

# A GPP-Based Sectionalization Toward a Fast Power Transmission System Restoration

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**Abstract.** High voltage transmission lines, in outdoor area, are in danger of extreme events such as tornadoes and hurricanes. Accordingly, terrible damage of transmission lines will cause a power grid blackout. Sectionalization as a part of a restoration process can make a power grid resilient by splitting it into multiple smaller areas. Then a diminutive portion of the total load is supplied at each area by black-start (BS) generation units with their self-start capability. To find the optimal sectionalization and perform a fast consumer electrification, a mathematical model is designed upon the association between the power transmission system sectionalization (PTSS) and graph-partitioning problem (GPP). The proposed GPP-based PTSS model finds the optimal sectionalization and restoration plan through a bi-level programming structure with sectionalization and restoration levels. Furthermore, pre-emptive goal programming (PEGP) supports the multiple objective terms of both levels. The model's efficiency is analyzed by IEEE 14- and 118-bus test systems.

**Keywords:** Graph partitioning problem · Power system restoration · Resilience · Sectionalization

## 1 Nomenclature

Sets		Parameters	
$g$	Index for generators, $g = \{1, \dots, NG\}$	$\omega$	Restoration time cost rate vector
$m$	Index for sections, $m = \{1, \dots, NG\}$	$c$	Marginal cost of generators
$b, \acute{b}$	Index for buses, $b = \{1, \dots, NB\}$	$\mathbf{KG}$	Bus-unit incident matrix
$d$	Index for demand loads, $d = \{1, \dots, ND\}$	$\mathbf{KL}$	Bus-line incident matrix
$t$	Index for time, $t = \{1, \dots, NT\}$	$\mathbf{KD}$	Bus-demand incident matrix
$l$	Index for transmission lines, $l = \{1, \dots, NL\}$	$\mathbf{D}$	Real power demand matrix

Parameters		Variables	
$P_g^{G,min} / P_g^{G,max}$	Maximal and minimal generating capacity of generation unit $g$	$P_{gt}^G$	Generated power of unit $g$ at time $t$
$RU_g^G / RD_g^G$	Ramp up/down rate of unit $g$	$\mathbf{LS} / \mathbf{LS}'$	Load shedding matrix per demand/section (hourly)
$P_l^{L,max}$	Power line capacity of line $l$	$s_{bm}$	State of bus $b$ at section $m$
$\mathbf{T}$	Total restoration time matrix	$P_{lt}^L$	Power flow on line $l$ at time $t$
$x_l$	Reactance of line $l$	$\theta_{bt}$	Phase angle of bus $b$ at time $t$
$VOLL$	Value of loss of load	$T_d^{Load}$	Load pick up time of demand $d$
$a_{bb'}$	Connection state between bus $b$ and $b'$	$\varphi_t$	Auxiliary current time equal to $t$ at time $t$
		$y_{bb'}$	Tie-line state between bus $b$ and $b'$
		$n_m$	The number of buses in section $m$

## 2 Introduction

An electric power system includes three sub-systems: power generation, power transmission, and power distribution. The transmission system as a network includes long transmission lines, transformers and outdoor substations, which are mostly located in open wide areas. It would most likely be affected by any extreme weathers or disasters such as ice or dust storms, hurricanes or earthquakes. Although these disruptions are rare, they have high impact on transmission networks resulting in cases of complete blackouts. For instance, Hurricane Sandy caused long and huge outages over 17 states of the United States in 2012. It also brought terrible monetary losses of over \$25 billion dollars to businesses affected [1].

Occurrence of blackouts because of shocks is almost inevitable and the repairing process usually takes time to bring the system back to its normal state. Hence, it is required to investigate in reducing the losses by either preventing the outages or enabling the system to be restored fast. In a restoration process, critical loads could be supplied by additional self-starting generation resources that decrease a large portion of loss of load in a power system. In this order, power networks are equipped with self-starting generation units called BS units. These units can contribute into the sectionalization process, to recover the system with a maximum resiliency [2]. The sectionalization method is a build-up approach. In a build-up restoration, some separated sections are made to be restored individually. Then, a quick restoration process can be conducted by restoring the sections in a parallel fashion. The power system restoration also can save time for the repair crew to fix or replace all damaged components. Therefore, the system can be reconfigured and synchronized to return to its normal state [3, 4].

The quality of a sectionalization-based restoration is dependent on the points of disjoints in the network that can make different topologies over the sections. Depending on the sectionalization pattern, restoration time and the amount of satisfied load might be different. Sectionalization can be performed by minimizing the restoration time [5] or the amount of unmet demand [6, 7]. It is also possible to find an optimal solution with minimum “electrical distance” within each section [8]. From the tie-lines’ perspective, minimizing the number of tie-lines could develop more robust sections and consequently a successful restoration [9]. A GPP approach divides a network into a given number of partitions while minimizing the disjoint edges [10]. Therefore, considering the power system network as a connected graph, its buses could represent the graph’s nodes and the transmission lines could be equivalent to the graph’s edges. Consequently, a joint model of PTSS and GPP can be presented to find the optimal parallel restoration solution.

In this study, a mathematical model is proposed to minimize multiple objective terms including total MWh load shedding, total restoration time and total number of disjoint edges. Furthermore, the power generation cost term is added to the objective to find a restoration plan at lower cost. In order to solve the multi-objective model, preemptive goal programming (PEGP) is used to support optimization of all objective terms at the same time. The model is subjected to the constraints of PTSS and GPP. The PTSS model includes the optimal power flow (OPF) model constraints to perform the restoration as well as sectionalization.

In order to reduce the complexity of the model, a decomposition is applied to divide the model into two levels: the upper level or sectionalization model and the lower level, which is called restoration model. An iterative optimization algorithm (IOA) is used beside PEGP to solve the model. This study brings following major contributions to the literature:

1. Merging the PTSS and GPP models to sectionalize a de-energized power grid.
2. Minimizing the total number of tie-lines considering the place of BS units.
3. Restoring most of the grid’s loads within the first hours of a restoration process.

The rest of the paper is structured as follows: Sect. 3 presents the mathematical optimization model. The solution methodology is explained in Sect. 4. Section 5 shows the numerical results and the conclusion is performed in Sect. 6.

### 3 Model Description

The GPP model minimizes the disjoint edges to split a graph into a given number of partitions as depicted in Fig. 1. The size of each partition could be given as an input or could be found optimally within the model solution. Hence, by combining PTSS and GPP the proposed model will optimize the section sizes while considering the capacity of power generation and minimization of load shedding in each section. Furthermore, cost of restoration time and power generation are minimized to achieve a fast and economic restoration. These objectives are subjected to GPP constraints and PTSS constraints. The GPP mathematical model individually is presented as follows:

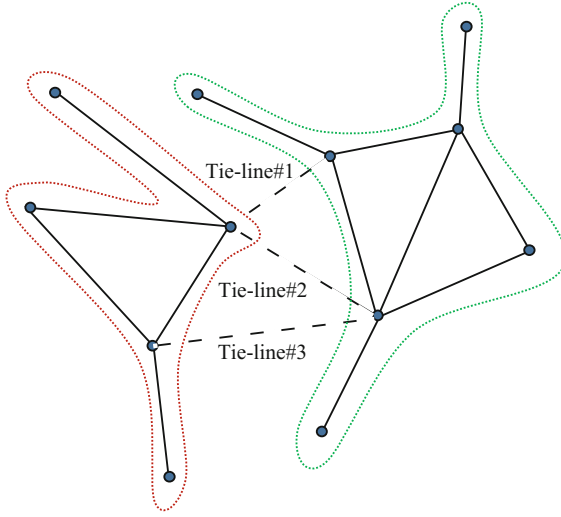
$$\min f^{GPP} = \sum_{b'=1}^{NB} \sum_{b=1}^{NB} y_{bb'} \cdot a_{bb'}. \quad (1)$$

$$y_{bb'} = 1 - \sum_{m=1}^{NS} s_{mb} \cdot s_{mb'}, \forall b, b'. \quad (2)$$

$$\sum_{m=1}^{NS} s_{mb} = 1, \forall b. \quad (3)$$

$$\sum_{b=1}^{NB} s_{mb} = n_m, \forall m. \quad (4)$$

$$\sum_{m=1}^{NS} n_m = NB. \quad (5)$$



**Fig. 1.** Graph partitioning example.

The PTSS model is constrained with upper and lower limits of power generation capacity (6), ramp up and ramp down limits of generation units (7). Transmission line power flows constraints are also presented with Eqs. (8)–(10). Since there is no power flow on tie-lines,  $y_{bb'}$  is considered in both (8) and (9) to ensure it. In this study, we set all buses with BS generation units as the reference buses in each section (10). The real power balance at each bus is given in equality constraint (11).

$$P_g^{G,\min} \leq P_{gt}^G \leq P_g^{G,\max}, \forall g, \forall t. \quad (6)$$

$$-RD_g^G \leq P_{gt}^G - P_{g(t-1)}^G \leq RU_g^G, \forall g, \forall t. \quad (7)$$

$$|P_{lt}^L| \leq P_l^{L,\max}(1 - y_{bb'}), \forall l \sim (b, b'), \forall t. \quad (8)$$

$$\left| P_{lt}^L - \frac{(\theta_{bt} - \theta_{b't})}{x_l} \right| \leq M \cdot y_{bb'}, \forall l \sim (b, b'), \forall t. \quad (9)$$

$$\theta_{ref,t} = 0, \forall t. \quad (10)$$

$$\mathbf{KG} \cdot \mathbf{P}^G - \mathbf{KL} \cdot \mathbf{P}^L + \mathbf{KD} \cdot \mathbf{LS} = \mathbf{KD} \cdot \mathbf{D}. \quad (11)$$

Furthermore, it is desired to minimize the total load shedding cost,  $f^{LS}$ , cost of restoration time,  $f^T$ , and total power generation cost,  $f^{Gen}$ . Therefore, the summation of these three terms is minimized in the objective of PTSS:

$$\min f^{LS} + f^T + f^{Gen} = VOLL \cdot \mathbf{LS} + \boldsymbol{\omega}^T \cdot (\mathbf{T} \circ \mathbf{s}) + \mathbf{c}^T \cdot \mathbf{P}^G. \quad (12)$$

Where  $VOLL$  is the value of loss of load which is equal to \$1000/MWh,  $\boldsymbol{\omega}$  is the outage cost vector (\$/h) and is predefined for each class of demand, and  $\mathbf{c}$  is the marginal cost of power generation (\$/MWh). There is a Hadamard product ( $\circ$ ) between  $\mathbf{T}$  and  $\mathbf{s}$  which means an element-wise multiplication of these two same-size matrices [11]. The matrix  $\mathbf{T}$  is a given restoration time which is approximated with the minimum possible operational delay to restore a load within each section and load pick-up time of that load which is explained in detail in Sect. 4.

To combine these two models and solve as a linear model, it is required to linearize Eq. (2) by replacing it with the following two inequalities:

$$-y_{bb'} - s_{mb} + s_{mb'} \leq 0, \forall (b, b') \sim l, \forall m. \quad (13)$$

$$-y_{bb'} + s_{mb} - s_{mb'} \leq 0, \forall (b, b') \sim l, \forall m. \quad (14)$$

In order to solve the model in a lower complexity, the combined GPP-PTSS model is recast as a bi-level programming.

### 3.1 Bi-level Programming

The GPP-PTSS model in a bi-level programming structure includes two models: upper and lower levels. The sectionalization would be done through the upper level, which is called “sectionalization” model. The sectionalization model includes GPP constraints as explained and for the sake of simplicity, the PTSS constraints are shown without the line power flow constraints. Therefore, GPP’s constraints take care of the power system’s network structure and the optimal power flow would be analyzed at lower level, which is called “restoration” model. In the restoration level, the sections’ pattern is fixed and the optimal values of load shedding and restoration time could be obtained.

To exclude the transmission line’s power flow from the sectionalization level it is required to revise the load balance Eq. (11). Without the line power flow, it is enough

to find the load balance per section hence, the network's line power flow limits could be replaced by the total load balance constraint at each section:

$$\mathbf{s} \cdot \mathbf{P}^G + \mathbf{LS}' = \mathbf{s} \cdot \mathbf{D}. \quad (15)$$

Where the first term in (15) is linearized by the given Lemma in [12].

Consequently, the total load shedding cost,  $f^{LS}$ , needs to be modified by replacing **LS** (load shedding per load) with **LS'** (load shedding per section), and replacing  $VOLL$  with  $\frac{NB \cdot VOLL}{NS}$ . Finally, the upper level minimizes the modified PTSS objective function and  $f^{GPP}$  with respect to linearized GPP's constraints and constraints (6), (7), and (11).

The restoration model is constrained by (6)–(11) to minimize the objective function (12). One of the main tasks of the restoration model is to find the optimal restoration time by finding the optimal period taken by BS units to supply the demand, which is called “load pick-up time” [13]. Constraint (16) is in charge of this task while in the objective function  $f^T$  is modified to  $-\omega^T \cdot \mathbf{T}^{Load}$ . The new  $f^T$  is negative due to the fact that constraint (16) will find the earliest time to have the zero-load shedding value. Therefore, to avoid  $T_d^{Load} = 0$ ,  $T_d^{Load}$  should be maximized in the objective function, which is equivalent to minimizing the negative value of it.

$$T_d^{Load} \leq \varphi_t + M \cdot LS_{dt}, \forall d, \forall t. \quad (16)$$

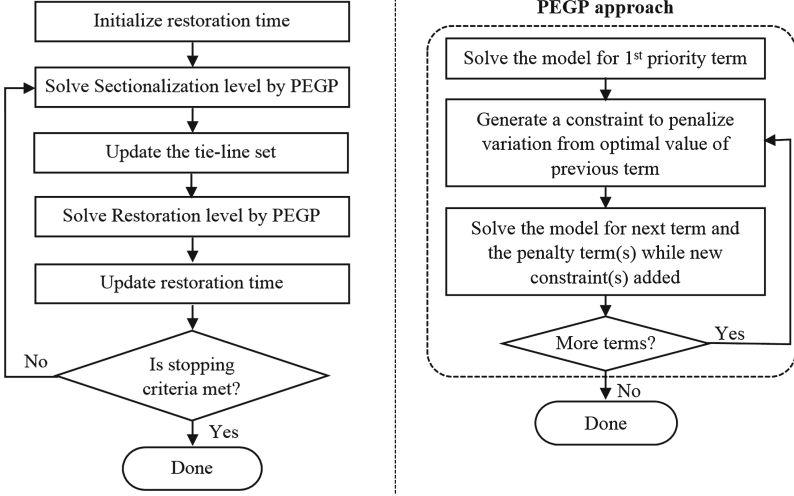
In the next section, the taken solution methodology to solve the proposed model is described.

## 4 Solution Methodology

A typical bi-level program can be solved via different approaches such as iterative optimization algorithm (IOA), mathematical programming with equilibrium constraints (MPEC), and penalty function methods. In this study, the IOA method has been applied and the result has been compared to the MPEC approach [14]. In addition, the PEGP method is taken to solve the model at each iteration by considering the multiple objective terms with different scales and priorities.

### 4.1 Iterative Optimization Algorithm

Iterative optimization method on a bi-level programming is a heuristic algorithm, which solves each level individually and sends updated variables under its control to the other level. The details of the method is illustrated by the flowchart in Fig. 2. As explained in Sect. 3, restoration time is composed of a constant and a variable terms. The constant term contains all delays including switching time, operators handling time, and unexpected delays that are assumed to be given [5]. The variable term is the load pick-up time ( $T_d^{Load}$ ) which is under control of the lower level model (restoration model) and it is initiated with  $T_d^{Load} = 0$ . Therefore, by the initial restoration time, the



**Fig. 2.** Iterative optimization algorithm supported by preemptive goal programming

IOA method can start from the upper level (sectionalization model) and iteratively solve the model until stopping criteria get satisfied. Here, the stopping criterion is defined based on the same sectionalization pattern at two consecutive iterations,  $\mathbf{s}^k$  and  $\mathbf{s}^{k-1}$ , as follows:

$$\|\mathbf{s}^k - \mathbf{s}^{k-1}\|_2 = 0. \quad (17)$$

## 4.2 Pre-emptive Goal Programming Method

Pre-emptive goal programming divides the goals into different priorities' sets and optimization would be started at the highest priority set, then the next sets are considered such that the optimal value of the previous set is preserved by limiting the feasible area to minimum violation from the optimal value set [15]. In the restoration model, different decision makers come with different priorities in terms of the objective function. In this paper, the importance of each objective term in GPP-based model are chosen as total number of disjoint edges, total load shedding cost, the cost of restoration, and the power generation cost.

At the first step, inequality constraint (18) is the first generated constraint:

$$f^{GPP} \leq f^{GPP*} + \varepsilon^{GPP}. \quad (18)$$

Where  $f^{GPP*}$  is the optimal value of  $f^{GPP}$  and  $\varepsilon^{GPP}$  is the deviation amount from the optimal value, which is penalized at the next objective function:

$$\min f^{LS} + \kappa^{GPP} \cdot \varepsilon^{GPP}. \quad (19)$$

In the new objective function  $\kappa^{GPP}$  is a fixed coefficient for the penalty term to ensure its minimization beside  $f^{LS}$ . The same approach is taken at the next steps until particular optimal value of all terms gives the final optimal solution.

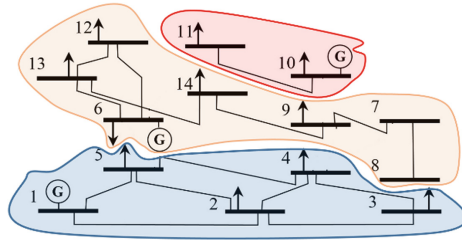
The efficiency of the proposed model is examined on two case studies in Sect. 5.

## 5 Numerical Results

To analyze the performance of the proposed model, the standard IEEE 14- and 118-bus test systems are chosen as small and large-scale test cases. The data regarding these two cases is available at [16]. The small-scale test case assumed to be equipped with three black start (BS) generation units on buses 1, 6, and 10 and the large-scale test case has eight BS units, pre-located on buses 10, 25, 49, 59, 69, 80, 89, and 100. The model is solved with CPLEX 12.3.0.0 under GAMS 24.8.3 on a PC with Intel Xeon 2.53 GHz, 12-core, and 128 GB of RAM. The both cases are solved within the first 24 h after a disturbance. It is also assumed that the systems' post-disturbance statuses are completely de-energized and given as-is.

### 5.1 Small-Scale Test System Results

Implementing the model on an IEEE 14-bus test system, the optimal sectionalized grid is illustrated by Fig. 3. The sections are well connected and formed by cutting just five lines from 20 lines of the 14-bus grid. The IOA approach converged at the first iteration. The load shedding percentage, restoration time, and line availability percentage are shown in Table 1. Since the IEEE 14-bus test system is not included by the selected comparable study [13], the respected results are produced by the authors of this paper based on the MPEC approach. Table 1 emerges the values of all three investigated elements achieved by GPP-IOA versus MPEC. The main observed reason for the better results by GPP-IOA solution approach is the stronger connectivity, 75% line availability, which causes the higher quality in the restoration process as well.



**Fig. 3.** The sectionalized IEEE 14-bus test system.

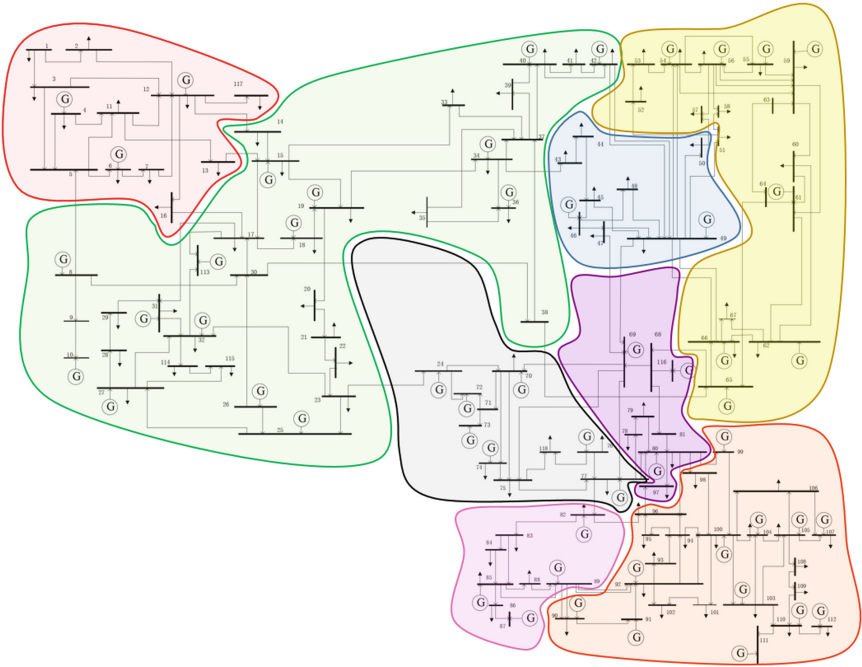


**Table 1.** Comparison between GPP-IOA and MPEC results.

Case	Solution approach	Load shedding %	Restoration time	Line availability%
Small-scale	GPP-IOA	26%	12.66 h	75%
	MPEC	39%	13.11 h	65%
Large-scale	GPP-IOA	18%	10.55 h	83%
	MPEC	26%	—	75%

### 5.2 Large-Scale Test System Results

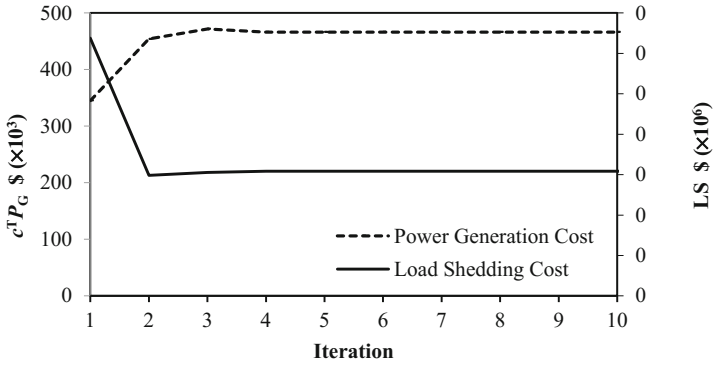
The optimal sectionalized grid for IEEE 118-bus test case is presented at Fig. 4.



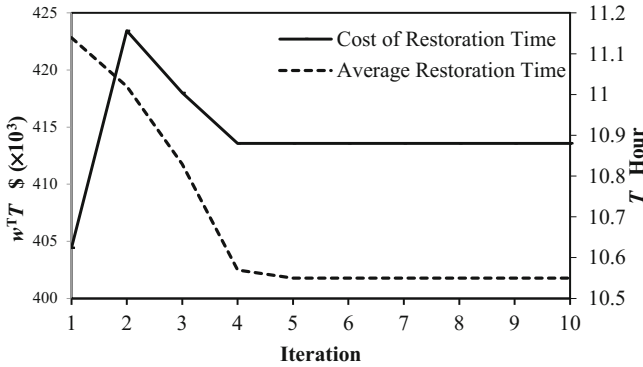
**Fig. 4.** The sectionalized IEEE 118-bus test system.

Figure 5 shows the total load shedding cost vs the total cost of power generation over iterations. The results as plotted in Fig. 5 shows a drop within the first iterations on load shedding cost while the required power is generated and jumped up as depicted by the cost of generation's curve.

The restoration time is also decreased sharply at the first four iterations as presented by Fig. 6 while a temporary pick value is observed at iteration 2 on the cost of restoration time and both have been stabled since iteration 5 with a restoration time of 10.55 h.



**Fig. 5.** Total cost of generation vs total load shedding cost on the large-scale case.



**Fig. 6.** Cost of restoration time and average restoration time on the large-scale case.

In general, the GPP-based model has a fast convergence and its performance is comparable with the selected study's results solved by MPEC with the same large-scale case study without GPP approach [13]. As presented in Table 1 the GPP model solved by IOA achieved better results in total load shedding value and available line percentages.

## 6 Conclusion

A power system subjected to a complete blackout must be restored in the lowest restoration time. In a resilient power system, a sectionalization approach would be utilized to have a fast and effective restoration. In this study, a graph partitioning technique is developed to build the sections. The proposed GPP-based PTSS presents a bi-level program solved via IOA and PEGP solution methodologies. The model's efficiency is examined with two case studies: IEEE 14- and 118-bus test systems and

the results have been compared with a MPEC method's results from another study. The comparison of the large-scale results shows 8% lower load shedding percentage and 8% higher line availability percentage via IOA solution method in contrast with the MPEC model from the selected study. In conclusion, the presented GPP-based PTSS model develops an efficient optimization framework which gives well-interconnected sections while most of demands are satisfied.

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