# Preface

## Scope

In an abstract form, the evolutional nonlinear system is a mathematical model that describes how physical, chemical, biological, economic, or even mathematical phenomena evolve in time. As a rule, it contains ordinary/partial/stochastic differential equations or inclusions that tell us how the system at hand changes "from one instant to the next." The main goal is to gain information about solutions of this system and then translate this mathematical information into the scientific context. The main challenge addressed by this book is to take this short-term information and obtain information about long-term overall behavior. The study of nonlinear systems has three parts: exact methods, quantitative methods and qualitative methods. But even if we solve the system symbolically, the question of computing values remains.

In this book, we concentrate on the following topics, specific for nonlinear systems:

(a) constructive existence results and regularity theorems for all weak (generalized) solutions;
(b) convergence results for solutions and their approximations in strongest topologies of the natural phase and extended phase spaces;
(c) uniform global behavior of solutions in time;
(d) pointwise behavior of solutions for autonomous problems with possible gaps by the phase variables.

With numerous applications including nonlinear parabolic equations of divergent form, parabolic problems with nonpolynomial growth, nonlinear stochastic equations of parabolic type, unilateral problems with possibly nonmonotone potential, nonlinear problems on manifolds with or without boundary, contact piezoelectric problems with nonmonotone potential, viscoelastic problems with nonlinear "reaction-displacement" and "reaction-velocity" laws as well as particular examples like a model of conduction of electrical impulses in nerve axons, a climate energy balance model, FitzHugh–Nagumo system, Lotka–Volterra system with diffusion, Ginzburg–Landau equations, Belousov–Zhabotinsky equations, and the 3D Navier–Stokes equations. This book is also distinguished with the solutions of a number of applied problems in physics, chemistry, biology, economics, etc.

## Contents

This book consists of three parts: Existence and Regularity Results, Quantitative Methods and Their Convergence (Part I), Convergence Results in Strongest Topologies (Part II), and Uniform Global Behavior of Solutions: Uniform Attractors, Flattening and Entropy (Part III). Part I presents several numerical methods for approximate solution of nonlinear systems, their convergence, and regularity results and also discusses recent advances in regularity problem for the 3D Navier–Stokes equations. Part II covers three major topics: (1) strongest convergence results for weak solutions of nonautonomous reaction–diffusion equations with Carathéodory's nonlinearity with applications to FitzHugh–Nagumo systems, Lotka–Volterra systems with diffusion, Ginzburg–Landau equations, Belousov–Zhabotinsky equations, etc; (2) strongest convergence results for weak solutions of feedback control problems with applications to impulse feedback control mechanical problems and mathematical problems of biology and climatology; and (3) strongest convergence results for weak solutions of differential-operator equations and inclusions with applications to nonlinear parabolic equations of divergent form, parabolic problems with nonpolynomial growth, nonlinear stochastic equations of parabolic type, general parabolic and hyperbolic problems, unilateral problems with possibly nonmonotone operators, etc. Part III discusses general methodology for the global qualitative and quantitative investigation of dissipative dynamical systems, first- and second-order operator differential equations and inclusions, and evolutional variational inequalities with possibly nonmonotone potential with several applications. Indirect Lyapunov method for autonomous dynamical systems, exponential attractors, and Kolmogorov entropy are also established. All case studies are closely related to theoretical Parts I and II and are examples of applications to solutions of problems (a) and (b).

## Audience

This book is aimed at practitioners working in the areas of nonlinear mechanics, mathematical biology, control theory, differential equations, nonlinear boundary value problems, and decision making. It can serve as a quick introduction into the novel methods of qualitative and quantitative analysis of nonlinear systems for the graduate students, engineers, and mathematicians interested in analysis and control of nonlinear processes and fields, mathematical modeling, and dynamical systems in infinite-dimensional spaces, to mention just a few. It can also be used as a supplementary reading for a number of graduate courses including but not limited to those of nonlinear PDEs, control and optimization, stochastic partial differential equations, advanced numerical methods, systems analysis, and advanced engineering economy.

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