An Overview and Re-interpretation of Paradoxes of Responsiveness

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Abstract One of the most obvious desiderata of democratic decision-making is that the political outcomes (policies, elected persons, legislation) be responsive to popular opinions. In representative forms of governance the responsiveness is not expected to pertain to every single outcome, but the very idea of going to the people seems to presuppose some degree of responsiveness. In social choice theory several notions that aim to capture aspects of responsiveness have been introduced and related to other desiderata of social choice. We shall discuss the most common notions and discuss their relevance in democratic decision making. We shall also look at the paradoxes related to non-responsiveness from a novel angle, viz. we try to determine their significance to the multiple criteria decision making (MCDM). It turns out that some methods of aggregating criterion performances of policy alternatives can be ruled out because of their bizarre behavior under some decision settings.

1 Introduction

Responsiveness is one of the most obvious desiderata in democratic rule. At the very least unresponsive rules of governing are certainly not acceptable as the very idea of democracy presupposes that the ruled, the people, can, by expressing by their opinions in legitimate manner, bring about changes in the way public policies are formulated and executed. Elections are the normal institutions to transform the popular views into public policies or other electoral outcomes. The most common of the latter are, of course, those that pertain to composition of parliaments or offices of the president. But what does unresponsiveness, then, mean? A clear example of an unresponsive voting rule is a constant one which results in a fixed outcome, say x, regardless of the distribution of the expressed opinions by the voters. I.e. no

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matter how the voters vote, x always wins. Clearly, constant rules make the act of voting meaningless in the instrumental sense, that is, as a way of influencing the way public policies are to be pursued. A similarly minimalistic way of defining responsiveness as the exclusion of constant rules is the condition on rules known as citizens' sovereignty. This requires that, given a set of alternatives A of k alternatives and any ranking R of those alternatives, there is a distribution of voter opinions over those alternatives so that R is the outcome resulting from the application of the rule to this distribution. This condition excludes blatant discriminations against some alternatives. This doesn't mean that rules that satisfy citizens' sovereignty are *eo ipso* intuitively responsive to the voter opinions. A case in point is the unanimity rule: the *status quo* alternative, say x, is selected, unless all individuals prefer another alternative, say y, to x. This rule clearly satisfies citizens' sovereignty, but is extremely biased towards the *status quo*.

In what follows we shall investigate some intuitively natural forms of responsiveness of choice rules. The forms will be looked upon as invulnerability to certain kinds of paradoxes. Our primary focus is on variations in the choice sets resulting from rules under changes in individual opinions. Two types of settings are of interest: first, those where the changes in individual opinions happen in a fixed electorate, and second, those where the changes involve enlarging the electorate itself by including new voters in the voter set. The former settings will be called fixed electorate paradoxes and properties, while the latter will be called paradoxes and properties in variable electorates.

2 What Is Responsiveness?

A natural way of approaching the responsiveness problem is to start from comparing the opinions of the electorate to the result of the choice rule. The question then becomes, how well or accurately the latter represents the former. In any given choice situation we could argue that the better the choice result represents the voter opinions, the more responsive the rule. It turns out that nearly all voting rules that transform n-tuples of individual complete and transitive preference relations (rankings) (n being the number of voters) into collective rankings can be seen as the most optimal, i.e. most responsive, rules. What makes them different is their underlying idea of a consensus state, that is, a situation involving no disagreement as to the outcome and the distance metric used in measuring the distance of any preference profile from a consensus state. Such a consensus state can be one where all voters have identical rankings over the alternatives or one where all voters rank the same alternative first or one where a given alternative is the Condorcet winner. Similarly, the distance measure can be the inversion metric counting the number of binary inversions of adjacent alternatives needed to transform one ranking to another or a discrete metric that simply counts those rankings that differ in some respects from one another (see [14, 18]). More precisely, the inversion distance between two rankings R_1 and

R_2 over k alternatives is the smallest number of swaps of two adjacent alternatives required to transform R_1 to R_2.

If the consensus state is one where all voters have an identical ranking over alternatives and if the distance between any two rankings is measured by the inversion metric, then the outcomes ensuing from the application of Kemeny's rule are optimal in the sense of minimizing the distance between the observed profile and the desired consensus state. Similarly, it has been shown by Nitzan that the Borda count outcome represents best the voter opinions if the distance measure is the inversion metric and if the consensus state is one where all voters are unanimous about which alternative should be ranked first [18]. Plurality voting, in turn, can be seen as the optimal representation of the voters' opinions if the distance measure is the discrete metric and the consensus state is the same as in the Borda count. The discrete distance between R_1 and R_2 is defined to be zero if $R_1 = R_2$ and unity, otherwise. Similarly, most voting systems can be defined as optimal distance minimizing rules (see [3, 5, 12, 14]).

Looking at voting rules as distance minimizing devices from a consensus state reveals essential similarities and differences in their underlying motivation. The picture that emerges from this comparison is, however, purely static: the state of consensus—understood in various senses—is being compared with the observed profile of reported preferences of voters. The voting outcome is 'a response' to 'the stimulus' provided by the preference profile. The reasonableness of the response boils down to the plausibility of the consensus states and distance measures associated with various rules. A more nuanced picture of responsiveness of rules emerges when we compare the responses or outcomes of rules under various changes in the stimuli, i.e. preference profiles.

3 Responsiveness in Fixed Electorates: Monotonicity

The very idea of going to the people would seem to imply that the more voters support an alternative, the better chances the latter has for becoming the chosen one. Expressed in this way, the idea allows for several non-equivalent specifications. Firstly, it may mean that if the number of voters supporting an alternative is increased, then the alternative has at least as large a probability of being elected than before the increase. But what do we mean by 'support of an alternative'? At least three different interpretations are possible:

- the number of voters ranking the alternative first is increased,
- the position of the alternative is improved *vis-à-vis* some others, or
- some voters who rank the alternative first join the original electorate.

In fixed electorates, the third interpretation is excluded. In fact, the second interpretation is most common in the theory of voting. It can be further divided into two main concepts: (i) monotonicity, and (ii) Maskin monotonicity. According to the former, the additional support for an alternative means that its position is improved in at least one voter's preference ranking, *ceteris paribus*, i.e. the positions of all other alternatives with respect to each other remain the same. In Maskin monotonicity, in contrast, the additional support means that the position of an alternative is improved *vis-à-vis* some other alternatives, but no restrictions are imposed in the mutual positions of other alternatives.

With these distinctions in mind we can define the best-known responsiveness concept, monotonicity as follows (cf. [10]).

Definition 1 Upward monotonicity. Suppose that in a given profile over a set A of alternatives, $x \in A$ wins when rule D is applied. Suppose now that the profile is modified so that the position of x is improved, *ceteris paribus*, in at least one voter's preference ranking. Now, D is monotonic if and only if x remains the winner in the modified profile.

More recently, Miller has suggested another monotonicity concept well in the spirit of the preceding one. He calls it downward monotonicity, in contradistinction to the above which he calls upward monotonicity [15].

Definition 2 Downward monotonicity. Suppose that in a profile over a set of alternatives A, $x \in A$ wins when D is applied. Suppose moreover that a group of voters change their mind and lower the position of another alternative, y, *ceteris paribus*. Then D is downward monotonic if and only if no such change makes y the winner in the modified profile.

The *ceteris paribus* proviso is essential here. In fact, it is the only thing that distinguishes the upward monotonicity concept from Maskin monotonicity. The latter is defined as follows (cf. [13]).

Definition 3 Suppose that in a given profile over a set A of alternatives, $x \in A$ wins when rule D is applied. Suppose now that the profile is modified so that the position of x with respect to any other alternative y is at least as high in all voters' ranking and perhaps strictly higher for some $z \in A$ and some individuals. Now, D is Maskin monotonic if and only if x remains the winner in the modified profile.

Although *prima facie* the definitions are not very different, their difference is quite dramatic when we apply them to the most common voting procedures. It turns out that none of them is Maskin monotonic, while many are monotonic. In this regard Maskin monotonicity resembles the well-known independence of irrelevant alternatives condition of Arrow's impossibility theorem. For a thorough discussion on the significance of Maskin monotonicity in implementation and choice theory, the reader is referred to [1, 2].

The difference in the two definitions above is illustrated in terms of plurality voting in Table 1. The plurality winner there is x. Suppose that the profile is modified so that y is lifted ahead of z by the voter who ranked z first and y is lifted ahead of both w and z by the voter represented by the right-hand column. These changes do not involve x. Moreover, suppose that the position of x is improved by lifting it ahead of z in the second column from the left and ahead of z and w in the right-most column. So, for the rule to be Maskin monotonic, x would have to remain the winner

Table 1 Plurality voting is not Maskin monotonic

2 voters	1 voter	1 voter	1 voter
x	y	z	w
y	z	y	z
z	x	x	y
w	w	w	x

in the modified profile as well. However, after these modifications the plurality voting elects *y* since it is ranked first by three voters out of five in the modified profile. So, the plurality voting is not Maskin monotonic. On the other hand, it is obvious that the plurality voting is upward monotonic since lifting the winner ahead of some other alternatives in a profile, *ceteris paribus*, either leaves the number of voters ranking each alternative first unchanged (if the modifications pertain to alternatives ranked lower than first both in the original and modified profile) or increases the number of voters ranking the original winner first by some positive number. So—since no other alternative will be ranked first by more voters than originally—the original winner remains the winner also in the modified profile. For an analysis of some other systems in terms of Maskin monotonicity see [19].

Clearly, if a rule is Maskin monotonic, it is also monotonic, but the converse is not true as the case of plurality voting demonstrates. Overall, the primary significance of the Maskin monotonicity is in implementation theory where it has been shown to be a necessary condition for Nash-implementation (see e.g. [2]). The well-known Muller-Satterthwaite theorem states that all weakly unanimous and Maskin monotonic choice rules are dictatorial [17]. Weak unanimity is also known as Pareto principle: if every voter strictly prefers *x* to *y*, then *y* is not chosen. (For a slightly different formulation of the theorem, see [2]). Obviously, Maskin monotonic and Pareto optimal voting rules—if they exist—are in the dubious company of dictatorial rules. Hence, to avoid confusion, the distinction between monotonic and Maskin monotonic rules should be made explicitly.

It is well-known that the plurality runoff as well as alternative vote are non-monotonic. In fact, they are non-monotonic in a very strong sense: there are profiles where additional support for a winning candidate may turn it into a non-winner *and* diminishing support for a candidate may render it a winner even though it wasn't one in the original profile. Miller calls this a double monotonicity failure and provides the following example (Table 2) [15].

Table 2 Plurality runoff and double monotonicity failure [15]

38 voters	32 voters	30 voters
y	x	z
z	y	x
x	z	y

Here x wins. Suppose now that 9–17 voters from the left-most group lift x ahead of both y and z, *ceteris paribus*. Then, the runoff takes place between x and z, whereupon z wins. This is just another instance of the upward monotonicity failure, but there is another one as well in the same profile. To wit, let three voters of the left-most group drop y to the second place, *ceteris paribus*. Then, x is dropped out of the runoff contest and the winner is y. Hence we have an instance of the double monotonicity failure.

In the definition of upward monotonicity the starting point is a setting where a winner's position is improved, *ceteris paribus*, and one finds out whether the improvement is necessarily accompanied with the winner maintaining its status. One could envisage another, more general, notion of monotonicity whereby an improvement of an alternative's position in some rankings, *ceteris paribus*, is never accompanied with a lower rank for it in the social ranking. This notion would cover not only situations where the focus is on what happens to the winning alternative once its position is improved, but also situations where one looks at the position of non-winners upon an improvement in their ranking in individual preferences, *ceteris paribus*. This more general notion of monotonicity is applicable to social welfare functions, while the above definition applies to social choice functions. Since above definition is by now standard we shall, however, adhere to it.

4 Responsiveness in Variable Electorates: The No-Show Paradox

What the general formulation of monotonicity in the end boils down to is that an improvement of an alternatives position *vis-à-vis* the others, *ceteris paribus*, never lowers the position of the alternative in the collective ranking. This concerns fixed electorates, i.e. those where the modifications occur within the same electorate. In variable electorates, in contrast, the electorate is assumed to expand as a result of new voters joining it. The responsiveness of the voting rule is then typically determined by whether or not it satisfies the property called participation. This property is defined by means of the no-show paradox. The latter occurs whenever a group of identically-minded voters is better off—in terms of the voting outcome—when it abstains than when it votes according to its preferences. Thus, the conclusion that an instance of the no-show paradox occurs is based on comparing two outcomes: one, say O_1, resulting from the application of a given procedure, F, to a set A of alternatives and to a profile R of the set N of voters over A, and the other, say O_2, resulting from applying F to A and to a profile that consists of R augmented by a group of voters each having identical preferences over A. Whenever the added voters prefer O_1 to O_2 we have an instance of the no-show paradox.

This definition includes, as special cases, the two types of no-show paradoxes outlined by Fishburn and Brams [11] (see also [8, 9]). To wit,

Table 3 Plurality runoff and no-show paradox

26 voters	47 voters	2 voters	25 voters
x	y	y	z
z	z	z	x
y	x	x	y

Definition 4 No-show paradox 1: The addition of identical ballots with x ranked last may change the winner from another candidate to x.

And the other type:

Definition 5 No-show paradox 2: One of the candidates elected could have ended a loser if additional people who ranked him in first place had actually voted.

These are clearly non-equivalent definitions with a common feature: the added voters are better off not voting in both cases (see also [16]). In the former definition the fact that they vote according to their preferences brings about the outcome that is their worst, while without their votes, the outcome would have been something more preferable. In the latter definition, the added voters' votes to their most preferred candidate turn their favorite into a non-winner, whereas with their abstaining, it would have won. The non-equivalence of the definitions is demonstrated by the plurality runoff (a.k.a. alternative vote or instant runoff) and the following 100-voter profile (Table 3).

Let us first assume that the group of 47 voters in the second column do not vote at all. Since there are then only 53 voters, none of the three candidates gets elected on the first round. Instead, the second round is arranged between x and z, whereupon z wins with 27 votes against 26. Suppose now that the 47-voter group joins the competition. Its lowest ranked alternative is x. With this group joining the profile is the one depicted in Table 3. Now the second round involves x and y resulting in the victory of x, the worst alternative of the 47-voter group. Thus, the situation is one described Definition 4. On the other hand, the conditions of Definition 5 cannot apply to the plurality runoff system. The reason is the following. Let x be the winner in the original profile and add a group of voters with identical preferences with x ranked first to obtain the augmented profile. Adding the group does not affect the distribution of voters who rank other alternatives first. Thus, whichever alternative was the runoff competitor of x in the original profile, will be its competitor in the augmented profile. Since x defeated its runoff competitor in the original profile, it will defeat it in the augmented one as well since its support has been increased, while that of its competitor hasn't. If x won in the first round in the original profile—i.e. no runoff was required—it clearly does so in the augmented one as well (see also [9]). Thus, Definitions 4 and 5 are non-equivalent.

The case of plurality runoff is instructive in another way as well, viz. it shows that although apparently similar the properties of non-monotonicity and vulnerability to no-show paradox are not identical. Plurality runoff is upward non-monotonic, but not vulnerable to the no-show paradox in the sense of Definition 5. It turns out that—although seemingly closely related—upward non-monotonicity and vulnerability to the no-show paradox in the sense of Definition 5 are largely logically independent: there are systems that are vulnerable and monotonic in this sense and there are vulnerable ones that are non-monotonic. An example of the former combination is Copeland's rule and of the latter, as was just seen in Table 3, the plurality runoff and alternative vote. Of systems that are invulnerable to the no-show paradox in the sense of Definition 5 and are upward monotonic one can mention the Borda count and plurality voting [19]. Finally, Campbell and Kelly provide a constructive proof that there are systems that are upward non-monotonic, but at the same time invulnerable to the no-show paradox in the sense Definition 5 [4].

5 Extreme Forms

The no-show paradoxes can take on various degrees of severity. In particular, it may turn out that the alternative ranked first by a sub-group of unanimous voters will be a winner if they abstain, but a non-winner if they vote according to their preferences, *ceteris paribus*. So, by voting this group may turn their favorite from a winner to a non-winner. This is called the P-TOP paradox by Felsenthal and Tideman [7]. This is obviously the extreme form of Definition 5. It is also known as the strong no-show paradox [20].

The extreme form of Definition 4, on the other hand, occurs when a group of unanimous voters voting according to their preferences where x is ranked lowest, *ceteris paribus*, brings about an outcome where x wins, while, had the group abstained, some other alternative would have won. So, by voting the group changes the outcome from something that is not their worst to something that is. This paradox is called the P-BOT paradox [7]. Table 4 illustrates the strong no-show paradox under Copeland's rule. The example has been originally devised by Fishburn and thereafter utilized by Richelson as well as by Felsenthal and Nurmi [8, 10, 21].

Here v defeats more alternatives in pairwise comparisons (with the majority rule) than any other alternative and is thus the Copeland winner. Suppose we add a voter with the preference ranking *vwxyz* to the original profile of Table 4. This modification leaves v's Copeland score unchanged, but makes w the Condorcet, and hence

Table 4 Copeland's rule and the P-TOP paradox

2 voters	1 voter	1 voter
w	z	v
v	y	z
x	x	y
y	w	x
z	v	w

Table 5 Copeland's rule and the P-BOT paradox

5 voters	4 voters
y	z
z	v
v	x
x	y

Copeland, winner. Thus, adding a voter ranking the original winner first to the profile promotes another alternative ahead of the original winner.

Table 5 illustrates the vulnerability of Copeland's rule to the other extreme form, the P-BOT paradox.

As y is the strong Condorcet winner, it is *eo ipso* the Copeland winner. If we now add three voters all having the the ranking: $xvyz$, this makes the 12-vote profile cyclic in terms of the majority comparisons. The Copeland winners are now z and v. Thus we have a weak version of the P-BOT paradox whereby the alternative ranked last by the added voters belongs to the choice set in the augmented profile, whereas it was not in the choice set in the original one.

Although we have used just Copeland's rule in the illustration of the paradoxes, it turns out that the P-TOP and P-BOT paradoxes are quite common among voting rules. Indeed, Pérez has shown that nearly all Condorcet extensions are vulnerable to either one or both extreme forms of the no-show paradox, the only exceptions being the Minmax and Young's rules [20]: the former is invulnerable to both P-TOP and P-BOT, while the latter is invulnerable to the P-BOT one [8].

6 The MCDM Context

The vulnerability to monotonicity failures is typically discussed in the context of voting. There these failures confront the voters with contradictory incentives. On the one hand, voting for one's favorite would seem precisely what is needed to increase the probability of his/her getting elected. On the other hand, depending on the procedure used the voting might jeopardize the favorite's chances of being elected or—worse still—might lead to the election of the worst possible candidate. But monotonicity failures can play a role in MCDM as well. To wit, let us assume that we have a choice situation involving n criteria and ordinal measurements of the performance of the k decision alternatives on those criteria. To make the decision one needs a rule which allows the determination of the best alternative or priority ranking over all alternatives to make a decision. The voting rules discussed above can then be used to find a solution to the choice problem. Would this new context affect the significance of the monotonicity failures?

Arguably not. If a non-monotonic aggregation of criterion measurements is being resorted to, this would mean that improving an alternative's position on some criteria *ceteris paribus* might exclude it from the set of chosen alternatives even though it would have been chosen had the improvement not been made. This would apply to upward non-monotonic systems. Similarly, in downward non-monotonic systems the worsening of an alternative's measurement value could lead to its choice. Hence, the use of a (downward or upward) non-monotonic criterion aggregation rule could lead to bizarre outcomes.

The same conclusion holds for monotonicity failures in variable criterion sets. Introducing new criteria—in itself a very common occurrence—may result in strange choices. Table 4 can be viewed from the MCDM angle as an illustration: with four criteria and measurements depicted in the table Copeland's rule results in v. However, if another criterion on which v is on top is added, another alternative—here w—is chosen. To make the case somewhat more concretely, suppose the alternatives are some devices (e.g. fighter jets) with several essential technical qualities (maximum speed, fuel consumption, agility, easiness of service) along which the order of preference can be formed. Suppose that five alternatives can be placed in the order of priority along these technical criteria as in Table 4. Using Copeland's rule v is chosen. Then someone suggests another criterion, overall cost of purchase and maintenance, and it turns out that v is best on this new criterion. If Copeland's rule is used in the new setting of five criteria, w emerges as the winner. This is undoubtedly somewhat counterintuitive.

Equally if not more bizarre is the setting exhibited in Table 5. In the original setting 5 out of 9 criteria place y at the top and by Copeland's rule it is chosen. If three criteria are added as described so that on all of them z is ranked last, the new choice set includes this last ranked alternative.

Although Copeland's rule has been used as an example above, these anomalies characterize many other other rules based on pairwise comparison of alternatives. In particular, it applies to most Condorcet extensions as was pointed out by Pérez. Using these rules in aggregation of criterion measurements is thus questionable.

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