

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Related Work	6
1.2	Outline and Prerequisites	7
1.3	Motivating Example: Analysis of Digital Systems	8
1.4	Motivating Example: Quantizer Design	11
 <b>Part I Random Variables</b>		
<b>2</b>	<b>Piecewise Bijective Functions and Continuous Inputs</b>	<b>17</b>
2.1	The PDF of $Y = g(X)$	18
2.2	The Differential Entropy of $Y = g(X)$	20
2.3	Information Loss in PBFs	21
2.3.1	Elementary Properties	21
2.3.2	Upper Bounds on the Information Loss	26
2.3.3	Computing Information Loss Numerically	29
2.3.4	Application: Polynomials	30
<b>3</b>	<b>General Input Distributions</b>	<b>35</b>
3.1	Information Loss for Systems with General Inputs	36
3.2	Systems with Infinite Information Loss	37
3.3	Alternative Proof of Theorem 2.1	40
<b>4</b>	<b>Dimensionality-Reducing Functions</b>	<b>43</b>
4.1	Information Dimension	44
4.1.1	Properties and Equivalent Definitions of Information Dimension	45
4.1.2	$d(X)$ -dimensional Entropy and Mixture of Distributions	49
4.1.3	Operational Characterization of Information Dimension	53

4.2	Relative Information Loss . . . . .	54
4.2.1	Elementary Properties . . . . .	55
4.2.2	Bounds on the Relative Information Loss . . . . .	56
4.2.3	Relative Information Loss for System Reducing the Dimensionality of Continuous Random Variables . . .	57
4.2.4	Relative Information Loss and Perfect Reconstruction . . .	60
4.2.5	Outlook: Relative Information Loss for Discrete-Continuous Mixtures . . . . .	62
4.3	Application: Principal Components Analysis . . . . .	64
4.3.1	The Energy-Centered Perspective . . . . .	65
4.3.2	PCA with Given Covariance Matrix . . . . .	67
4.3.3	PCA with Sample Covariance Matrix . . . . .	68
5	<b>Relevant Information Loss</b> . . . . .	73
5.1	Definition and Properties . . . . .	73
5.1.1	Elementary Properties . . . . .	75
5.1.2	An Upper Bound on Relevant Information Loss . . . . .	78
5.2	Signal Enhancement and the Information Bottleneck Method . . .	80
5.3	Application: PCA with Signal-and-Noise Models . . . . .	82
 <b>Part II Stationary Stochastic Processes</b>		
6	<b>Discrete-Valued Processes</b> . . . . .	93
6.1	Information Loss Rate for Discrete-Valued Processes . . . . .	94
6.2	Information Loss Rate for Markov Chains . . . . .	96
6.3	Outlook: Systems with Memory . . . . .	99
6.3.1	Partially Invertible Systems . . . . .	100
6.3.2	Application: Fixed-Point Implementation of a Linear Filter . . . . .	102
7	<b>Piecewise Bijective Functions and Continuous Inputs</b> . . . . .	105
7.1	The Differential Entropy Rate of Stationary Processes . . . . .	105
7.2	Information Loss Rate in PBFs . . . . .	106
7.2.1	Elementary Properties . . . . .	106
7.2.2	Upper Bounds on the Information Loss Rate . . . . .	108
7.2.3	Application: AR(1)-Process in a Rectifier . . . . .	109
7.3	Outlook: Systems with Memory . . . . .	111
8	<b>Dimensionality-Reducing Functions</b> . . . . .	115
8.1	Relative Information Loss Rate . . . . .	115
8.2	Application: Downsampling . . . . .	118
8.2.1	The Energy-Centered Perspective . . . . .	119
8.2.2	Information Loss in a Downsampling Device . . . . .	119
8.3	Outlook: Systems with Memory . . . . .	123

- 9    Relevant Information Loss Rate . . . . .** 127
  - 9.1   Definition and Properties . . . . . 127
    - 9.1.1   Upper Bounds on the Relevant Information Loss Rate. . . 129
  - 9.2   Application: Anti-aliasing Filter Design for Downsampling. . . . 130
    - 9.2.1   Anti-aliasing Filters for Non-Gaussian Processes . . . . . 132
    - 9.2.2   FIR Solutions for Information Maximization . . . . . 135
- 10   Conclusion and Outlook. . . . .** 137
- References. . . . .** 141

Information Loss in Deterministic Signal Processing  
Systems

Geiger, B.C.; Kubin, G.

2018, XIII, 145 p. 16 illus., 9 illus. in color., Hardcover

ISBN: 978-3-319-59532-0