

## Chapter 2

# Radio Resource Allocation for Device-to-Device Underlay Communications

### 2.1 Introduction

With the increasing demand for local traffic, device-to-device (D2D) communications under the control of evolved Node B (eNB) have recently received a great deal of attention [1–4]. Reusing the same spectrum as in, not as for the cellular communications, user equipments (UEs) in a cellular network in proximity can set up direct transmissions, which potentially increases the overall spectral efficiency [5]. In the Third Generation Partnership Project (3GPP), UEs are provided with a resource pool (time and frequency) in which they attempt to receive scheduling assignments, and eNB controls whether UEs may apply scheduled or autonomous D2D transmissions [6]. However, D2D communications generate interference to the cellular network if the radio resources are not properly allocated [7–9]. In addition, multiple D2D pairs in the same channel also create mutual interference [10]. Thus, interference management becomes one critical issue for D2D communications underlaying cellular networks.

In the literature, much attention has been paid to manage the interference in D2D networks. The studies in [11] propose a radio resource allocation algorithm using fractional frequency reuse to alleviate the interference between D2D pairs and cellular UEs. The work in [12, 13] tackles the interference management issues from the economy perspectives. In [12], the authors formulate the resource allocation problem as a reverse iterative combinatorial auction game, and propose a joint radio resource and power allocation method to increase energy efficiency. In [13], a sequential second price auction mechanism is designed to allocate the spectrum resources for D2D communications with multiple user pairs.

As shown in the literature, though D2D communication may generate additional interference to cellular systems, it improves the system throughput with proper interference management [14]. Therefore, the allocation of radio resources for D2D underlay communications needs further studies for efficient solutions with low

complexity. Graph theory is a useful tool to solve this kind of resource allocation problems in wireless communications [15, 16]. With graph theory, cellular UEs and D2D pairs are modeled as vertices in a graph, and the interference links between the UEs are constructed as edges [17, 18]. In [17], the weight of edges is used to represent the interference between two vertices, and the channel allocation is to iteratively gather vertices from the corresponding channel, taking both the interference value and the cluster value into account. In [18], the system model is constructed as a weighted bipartite graph, and the channel allocation problem is formulated as a matching problem to maximize the capacity.

However, it is worth mentioning that the conception of edge in graph theory might not be sufficient in modeling the interference relation due to the cumulative effect of the interference. Specifically, the interference from several vertices may constitute a strong interferer, even though the interference from each individual vertex is weak [19, 20]. When the cumulative interference from neighboring D2D pairs or cellular UEs exceeds a threshold, it may reduce the communication quality of all the users. Hence, it is necessary to take into account the cumulative impact of multiple interference sources to the cellular UEs and D2D pairs as victims.

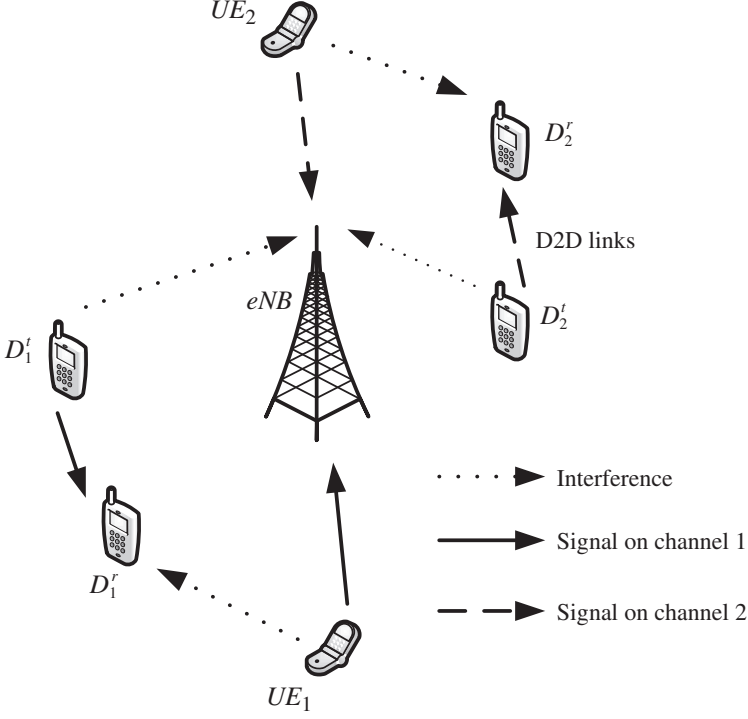
To this end, in this paper, we use the hypergraph to solve the interference management problem for D2D communication underlaying cellular networks. A hypergraph is a generalization of an undirected graph, in which the hyperedges are any subsets of the given set of vertices, instead of exactly two vertices defined in the traditional graph [21]. In wireless networks, the hypergraph achieves better approximation accuracy than the traditional graph as it effectively captures the cumulative interference. As such, the system capacity can be further improved by the hypergraph based method, compared to the traditional graph approach [22].

The goal of this chapter is to develop a resource allocation model for D2D communications underlaying cellular networks, in which the data sum-rate of D2D pairs and cellular UEs are maximized. We first formulate a resource allocation problem for multiple D2D pairs sharing channel resources with one cellular UE to maximize the cell capacity. Subsequently, we study the resource allocation problem using hypergraph theory. A hypergraph coloring method with low complexity is proposed to address the channel allocation for both D2D pairs and cellular UEs, which effectively increases the cell capacity. Simulation results show that the proposed hypergraph based method can achieve a performance very close to the optimal result, and perform much better than the traditional graph based method.

## 2.2 System Model and Problem Formulation

### 2.2.1 System Model

As shown in Fig. 2.1 [23], we consider an uplink transmission scenario in a cellular network that consists of  $N$  cellular UEs and  $M$  D2D pairs. We denote a cellular UE by  $U_n$ ,  $1 \leq n \leq N$ , and a D2D pair by  $D_m$ ,  $1 \leq m \leq M$ . Here, we use  $D'_m$  to represent



**Fig. 2.1** System model for D2D communications underlaying cellular network when sharing uplink resource

the transmitter of D2D pair  $D_m$ , and  $D_m^r$  to represent the receiver of D2D pair  $D_m$ . Orthogonal Frequency Division Multiple Access (OFDMA) is employed to support multiple access for both the cellular and D2D communications, where a set of  $K$  channels are available for resource allocation. In this system, the eNB coordinates the resource allocation between cellular UEs and D2D pairs. We assume that D2D pairs transmit with the power denoted by  $P^d$ , and cellular UEs use the transmission power  $P^c$ .

The channel is modeled as a Rayleigh fading channel, and the channel gains can be calculated by

$$\begin{cases} g_n^c = L_n^c h_n^c, & \text{cellular link from } U_n \text{ to eNB;} \\ g_m^{t,r} = L_m^{t,r} h_m^{t,r}, & \text{D2D link from } D_m^t \text{ to } D_m^r; \\ g_m^t = L_m^t h_m^t, & \text{link from } D_m^t \text{ to eNB;} \\ g_{n,m}^{c,r} = L_{n,m}^{c,r} h_{n,m}^{c,r}, & \text{link from } U_n \text{ to } D_m^r; \\ g_{i,m}^{t,r} = L_{i,m}^{t,r} h_{i,m}^{t,r}, & \text{link from } D_i^t \text{ to } D_m^r, \end{cases} \quad (2.1)$$

where  $L_n^c, L_m^{t,r}, L_m^t, L_{n,m}^{c,r}$ , and  $L_{i,m}^{t,r}$  denote the corresponding distance-dependent path loss, and  $h_n^c, h_m^{t,r}, h_m^t, h_{n,m}^{c,r}$ , and  $h_{i,m}^{t,r}$  denote the fading channel, respectively,  $1 \leq n \leq N$ ,  $1 \leq m \leq M$ ,  $1 \leq i \leq M$ , and  $i \neq m$ . The thermal noise satisfies independent Gaussian distribution with zero mean and variance  $\sigma^2$ .

The instantaneous SINR of the received signal at the eNB from cellular UE  $U_n$  over channel  $k$  can be written as

$$\gamma_n^c = \frac{P^c g_n^c}{\sigma^2 + \sum_{m \in \mathcal{C}_k} P^d g_m^t}, \quad (2.2)$$

and the instantaneous SINR at the D2D receiver  $D_m^r$  over channel  $k$  is given by

$$\gamma_m^d = \frac{P^d g_m^{t,r}}{\sigma^2 + \sum_{n \in \mathcal{C}_k} P^c g_{n,m}^{c,r} + \sum_{i \neq m, i \in \mathcal{C}_k} P^d g_{i,m}^{t,r}}, \quad (2.3)$$

where  $\mathcal{C}_k$  represents the set of cellular UEs and D2D pairs to which channel  $k$  is allocated.

### 2.2.2 Problem Formulation

We assume that a channel can be allocated to at most one cellular UE, and a maximum of one channel can be utilized by a D2D pair or a cellular UE. For convenience, we denote the channel allocation matrix by

$$S_{(N+M) \times K} = \begin{pmatrix} A_{N \times K} \\ B_{M \times K} \end{pmatrix}, \quad (2.4)$$

where  $A_{N \times K} = [\alpha_{n,k}]$  represents the channel allocation matrix for the cellular UEs, and  $B_{M \times K} = [\beta_{m,k}]$  stands for the channel allocation matrix for the D2D pairs,  $1 \leq n \leq N$ ,  $1 \leq m \leq M$ ,  $1 \leq k \leq K$ . The value of  $\alpha_{n,k}$  and  $\beta_{m,k}$  are defined as

$$\alpha_{n,k} = \begin{cases} 1, & \text{when channel } k \text{ is allocated to } U_n, \\ 0, & \text{otherwise,} \end{cases} \quad (2.5)$$

and

$$\beta_{m,k} = \begin{cases} 1, & \text{when channel } k \text{ is allocated to } D_m, \\ 0, & \text{otherwise.} \end{cases} \quad (2.6)$$

Our objective is to maximize the cell capacity by optimizing the channel allocation variables  $\{\alpha_{n,k}; \beta_{m,k}\}$  for the cellular UEs and D2D pairs, which can be formulated as

$$\max \sum_{k=1}^K \left[ \sum_{n=1}^N \log_2(1+\gamma_n^c) \alpha_{n,k} + \sum_{m=1}^M \log_2(1+\gamma_m^d) \beta_{m,k} \right] \quad (2.7)$$

$$s.t. \quad \begin{cases} \sum_{n=1}^N \alpha_{n,k} \leq 1, \\ \sum_{k=1}^K \alpha_{n,k} \leq 1, \quad \sum_{k=1}^K \beta_{m,k} \leq 1, \end{cases} \quad (2.8)$$

where  $\gamma_n^c$  and  $\gamma_m^d$  are given in (2.2) and (2.3), respectively. Constraints in (2.8) imply that each channel can be allocated to at most one cellular UE, and a maximum of one channel can be utilized by each D2D pair or each cellular UE.

Note that the aforementioned resource allocation problem in (2.7) is a NP-hard combinatorial optimization problem with nonlinear constraints, graph coloring is an approximate and efficient method for such a resource allocation problem [24]. Thus, we formulate the channel resources as  $K$  different colors, the cellular UEs as  $N$  (cellular) vertices, and the D2D pairs as  $M$  (D2D) vertices in the plane. Consequently, the channel allocation problem is transformed into a coloring problem of the vertices with fixed colors [25]. In the following two subsections, we will demonstrate the graph and the hypergraph based methods, respectively.

## 2.3 Traditional Graph Based Channel Allocation

Before introducing the hypergraph based channel allocation method, we describe the conventional graph based method. In a graph, vertices represent the cellular UEs and the D2D pairs, and edges indicate that the interference between connected vertices does not allow them to use the same channel simultaneously [26]. The graph based method contains the graph construction and the channel allocation algorithm as follows.

### 2.3.1 Graph Construction

We transform the interference information into a graph. A cellular UE  $U_n$  and a D2D pair  $D_m$  are connected by an edge which satisfies that the wanted signal ratio to the interference is below a threshold:

$$\frac{P^c g_n^c}{P^d g_m^t} < \delta_c; \text{ at the eNB receiver,} \quad (2.9)$$

or

$$\frac{P^d g_m^{t,r}}{P^c g_{n,m}^{c,r}} < \delta_d; \text{ at the D2D receiver } D_m, \quad (2.10)$$

where  $\delta_c$  and  $\delta_d$  are the thresholds selected to determine the severity of the interference at the eNB and the receiver of a D2D pair, respectively. Two D2D pairs  $D_i$  and  $D_m$  are connected by an edge if

$$\frac{g_m^{t,r}}{g_{i,m}^{t,r}} < \delta_d; \text{ at the D2D receiver } D_m, \quad (2.11)$$

which indicates that if the interference from another D2D pair is strong, these two D2D pairs cannot share the same channel. Besides, two cellular UEs  $U_i$  and  $U_j$  always form an edge for the assumption that two cellular UEs cannot share the same channel. In this way, an interference graph is constructed.

### 2.3.2 Channel Allocation Algorithm

After the graph construction, we use the greedy coloring algorithm in [24] to color the constructed graph. We define the available color set by all the colors except the colors used in the connected vertices. The algorithm successively colors the vertices in a color randomly chosen in the corresponding available color set, in the descending order of degree. If the available color set becomes empty, the vertex remains uncolored. In this way, the cellular UEs and the D2D pairs are classified into clusters with different colors, where the colors represent the channels. Finally, the channels are allocated to the D2D pairs and cellular UEs with mutual interference below the given threshold. These detailed algorithms are shown in Table 2.1.

## 2.4 Hypergraph Based Channel Allocation

In the traditional graph based method of Sect. 2.3, the edge connecting two vertices is not sufficient to model the interference in a wireless network, because some weak interferers together may constitute a strong cumulative interferer to affect the link quality. In this subsection, the hypergraph method, in which a hyperedge contains several vertices, is used for interference modeling.

**Table 2.1** Graph based resource allocation method**Stage I: Graph Construction**

- \* Cellular UEs  $U_n$  and  $U_j$  form an edge,  $\forall U_n, U_j$ , where  $n \neq j$ .
- \* A cellular UE  $U_n$  and a D2D pair  $D_m$  form an edge if they satisfy (2.9) or (2.10).
- \* D2D pairs  $D_i$  and  $D_m$  form an edge if they satisfy (2.11).

**Stage II: Graph Coloring Algorithm**

- \*  $i = 1$ . Find a vertex of the maximum degree and label it  $x_i$ .
- \* **repeat**
- 1. Set  $i = i + 1$ . Select from the unexamined subgraph a vertex  $x$  which has the maximum degree, and label it  $x_i$ .
- 2. Break the edges which connect to vertex  $x_i$ ;
- \* **until** All the vertices in the graph are examined.
- \* Starting from  $i = 1$ , select a color randomly from the available color set to color  $x_i$ . If the available color set is empty, leave the vertex  $x_i$  uncolored.

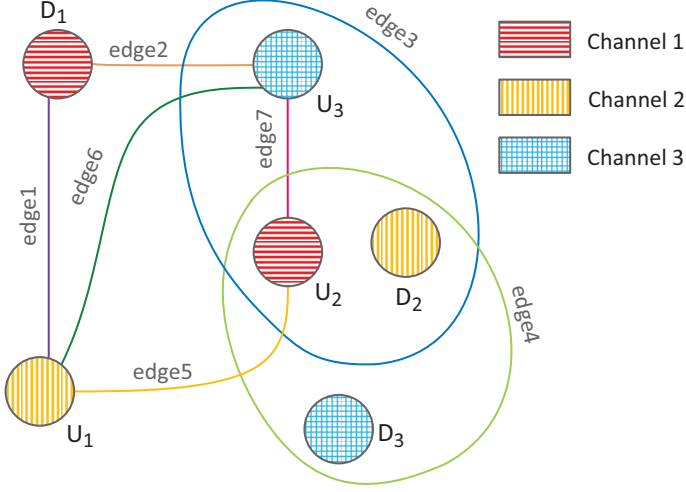
### 2.4.1 Hypergraph Construction

In the following, we will present a hypergraph based channel allocation method to solve the resource allocation problem. The first step is to construct the hypergraph for the mutual interference between D2D pairs and cellular UEs, and the next one is to color the constructed hypergraph. By hypergraph coloring, different subsets of cellular UEs and D2D pairs are generated, where one subset corresponds to one channel. Finally, orthogonal channels are assigned to each subset, which means that the cellular UE and D2D pairs in the subset share the channel.

In the hypergraph construction, we define two kinds of interferers. The first kind is *independent interferer*, and the second one is *cumulative interferer*. We define that the independent interferers of a D2D receiver or the eNB receiver are the D2D pairs and cellular UEs which decrease the received SINR independently. The cumulative interferers decrease SINR notably when combined in the receiver. We construct the hypergraph by the following steps.

#### 2.4.1.1 Independent Interferer Recognition

The first step is to select the independent interferers. Under the assumption that a maximum of one channel can be utilized by a cellular UE, one cellular UE can be regarded as an independent interferer of another, and thus, they form an edge. This step is to avoid the severe interference which originates from two UEs sharing the same channel. We give an example in Fig. 2.2 with three cellular UEs and three D2D pairs which are denoted by  $U_1, U_2, U_3, D_1, D_2$  and  $D_3$ , respectively. According to the aforementioned construction, cellular  $U_1, U_2$  and  $U_3$  form edge 5, edge 6 and edge 7.



**Fig. 2.2** An example of the hypergraph modeling

Next, we search the independent interferers for each UE, and construct the corresponding edges. Similar to the graph based method, for the cellular UEs, we follow the pairwise comparison as we have done in Sect. 2.3 to select the independent interferers. If cellular UE  $U_n$  and D2D pair  $D_m$  satisfy (2.9) or (2.10), they form an edge. Similarly, we also make the pairwise comparison for the D2D pairs to select independent interferers. If D2D pairs  $D_i$  and  $D_m$  satisfy (2.11), they form an edge as well. As shown in Fig. 2.2,  $U_1$  and  $D_1$  form edge 1, and  $U_3$  and  $D_1$  form edge 2. In the next paragraph, we construct the hyperedges, accounting for the cumulative interference from different users.

#### 2.4.1.2 Cumulative Interferer Recognition

After all the independent interferers are determined, the next step is to find the cumulative interferers, and construct the hyperedge. The cumulative interference is gathered from more than one UEs, except the independent interferers. We select a number of UEs, and compare the cumulative interference with an interference threshold  $\eta$  to verify whether they become interferers if cumulated. For instance, we select  $Q$  UEs, including cellular and D2D interferers, and then compare the cumulative interference to the wanted signal to determine whether they together form a hyperedge. For a cellular UE  $U_n$ , if the wanted signal to the cumulative interference ratio is below a threshold  $\eta_c$ , the cumulative interferers and the cellular UE together form a hyperedge, i.e.,

$$\frac{P^c g_n^c}{\sum_{m=1}^G P^d g_m^d} < \eta_c; \text{ at the eNB receiver.} \quad (2.12)$$

And for a D2D pair  $D_m$ , if the wanted signal to the cumulative interference ratio is below a threshold  $\eta_d$ , the cumulative interferers and the D2D pair together form a hyperedge, i.e.,

$$\frac{P^d g_m^{t,r}}{\sum_{j=1}^{F_m} P^c g_{j,m}^{c,r} + \sum_{i=1}^{Z_m} P^d g_{i,m}^{t,r}} < \eta_d; \text{ at the D2D receiver.} \quad (2.13)$$

Here,  $F_m$  and  $Z_m$  are the number of the cellular and D2D interferers in the hyperedge, respectively, i.e.,  $Z_m + F_m = Q$ . As the example shown in Fig. 2.2,  $U_2$ ,  $D_2$  and  $D_3$  form edge 4, and  $U_2$ ,  $U_3$  and  $D_2$  form edge 3.

It is worth mentioning that the value of  $Q$  is optional. Here, we only consider constructing the hypergraph with  $Q$  equal to 2, because it would be sufficient for the modeling. The hyperedge is to select  $Q$  UEs which generate severe interference to the examined UE, and judge whether the interference meets the criteria. With a higher value of  $Q$ , the complexity of the construction will increase. However, from the simulation results in Sect. 2.6, the cell capacity will increase less than 1% when the value of  $Q$  adds by 1. To achieve a compromise between cell capacity and computational complexity, we construct the hypergraph with  $Q = 2$ .

By definition, the union of hyperedges need to be the vertex set  $X$ . A special case may occur, where one vertex is neither an independent interferer of any UE, nor any of the cumulative interferers. In such a case, the union of hyperedges is not equal to the vertex set  $X$ . The vertex which is not in any hyperedge forms a hyperedge itself. In this way, the union of hyperedges is equal to vertex set  $X$ . After all these steps, hypergraph  $H$  can be constructed.

### 2.4.2 Hypergraph Coloring Algorithm

After hypergraph construction, hypergraph  $H$  can be colored. A color in the hypergraph corresponds to a channel, and coloring vertices is equivalent to allocating a channel to the D2D pairs and cellular UEs. Similar to the graph coloring in Sect. 2.3, the vertices contained in the same hyperedge cannot be colored by the same color. In this way, the cumulative interference can be alleviated.

Since coloring of the hypergraph is NP-hard, there is no computationally efficient algorithm to obtain the optimal solution [28]. Coloring algorithms have been proposed to color a hypergraph efficiently in [27]. The one mentioned in [28] is a greedy algorithm to color the hypergraph which is colorable. This implies that there exists a sufficient number of colors to color the hypergraph. However, in the OFDMA network, the condition may not be fulfilled, because the number of vertices may change as a function of cell load, while the number of channels is fixed. If the network is heavily loaded, it is not possible to color the whole hypergraph. In the light of these observations, we propose to modify the greedy method in Table 1.1 to meet the needs in an OFDMA cell.

**Table 2.2** Hypergraph based resource allocation method**Stage I: Hypergraph Construction**

\* One cellular UE can be regarded as the independent interferer of another, and thus, two cellular UEs form an edge.

**\* repeat**

1. Compare the SINR with the threshold  $\delta$  to select independent interferers.
  - 1) For a cellular UE  $U_n$ , if it satisfies (2.9), the D2D pair  $D_m$  is an independent interferer.
  - 2) For a D2D pair  $D_m$ , if it satisfies (2.10) or (2.11), the cellular UE  $U_n$  or D2D pair  $D_i$  is an independent interferer as well.
2. Form edges with the independent interferers.

\* **until** All UEs find their independent interferers.

**\* repeat**

1. Compare the SINR with the threshold  $\eta$  to find cumulative interferers.
  - 1) For a cellular UE  $U_n$ , if it satisfies (2.12), the D2D pairs are the cumulative interferers.
  - 2) For a D2D pair  $D_m$ , if it satisfies (2.13), the cellular UEs and D2D pairs are the cumulative interferers.
2. Form hyperedges with the cumulative interferers.

\* **until** All UEs find their cumulative interferers.

\* The vertex which is not in any hyperedge or edge forms a hyperedge itself.

**Stage II: Hypergraph Coloring Algorithm**

\*  $i = n$ ,  $H_n = H$ . Find a vertex of the minimum monodegree in  $H_n$  and label it  $x_n$ .

**\* repeat**

1. Set  $i = i - 1$ , and strongly delete the vertex  $x_{i+1}$  and form an induced sub-hypergraph  $H_i = H_{i+1} - x_{i+1}$ .
2. Find a vertex of the minimum monodegree in  $H_i$  and label it  $x_i$ .

\* **until**  $i = 0$ .

\* Starting from  $i = 1$ , color the vertex  $x_i$  in a color randomly selected from the corresponding available color set, successively. When the available color set is empty, remain the vertex  $x_i$  uncolored.

The modified method is presented in Table 2.2. The difference between this modified method and the greedy method in [28] lies in the number of colors. The modified method uses a fixed number of colors instead of the lowest number of colors in [28]. According to graph based resource allocation method, the D2D pairs and the cellular UEs have equivalent opportunity in resource allocation. When the D2D pairs have better channel conditions, the D2D pairs can be allocated to channels instead of the cellular UEs.

It is worth mentioning that hypergraph coloring is a method to obtain the sub-optimal solution in polynomial time. According to the description of hypergraph based resource allocation method, the vertex with maximum monodegree is colored first. This implies that the UE which generate the largest interference are allocated to

the channels first, then other UEs can utilize other channels to avoid the interference. In this way, more UEs can be allocated to channels, and hence the capacity increases. Hypergraph coloring is therefore a greedy method to obtain a sub-optimal solution.

## 2.5 Property Analysis

In this section, we first evaluate the performance of the hypergraph based method, and then address the computational complexity of both the graph and the hypergraph methods.

**Proposition 2.1** *When the number of cellular UEs and the number of channels are fixed, the cell capacity will first increase and then become saturated as the number of D2D pairs increases.*

*Proof* For a large number of D2D pairs, we assume that the monodegree of D2D pair  $x$  is the lowest. If the monodegree of D2D pair  $x$  is higher than the number of colors  $K$ , the D2D pair  $x$  cannot be colored, i.e., allocated to the channels. When the number of D2D pairs grows, the traditional graph and the hypergraph methods only select those D2D pairs, which generate less interference to replace the previous candidates. This is the reason why the capacity becomes saturated with the increasing number of D2D pairs.  $\square$

**Proposition 2.2** *We divide the vertex set  $X$  of hypergraph  $H$  into cellular set  $X_c$  and D2D set  $X_d$ . When the number of cellular UEs increases by 1, the cellular UEs and D2D pairs form a new hypergraph  $H'$ . If  $M(H) = \max_{Y \subseteq X_c} \min_{x \in Y} m(x, H \setminus Y)$ , then  $M(H') = M(H) + 1$ ; Otherwise,  $M(H) \leq M(H') \leq M(H) + 1$ .*

*Proof* In hypergraph construction, if the number of vertices increases by 1, the monodegree of the other vertices will increase by at most 1. The reason is that once two vertices form an edge, one vertex will not be the cumulative interferer of the other, and they cannot form a hyperedge. In addition, any two cellular UEs are bound to form an edge, and thus, if the number of cellular UEs increases by 1, the monodegree of each cellular UE will increase by 1 as well.

Under the assumption  $M(H) = \max_{Y \subseteq X_c} \min_{x \in Y} m(x, H \setminus Y)$ , cellular UE  $x$  is the vertex which has the maximum value of the minimum monodegree. According to the aforementioned analysis, if the monodegree of cellular UE  $x$  increases by 1, then the monodegree of the other vertices will increase by at most 1. Thus, cellular UE  $x$  is still the vertex which has the maximum value of the minimum monodegree, and  $M(H') = M(H) + 1$ . Otherwise, a D2D pair  $x$  is the vertex which has the maximum value of the minimum monodegree. If the mutual interference between D2D pair  $x$  and the new cellular UE cannot form an edge nor a hyperedge,  $M(H') = M(H)$ . Therefore, if the vertex is not a cellular UE,  $M(H) \leq M(H') \leq M(H) + 1$ .  $\square$

**Proposition 2.3** *The maximum value of the minimum monodegree generated by the greedy hypergraph coloring algorithm is equal to  $M(H)$ .*

*Proof* According to Definition 1.19, the maximum value of the minimum monodegree over all vertices  $d$  generated by hypergraph based resource allocation method needs to satisfy that  $d \leq M(H)$ . On the other hand, hypergraph based resource allocation method strongly deletes the vertex of the minimum monodegree, and there must be an induced sub-hypergraph  $H \setminus Y_0$  obtained by also strongly deleting those vertices in  $Y_0$ . For a vertex  $y \in Y_0$ ,

$$m(y, H \setminus Y_0) = \min_z m(z, H \setminus Y_0) = M(H). \quad (2.14)$$

In the generic step  $l \geq 1$  of hypergraph based resource allocation method, the first vertex is deleted from set  $Y_0$  such that the minimum monodegree of the induced hypergraph  $H \setminus Y_0$  is equal to  $M(H)$ . Thus,  $H \setminus Y_0$  is an induced sub-hypergraph of  $H_l$ . The minimum monodegree  $m(x_l, H_l)$  of  $H_l$  is higher than that of  $H \setminus Y_0$ . Therefore,

$$M(H) = m(y, H \setminus Y_0) \leq m(x_l, H_l) \leq d. \quad (2.15)$$

□

**Proposition 2.4** *For any hypergraph  $H = (X, E)$ , we have  $\chi(H) \leq M(H) + 1$ .*

*Proof* From Proposition 2.3, the upperbound of the minimum monodegree obtained by hypergraph based resource allocation method is equal to  $M(H)$ . In the coloring process, vertex  $x_i$  will be contained by at most  $M(H)$  hyperedges. In the case where the vertices in these hyperedges are colored differently, the number of required colors is largest. Thus, in the coloring process, the number of colors used is not less than  $M(H)$ . In addition, these hyperedges have the unique common vertex  $x_i$ . Thus, the next new color is needed for this vertex  $x_i$ . □

Proposition 2.4 indicates that if the number of colors is larger than  $M(H)$ , all the vertices can be colored.

**Proposition 2.5** *The hypergraph based channel allocation method takes cubic polynomial time, in comparison to the graph based channel allocation method, which takes quadratic polynomial time.*

*Proof* According to Table 2.1, the graph based resource allocation method can be processed in a greedy manner. For the graph based method, the complexity of calculating the interference of the D2D pairs and cellular UEs is proportional to  $O(MN + N^2)$ . For graph coloring, it is necessary to go through all the vertices and break at most  $(M + N)(M + N - 1)$  edges. The computational complexity of the graph based channel allocation is quadratic given by

$$C_G \propto O((M + N)^2). \quad (2.16)$$

According to Table 2.2, the hypergraph based resource allocation method is processed in a greedy manner as well. For the hypergraph based method, the

complexity of finding the independent interferers is equal to the graph based method, i.e., proportional to  $O(MN + N^2)$ . The complexity of finding the cumulative interferers is proportional to  $O((M + N)^2)$ . For hypergraph coloring, there exist at most  $(M + N - 1)$  edges and  $(M + N - 1)(M + N - 2)$  hyperedges, and the method goes through all the vertices and breaking at most  $((M + N)(M + N - 1)(M + N - 2))$  edges. The computational complexity of the hypergraph based channel allocation method is cubic given by

$$C_H \propto O((M + N)^3). \quad (2.17)$$

□

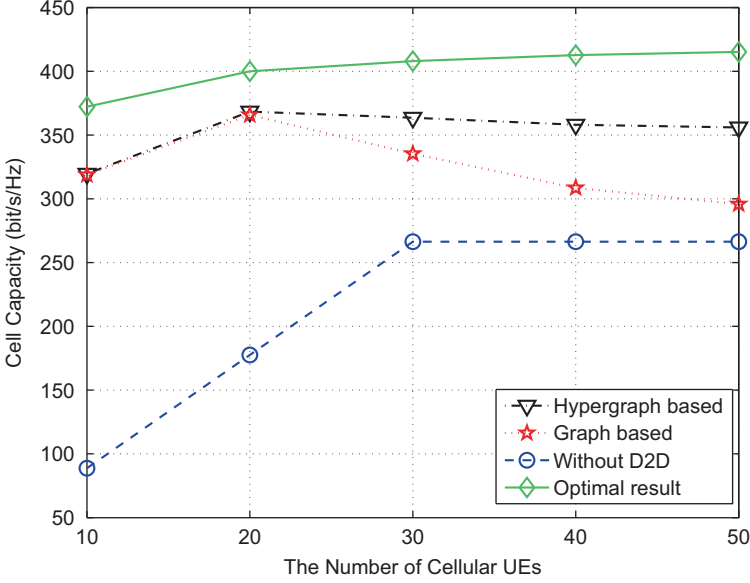
## 2.6 Simulation Results

In this section, we present the simulation results of the hypergraph based method in Table 2.2, in comparison to the graph based method in Table 2.1, and the scenario without D2D, where all the UEs are in the cellular mode. We investigate the relation of the cell capacity to the number of cellular UEs and D2D pairs under two conditions: (1) the number of channels is sufficient for orthogonal access; (2) the number of channels is not sufficient for orthogonal access. For the simulations, we consider a single cell scenario, where cellular communications and D2D communications co-exist, and they can share the channels. The cellular UEs and D2D pairs are distributed randomly in a cell, where the communication distance of each D2D pair cannot exceed a given maximum distance. In this simulation, we use the Shannon capacity model to evaluate the cell capacity. In addition, we focus on the frequency domain, and there is no time multiplexing. The simulation parameters are given in Table 2.3.

In Fig. 2.3, we show the cell capacity as a function of  $N$  cellular UEs with  $M = 20$  D2D pairs, and  $K = 30$  channels. We can see that the cell capacity with the graph or hypergraph based method increases at first and then decreases. When  $N \leq$

**Table 2.3** Parameters for simulation

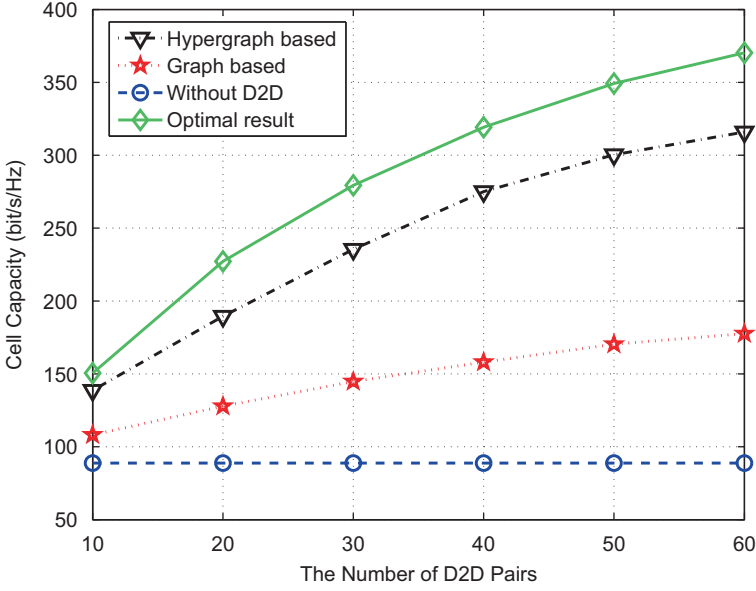
|                                    |               |
|------------------------------------|---------------|
| Cellular layout                    | Isolated cell |
| Cell radius                        | 500 m         |
| Maximum D2D pair distance          | 20 m          |
| Cellular UE's transmit power $P^c$ | 23 dBm        |
| D2D's transmit power $P^d$         | 13 dBm        |
| Carrier frequency                  | 2.3 GHz       |
| Transmission bandwidth             | 20 MHz        |
| Noise figure                       | 5 dB          |
| Threshold $\delta_c = \eta_c$      | 20 dB         |
| Threshold $\delta_d = \eta_d$      | 20 dB         |
| Path loss model                    | UMi in [29]   |



**Fig. 2.3** The cell capacity with the number of cellular UEs  $N$  for  $K = 30$ , and  $M = 20$

20, the cell capacity obtained by the hypergraph based method is almost the same as that obtained by the graph based method, because of low mutual interference. Besides, the cell capacity increases as the number of cellular UEs grows due to the channel sharing. When  $N > 20$ , the mutual interference becomes large and leads to the decrease in the cell capacity. In addition, for the hypergraph based method, the mutual interference is alleviated by allocating orthogonal channels, since the cumulative interferers are well modeled. Thus, when  $N = 50$ , the cell capacity obtained by the hypergraph based method is 60 bit/s/Hz higher than the graph based method. Compared to the graph based method, the capacity obtained by the hypergraph based method is closer to the optimal result.

Figure 2.4 illustrates the cell capacity as a function of the number of D2D pairs  $M$  with  $N = 10$ , and  $K = 10$ . The cell capacity increases as the number of D2D pairs grows, since more D2D pairs are allocated to channels. In addition, it shows that when  $M > 40$ , the increase of the cell capacity slows down. This indicates that when the number of D2D pairs becomes larger than 40, the cell capacity will be limited by the number of channels. Under the assumption that the UEs in the same edge or hyperedge cannot utilize the same channel, the cell capacity finally becomes saturated, because the number of channels is not sufficient. Simulation results are consistent with Proposition 2.1. When  $M = 20$ , the cell capacity with the hypergraph based method is about 63 bit/s/Hz higher than that with the graph based method, and the gap becomes 130 bit/s/Hz when  $M = 50$ . The reason is that when the number of D2D pairs grows, more UEs will share the same channel, leading to larger mutual interference. The hypergraph models cumulative interference with

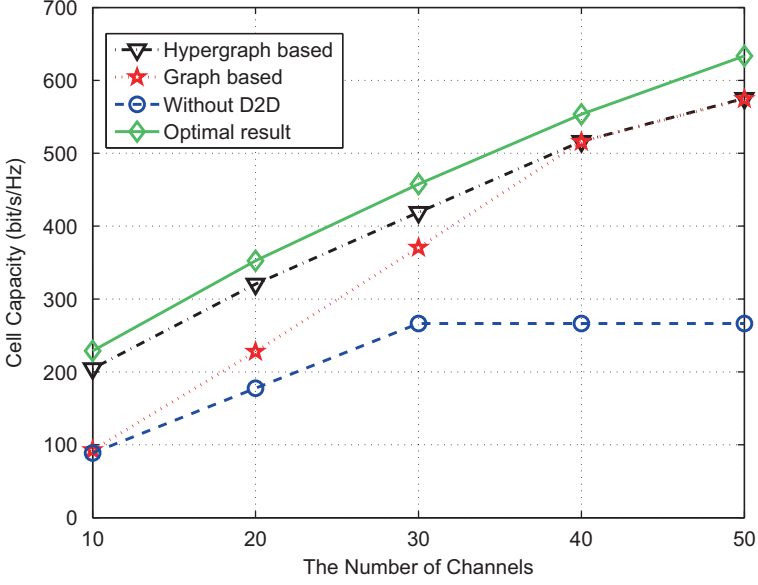


**Fig. 2.4** The cell capacity with the number of D2D pairs  $M$  for  $K = 10$ , and  $N = 10$

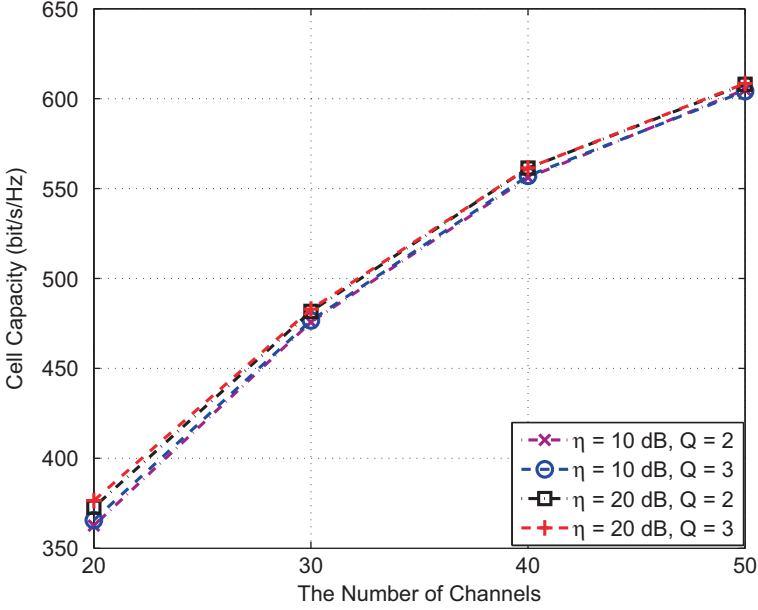
sufficient accuracy, the mutual interference gets alleviated well. Therefore, the gap between the cell capacity using the hypergraph based method and the graph based method increases. From Figs. 2.3 and 2.4, if the number of channels is fixed, it can be observed that the effect of cumulative interference modeling is more significant when the number of UEs is larger.

In Fig. 2.5, we provide the cell capacity as a function of the number of channels  $K$  with  $M = 30$  and  $N = 30$ . When the number of channels grows, the more UEs can be allocated to channels for communication. Therefore, the cell capacity increases as the number of channels grows. The cell capacity obtained by the hypergraph based method is about 90 bit/s/Hz higher than that obtained by the graph based method when  $K = 20$ . This implies that the hypergraph can model the interference with sufficient accuracy and hence alleviates it. When  $K = 50$ , the cell capacity with the graph based method is narrowly close to that with the hypergraph based method. The reason is that the number of channels becomes larger, and hence the number of cumulative interferers decreases.

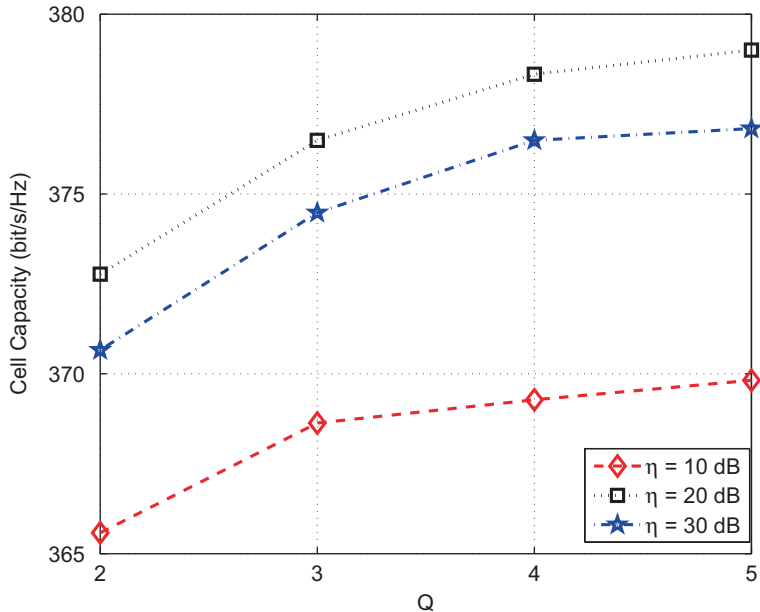
In Fig. 2.6, we compare the cell capacity with different numbers of cumulative interferers in a hyperedge  $Q$  and selection thresholds  $\eta_c$  and  $\eta_d$ . Here, we assume that  $\eta_c = \eta_d = \eta$ . The cell capacity with  $Q = 3$  is about 3 bit/s/Hz higher than that with  $Q = 1$  when  $K = 20$ . Therefore, we can conclude that the cell capacity increases less than 1% when the value of  $Q$  increases. However, the increase of  $Q$  will bring significant increase on the computational complexity. The increase of the cell capacity may not make up for the increase in complexity. Therefore, we construct the hypergraph with  $Q = 2$ .



**Fig. 2.5** The cell capacity with the number of channels  $K$  for  $M = 30$ , and  $N = 30$



**Fig. 2.6** Comparison of the cell capacity with the number of channels  $K$  for  $M = 20$ , and  $N = 40$



**Fig. 2.7** Comparison of the cell capacity with  $Q$  for  $M = 20$ ,  $N = 40$ , and  $K = 20$

In Fig. 2.7, we provide the cell capacity as a function of the value of  $Q$  under the assumptions that  $M = 30$ ,  $N = 30$ , and  $K = 30$ . As the value of  $Q$  increases, more cumulative interferers in a hyperedge would make it easier to form a hyperedge. Therefore, the cell capacity increases because the cumulative interference is well eliminated. Although the cell capacity will increase, the increase might not make up the increase in complexity. Therefore, we construct the hypergraph with  $Q = 2$ . In addition, with the same value of  $Q$ , if the threshold becomes high, the cell capacity decreases because the hyperedge will be hard to form. If the threshold becomes low, the number of hyperedges will increase. Under the assumption that the UEs in the same hyperedge cannot use the same channel, fewer UEs will be allocated to channels, and hence the cell capacity decreases.

## 2.7 Summary

In this chapter, we investigate channel allocation by a hypergraph method which coordinates the interference among D2D pairs and cellular UEs in order to increase the cell capacity using D2D underlay communications. We formulate the channel allocation problem as a hypergraph coloring problem to maximize the cell capacity. We also present a greedy coloring algorithm with polynomial complexity proportional to  $O((M + N)^3)$ , where  $N$  and  $M$  respectively represent the number

of cellular users and D2D pairs. The analysis indicates that proper allocation of D2D pairs can actually increase the cell capacity. The throughput of D2D pairs first increases and then saturates with the increasing number of D2D pairs. Simulation results show that the studied hypergraph based channel allocation method increases the cell capacity by 33% compared to the traditional graph based method with  $N = 50$ ,  $M = 20$  and  $K = 30$ , where  $K$  is the number of available channels.

## References

1. K. Doppler, M. Rinne, C. Wijting, C. Ribeiro, and K. Hugl, "Device-to-Device Communication as an Underlay to LTE-Advanced Networks," *IEEE Commun. Mag.*, vol. 7, no. 12, pp. 42–49, Dec. 2009.
2. P. Jänis, C. Yu, K. Doppler, C. Ribeiro, C. Wijting, K. Hugl, O. Tirkkonen, and V. Koivunen, "Device-to-Device Communication Underlying Cellular Communications Systems," *Int. J. Commun. Network Syst. Sci.*, vol. 2, no. 3, pp. 169–178, Jun. 2009.
3. H. Nishiyama, M. Ito, and N. Kato, "Relay-by-Smartphone: Realizing Multihop Device-to-Device Communications," in *IEEE Commun. Mag.*, vol. 52, no. 4, pp. 56–65, Apr. 2014.
4. J. Liu, S. Zhang, N. Kato, H. Ujikawa, and K. Suzuki, "Device-to-Device Communications for Enhancing Quality of Experience in Software Defined Multi-Tier LTE-A Networks," in *IEEE Network*, vol. 29, no. 4, pp. 46–52, Jul. 2015.
5. C. Xu, L. Song, and Z. Han, "Resource Management for Device-to-Device Underlay Communication," *Springer Briefs in Computer Science*, 2014.
6. 3GPP, "3rd Generation Partnership Project; Technical Specification Group RAN; Study on LTE Device to Device Proximity Services (ProSe) - Radio Aspects (Release 12)," TR 36.843 V12.0.1, Mar. 2014.
7. D. Feng, L. Lu, Y. Wu, G. Li, G. Feng, and S. Li, "Device-to-Device Communications Underlying Cellular Networks," *IEEE Trans. Commun.*, vol. 61, no. 8, pp. 3541–3551, Aug. 2013.
8. P. Phunchongharn, E. Hossain, and D. I. Kim, "Resource Allocation for Device-to-Device Communications Underlying LTE-advanced Networks," *IEEE Wireless Commun.*, vol. 20, no. 4, pp. 91–100, Aug. 2013.
9. H. Min, W. Seo, J. Lee, S. Park, and D. Hong, "Reliability Improvement Using Receive Mode Selection in the Device-to-Device Uplink Period Underlying Cellular Networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 413–418, Feb. 2011.
10. L. B. Le and E. Hossain, "Resource Allocation for Spectrum Underlay in Cognitive Radio Networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5306–5315, Dec. 2008.
11. H. S. Chae, J. Gu, B.-G. Choi, and M. Y. Chung, "Radio Resource Allocation Scheme for Device-to-Device Communication in Cellular Networks Using Fractional Frequency Reuse," in *Proc. APCC*, Sabah, Malaysia, Oct. 2011.
12. F. Wang, C. Xu, L. Song, Z. Han, and B. Zhang, "Energy-Efficient Radio Resource and Power Allocation for Device-to-Device Communication Underlying Cellular Networks," in *Proc. WCSP*, Huangshan, China, Oct. 2012.
13. C. Xu, L. Song, Z. Han, Q. Zhao, X. Wang, and B. Jiao, "Interference-Aware Resource Allocation for Device-to-Device Communications as an Underlay Using Sequential Second Price Auction," in *Proc. IEEE ICC*, Ottawa, Canada, Jun. 2012.
14. C.-H. Yu, K. Doppler, C. B. Ribeiro, and O. Tirkkonen, "Resource Sharing Optimization for Device-to-Device Communication Underlying Cellular Networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 8, pp. 2752–2763, Aug. 2011.
15. H. Tamura, M. Sengoku, K. Nakano, and S. Shinoda, "Graph Theoretic or Computational Geometric Research of Cellular Mobile Communications," in *Proc. IEEE ISCAS*, Orlando, FL, Jun. 1999.

16. A. Checco and D. J. Leith, "Learning-Based Constraint Satisfaction With Sensing Restrictions," *IEEE J. Sel. Topics Signal Process.*, vol. 7, no. 5, pp. 811–820, Oct. 2013.
17. R. Zhang, X. Cheng, L. Yang, and B. Jiao, "Interference-Aware Graph Based Resource Sharing for Device-to-Device Communications Underlying Cellular Networks," in *Proc. IEEE WCNC*, Shanghai, China, Apr. 2013.
18. H. Zhang, T. Wang, L. Song, and Z. Han, "Graph-based Resource Allocation for D2D Communications Underlying Cellular Networks," in *Proc. IEEE/CIC ICC*, Xi'an, China, Aug. 2013.
19. S. Sarkar and K. N. Sivarajan, "Hypergraph Models for Cellular Mobile Communication Systems," *IEEE Transactions on Vehicular Technology*, vol. 47, no. 2, pp. 460–471, May. 1998.
20. Q. Li, G. Kim, and R. Negi, "Maximal Scheduling in A Hypergraph Model for Wireless Networks," in *Proc. IEEE ICC*, Beijing, China, May. 2008.
21. A. A. Zykov, "Hypergraphs," in *Russian Mathematical Surveys*, vol. 29, no. 6, pp. 89–156, 1974.
22. Q. Li and R. Negi, "Maximal Scheduling in Wireless Ad Hoc Networks With Hypergraph Interference Models," in *IEEE Trans. Veh. Technol.*, vol. 61, no. 1, pp. 297–310, Jan. 2012.
23. H. Zhang, L. Song, and Z. Han, "Radio Resource Allocation for Device-to-Device Underlay Communication Using Hypergraph Theory," *IEEE Trans. Wireless Commun.*, vol. 15, no. 7, pp. 4852–4861, Jul. 2016.
24. R. Y. Chang, Z. Tao, J. Zhang, and C.-C. J. Kuo, "A Graph-based Approach to Multi-cell OFDMA Downlink Resource Allocation," in *Proc. IEEE GLOBECOM*, New Orleans, LA, Nov. 2008.
25. T. R. Jensen and B. Toft, *Graph Coloring Problems*, Wiley-Interscience, New York City, NY, 1995.
26. D. Tsoalkas, E. Liotou, N. Passas, and L. Merakos, "A Graph-Coloring Secondary Resource Allocation for D2D Communications in LTE Networks," in *Proc. IEEE CAMAD*, Barcelona, Spain, Sep. 2012.
27. V. I. Voloshin, *Coloring Mixed Hypergraphs: Theory, Algorithms and Applications*. American Mathematical Society, Providence, Rhode Island, 2002.
28. V. I. Voloshin, "Hypergraph Coloring" in *Introduction to Graph and Hypergraph Theory*, pp. 193–262, Nova Science Publishers, New York City, NY, 2008.
29. ITU-R, "Guidelines for Evaluation of Radio Interface Technologies for IMT-Advanced," Report ITU-R M.2135, 2008.

Hypergraph Theory in Wireless Communication  
Networks

Zhang, H.; Song, L.; Han, Z.; Zhang, Y.

2018, IX, 62 p. 26 illus., 14 illus. in color., Softcover

ISBN: 978-3-319-60467-1