from a particular supplier is selected more on the supply non-disruption likelihood than on its purchasing cost or defect rate.*

- *The suppliers associated with the highest disruption rates are rarely selected.*

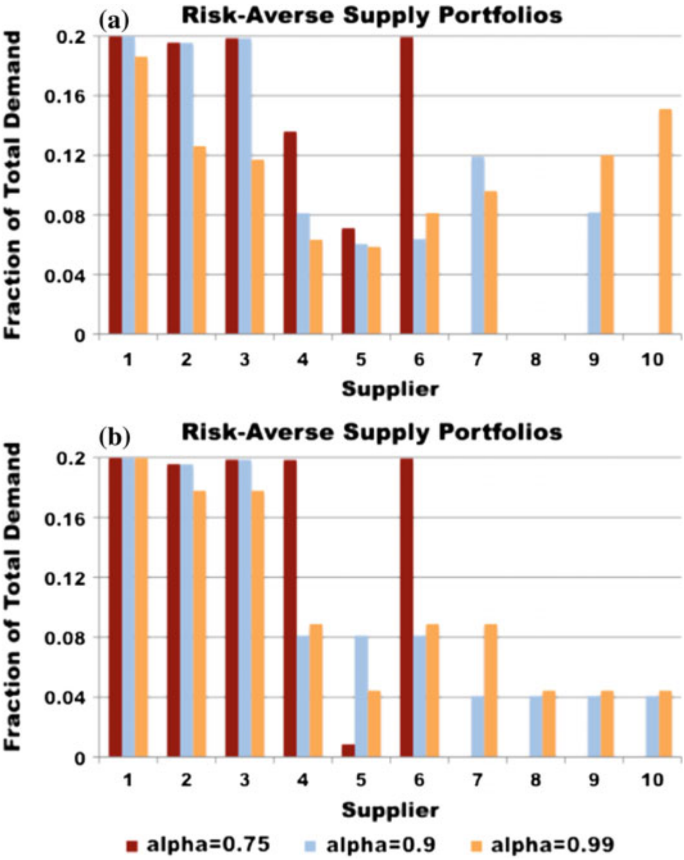

Fig. 2.6 Risk-averse supply portfolios: **a** model **SP_CV(c)**, **b** model **SP_CV(sl)**

- *In most cases the number of selected suppliers increases with the confidence level α, which indicates that the impact of disruption risks is mitigated by diversification of the supply portfolio.*
- *The greater is the range of disruption probabilities, the higher are both expected and worst-case costs.*
- *The closer are disruption rates for different suppliers, the closer are the corresponding quantities of ordered parts in the optimal portfolio.*

The probability measure for the scenario-based optimization under uncertainty is concentrated in finitely many points and the resulting discrete distribution of cost may have different effect on the optimal portfolio. In particular, the well known misbehaviour in the dependence of VaR on the confidence level can as well be encountered when CVaR is applied as a risk measure. For instance, if a large probability atom is concentrated at some cost, a slight increase of the confidence level may results in a significant change in VaR as well as in the optimal portfolio, while only a slight change of CVaR may occur. Such an instability of the optimal portfolio due to the discontinuity in the distribution function may be distressing in practice, when a slightly higher confidence level is required.

On the other hand the computational results indicate that the smaller is the number of concentration points and the greater are probability atoms concentrated at those points, the greater can be the positive difference $F(VaR) - \alpha$, i.e., the smaller than $1 - \alpha$ can be the probability of outcomes with cost higher (service level lower) than VaR.

The computational experiments prove that the proposed exact solution approach based on MIP approach provides the decision maker with a simple tool for evaluating the relationship between expected and worst-case costs. For a finite number of scenarios, the proposed models allow the evaluation of worst-case costs and shaping of the resulting cost distribution through the selection of optimal supply portfolio. The optimal risk averse supply portfolio can be found within CPU seconds for a limited number of scenarios considered, using commercially available solvers for MIP.

2.7 Notes

The supply chain risk management has been extensively studied over the past decade. Research addresses the two risk levels (e.g., Tang 2006): operational risks or disruption risks. Operational risks are referred to the inherent uncertainties arising from the problems of coordinating supply and demand such as uncertain customer demand, uncertain supply, and uncertain cost. Disruption risks are referred to the major disruptions to normal activities caused by natural and man-made disasters such as earthquakes, floods, hurricanes, etc., or equipment breakdowns, economic crises such as currency evaluation, labor strikes, terrorist attacks. In most cases, the business impact associated with disruption risks is much greater than that of the operational risks. In practice four basic approaches can be applied to mitigate the impact of supply chain risks (Tang 2006): supply management, demand management, product management, and information management. In particular, to ensure efficient supply of materials along a supply chain, supply chain management deals with selection of a supply portfolio, i.e., supplier selection and order quantity allocation under uncertain quality of supplied materials and reliability of on-time delivery. The supplier selection and order quantity allocation problem is a complex stochastic combinatorial optimization problem, however the research on supplier selection under disruption risks is limited. For example, chance-constrained programming models were developed by Kasilingam and Lee (1996) to account for stochastic demand and by Wu and Olson (2008) to consider expected losses from quality acceptance inspection or late delivery. Parlar and Perry (1996) present a continuous time model in which the availability of each of the m suppliers is uncertain because of disruptions such as equipment breakdown. By considering the case that each supplier is either "on" or "off", there are 2^m possible number of states for the whole system. For each of these 2^m states, they analyze a state-specific (q, Q) ordering policy so that the buyer would order Q units when the on-hand inventory reaches q. The risks associated with a supplier network was studied by Berger et al. (2004), who considered catastrophic super events that affect all suppliers, as well as unique events that impact only one single supplier, and then a decision-tree based model was presented to help determine the optimal number of suppliers needed for the buying firm. Ruiz-Torres and Mahmoodi (2007) considered unequal failure probabilities for all the suppliers. Berger and Zeng (2006) studied the optimal supply size in a single or multiple sourcing strategy context, under a number of scenarios that are determined by various financial loss functions, the operating cost functions and the probabilities of all the suppliers being down. Yu et al. (2009) considered the impacts of supply disruption risks on the choice between the single and dual sourcing methods in a two-echelon supply chain with a non-stationary and price-sensitive demand. Yue et al. (2010) introduced frontier sourcing portfolios to support manufacturers sourcing decisions, which consider the cost and probability of finishing the order on time. Ravindran et al. (2012) developed multi-criteria supplier selection models incorporating supplier risks. In the multi-objective formulation, price, lead-time, disruption risk due to natural event and quality risk are explicitly considered as four conflicting objectives that have to be minimized simultaneously. Four different variants of goal programming were used to solve the multi-objective optimization problem. Xanthopoulos et al. (2012) developed newsvendor-type inventory models for capturing the trade-off between inventory policies and disruption risks in a dual-sourcing supply chain network, where both supply channels are subject to disruption risks. The models were developed for both risk-neutral and risk-averse decision-making. Li and Zabinsky (2011) developed a two-stage stochastic programming model and a chance-constrained programming model to determine a minimal set of suppliers and optimal order quantities. Both models include several objectives and strive to balance a small number of suppliers with the risk of not being able to meet demand. The stochastic programming model is scenario-based and uses penalty coefficients whereas the chance-constrained programming model assumes a probability distribution and constrains the probability of not meeting demand. Hammami et al. (2014) proposed a scenario-based stochastic model for supplier selection in the presence of uncertain fluctuations of currency exchange rates and price discounts.

The vast majority of the decision models are mathematical programming models either single objective, e.g., Kasilingam and Lee (1996), Basnet and Leung (2005), Sawik (2005) or multiple objectives, e.g., Weber and Current (1993), Xia and Wu (2007), Demirtas and Ustun (2008), Ustun and Demirtas (2008). The models developed for supplier selection and order allocation can be either single-period models (e.g., Weber and Current 1993, Demirtas and Ustun 2008) that do not consider inventory management or multi-period models (e.g., Ghodsypour 2001, Basnet and Leung

2005, Ustun and Demirtas 2008, Che and Wang 2008) which consider the inventory management by lot-sizing and scheduling of orders.

The material presented in this chapter is based on research reported by Sawik (2011b, c), who proposed a portfolio approach for the supplier selection and order quantity allocation under disruption risks and under operational risks, respectively. The author applied the two popular in financial engineering percentile measures of risk, value-at-risk (VaR) and conditional value-at-risk (CVaR) (e.g., Sarykalin et al. 2008) for managing the risk of supply disruptions or supply delays. The proposed models were further enhanced in this chapter for maximization of expected or expected worst-case service level and for a multi-region sourcing subject to regional disruption risks.

Various simplifying assumptions that have been used in the models presented in this chapter can be relaxed. For example, it has been assumed that each supplier is capable of manufacturing all required part types. In a more general setting, each supplier may only be prepared to manufacture a subset of part types and provide with the parts the corresponding subset of customer orders. The proposed models can be enhanced also for a discount environment, where the suppliers offer discounts based on quantity or business volume of ordered parts, e.g., Sawik (2010). A critical issue that need to be considered before any practical application of the proposed models is attempted, however, is the estimation of probabilities and the resulting costs associated with each type of disaster event, for which different approaches are suggested in the literature, such as expert systems, game theory, utilization of large simulation models, etc. (e.g., Knemeyer et al. 2009).

Problems

2.1 Modify the probability for disruption scenarios (2.2) to account for correlated regional disruptions that may affect simultaneously suppliers in different regions.

2.2 Modify the SMIP models presented in this chapter for multiple part types with subsets of part types required for each customer order and subsets of suppliers available for each part type.

2.3 Enhance the SMIP models presented in this chapter for selection of static supply portfolio
(a) with total quantity discount for all ordered parts.
(b) with total business volume discount.
(see, Sawik 2010).

2.4 Mixed mean-risk static supply portfolio
(a) Modify model **SP_ECV(c)** to optimize expected cost and CVaR of service level and model **SP_ECV(sl)** to optimize expected service level and CVaR of cost.
(b) How should the values of the optimized objective functions be scaled into the interval [0,1] to avoid dimensional inconsistency among the two objectives and how

should the trade-off parameter be selected?
(c) How would you interpret the mixed mean-risk supply portfolio?

2.5 Explain why for a greater range of disruption probabilities, both expected and expected worst-case costs are higher.

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