Chapter 2
Utility Functions and Resource Allocation for Spectrum Sharing

2.1 User Applications Utility Functions

The user satisfaction with the provided service can be expressed using utility functions that represent the degree of satisfaction of the user function of the rate allocated by the cellular network [1–9]. We assume that the applications utility functions $U(r)$ are strictly concave or sigmoidal-like functions [10–16].

These applications utility functions have the following properties:

- $U(0) = 0$ and $U(r)$ is an increasing function of r.
- $U(r)$ is twice continuously differentiable in r and bounded above.

We use the normalized sigmoidal-like utility function, same as the one presented in [5, 17, 18], that is

$$U(r) = c\Big(\frac{1}{1 + e^{-a(r-b)}} - d\Big), \tag{2.1}$$

where $c = \frac{1+e^{ab}}{e^{ab}}$ and $d = \frac{1}{1+e^{ab}}$ so it satisfies $U(0) = 0$ and $U(\infty) = 1$ [16]. The normalized sigmoidal-like function has an inflection point at $r^{\text{inf}} = b$. In addition, we use the normalized logarithmic utility function, used in [1, 2, 19–21], that can be expressed as

$$U(r) = \frac{\log(1 + kr)}{\log(1 + kr_{\max})}, \tag{2.2}$$

where $r_{\max}$ gives 100% utilization and k is the slope of the curve that varies based on the user application. So, it satisfies $U(0) = 0$ and $U(r_{\max}) = 1$.

Figure 2.1 shows an example of sigmoidal and logarithmic utility functions. It shows three normalized sigmoidal-like utility functions that are expressed by Eq. (2.1) with different parameters $a = 5$, $b = 10$ which is an approximation to a

© Springer International Publishing AG 2018

H. Shajaiah et al., *Resource Allocation with Carrier Aggregation in Cellular Networks*, DOI 10.1007/978-3-319-60540-1_2

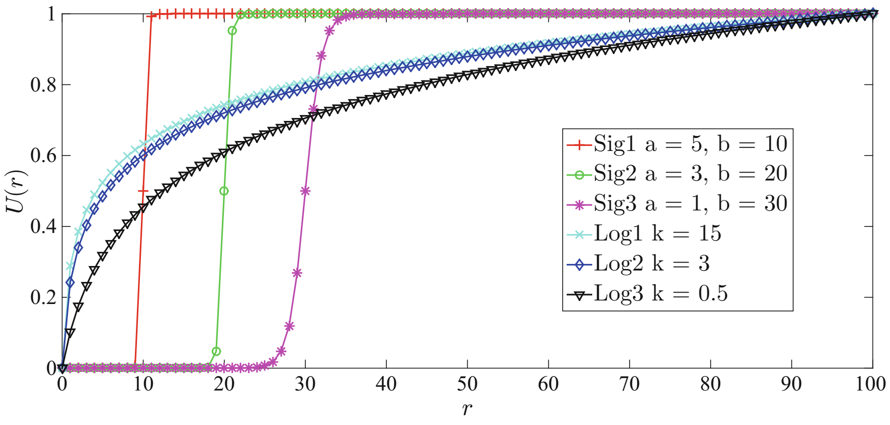

Fig. 2.1 Logarithmic and sigmoidal utility functions $U(r)$ representing delay-tolerant and real-time applications, respectively

step function at rate $r = 10$ (e.g., VoIP), $a = 3, b = 20$ which is an approximation of an adaptive real-time application with inflection point at rate $r = 20$ (e.g., standard definition video streaming), $a = 1$, $b = 30$ which is also an approximation of an adaptive real-time application with inflection point at rate $r = 30$ (e.g., high definition video streaming). In addition Fig. 2.1 shows three logarithmic functions that are expressed by Eq. (2.2) with $r_{\text{max}} = 100$ and different k parameters which are approximations for delay-tolerant applications (e.g., FTP). We use $k = \{15, 3, 0.5\}$. It is noticeable that real-time applications require a minimum rate, i.e., the inflection point, after that rate the application QoS is fulfilled to a large extent whereas logarithmic utility functions provide some QoS at low rates which is suitable for the delay-tolerant applications nature. Realistic values of a, b, and k for real mobile applications, e.g., youtube and FTP, are shown in [16, 18, 22].

2.2 Utility Proportional Fairness Resource Allocation

In proportional fairness resource allocation model, each user must be allocated some rate. This is guaranteed as allocating zero rate to any user will set the efficiency of the network to zero. The proportional fairness model is presented in the following equation:

$$r_i = \arg\max_{r_i} \prod_{i=1}^{N} U_i(r_i) \tag{2.3}$$

where $U_i(r_i)$ is the utility function of the ith user allocated resource r_i and N is the number of users. The objective function in Eq. (2.3) ensures non-zero resource allocation for all users which guarantees minimum QoS for all users. Frank Kelly algorithm [23] can be used to achieve rate allocation with the fairness model. It achieves Pareto optimal resource allocation across the network while using a proportional fairness approach to distribute all network resources where Pareto optimal or Pareto efficient solutions are the solutions that distribute all of the network resources; i.e., also referred to as the Pareto front [24–26]. Frank Kelly algorithm uses an iterative process to determine the rate that needs to be allocated to each user as well as the price the network should charge each user for the allocated resources. In the next chapters we will be using methods that are based on the Frank Kelly algorithm to solve different proportional fairness resource allocation formulations.

2.3 Utility Proportional Fairness Resource Allocation with Carrier Aggregation

In this book, a utility proportional fairness (UPF) resource allocation optimization framework is proposed to allocate multi-carrier resources optimally among active mobile users from their all in range carriers based on carrier aggregation scenario [27–31]. Throughout the next chapters we present different resource allocation methods for multi-carrier wireless systems. First, we present a multi-stage RA approach which uses a utility proportional fairness RA optimization problem to allocate each carrier resources separately in a multi-stage basis while taking into consideration the resources allocated to each user from other carriers every time the RA optimization problem is executed. The UPF resource allocation optimization problem that we use in the multi-stage RA with CA approach is given by

$$
\begin{aligned}
\max_{\mathbf{r}^j} \quad & \prod_{i=1}^{M_j} U_i\left(r_i^j + c_i^j\right) \\
\text{subject to} \quad & \sum_{i=1}^{M_j} r_i^j \leq R_j, \\
& r_i^j \geq 0, \quad i = 1, 2, \ldots, M_j, \\
& c_i^j = \sum_{n=1, n\neq j}^{K} v_i^n r_i^{n,\text{opt}}, \\
& v_i^n = \begin{cases} 1, & \text{the } i\text{th UE } \in \mathscr{M}_n \\ 0, & \text{the } i\text{th UE } \notin \mathscr{M}_n \end{cases}
\end{aligned}
\tag{2.4}
$$

where optimization problem (2.4) is carrier j RA optimization problem, $\mathscr{M}_j$ is the set of users located under the coverage area of the jth eNodeB and $M_j = |\mathscr{M}_j|$ is the number of users in the set $\mathscr{M}_j$, $\mathbf{r}^j = \{r_1^j, r_2^j, \ldots, r_{M_j}^j\}$, R_j is the jth carrier available resources, c_i^j is equivalent to the total rates allocated to the ith user by other carriers in its range, v_i^n is equivalent to 1 if the ith UE $\in \mathscr{M}_n$ and is equivalent to 0 if the ith UE $\notin \mathscr{M}_n$, and $r_i^{n,\text{opt}}$ is the optimal rate allocated to the ith user by the nth carrier.

In order to consider the case when it is required to treat users differently when assigning the network resources, we introduced a user discrimination feature to the resource allocation framework such that certain group of users (e.g., public safety users in systems that consider spectrum sharing between public safety and commercial users) are given priority when allocating the network resources. Furthermore, we developed resource allocation with CA methods to allocate multi-carrier resources based on user discrimination and used UPF optimization problem to calculate the allocated resources.

2.4 MATLAB Code

The following code plots the applications' utility functions shown in Fig. 2.1.

```
n = 0:100;
L=length(n);
%%Sigmoidal Utlity Function with a=5 and b=10%%
U1 = zeros(L,1);
for r = 1:L
    U1(r) = ((1+exp(5*10))/exp(5*10))*((1/(1+exp(-5*(n(r)-10)))-(1/(1+exp(5*10)))));
end
plot(n,U1, '-r+');
hold on
%%Sigmoidal Utility Function with a=3 and b=20%%
U2 = zeros(L,1);
for r = 1:L
    U2(r) = ((1+exp(3*20))/exp(3*20))*((1/(1+exp(-3*(n(r)-20)))-(1/(1+exp(3*20)))));
end
plot(n,U2, '-go');
hold on
%%Sigmoidal Utility Function with a=1 and b=30%%
U3 = zeros(L,1);
for r = 1:L
    U3(r) = ((1+exp(1*30))/exp(1*30))*((1/(1+exp(-1*(n(r)-30)))-(1/(1+exp(1*30)))));
end
plot(n,U3, '-m*');
hold on
%%Logarithmic Utility Function with k=15%%
```

```
U4 = zeros(L,1);
for r = 1:L
    U4(r) = log(1+(15*n(r)))/log(1+(15*100));
end
plot(n,U4, '-cx');
hold on
%%Logarithmic Utility Function with k=3%%
U5 = zeros(L,1);
for r = 1:L
    U5(r) = log(1+(3*n(r)))/log(1+(3*100));
end
plot(n,U5, '-bd');
hold on
%%Logarithmic Utility Function with k=0.5%%
U6 = zeros(L,1);
for r = 1:L
    U6(r) = log(1+(0.5*n(r)))/log(1+(0.5*100));
end
plot(n,U6, '-kv');

ylim([0 1]);
I=legend('Sig1 a = 5, b = 10','Sig2 a = 3, b = 20', 'Sig3 a =
     1, b = 30', 'Log1 k = 15','Log2 k = 3', 'Log3 k = 0.5','
    Interpreter','latex');
set(I,'interpreter','latex');
xlabel('$r$','Interpreter','latex');
ylabel('$U(r)$','Interpreter','latex');
```

References

1. G. Tychogiorgos, A. Gkelias, K.K. Leung, Utility-proportional fairness in wireless networks, in *PIMRC* (IEEE, New York, 2012), pp. 839–844
2. S. Shenker, Fundamental design issues for the future internet. IEEE J. Sel. Areas Commun. **13**, 1176–1188 (1995)
3. Y. Wang, A. Abdelhadi, T.C. Clancy, Optimal power allocation for LTE users with different modulations, in *2016 Annual IEEE Systems Conference (SysCon)*, April 2016, pp. 1–5
4. M. Ghorbanzadeh, A. Abdelhadi, C. Clancy, Distributed resource allocation, in *Cellular Communications Systems in Congested Environments* (Springer, Berlin, 2017), pp. 61–91
5. J.-W. Lee, R.R. Mazumdar, N.B. Shroff, Downlink power allocation for multi-class wireless systems. IEEE/ACM Trans. Netw. **13**, 854–867 (2005)
6. A. Kumar, A. Abdelhadi, T.C. Clancy, A delay efficient multiclass packet scheduler for heterogeneous M2M uplink, in *IEEE MILCOM* (2016)
7. A. Kumar, A. Abdelhadi, T.C. Clancy, An online delay efficient packet scheduler for M2M traffic in industrial automation, in *IEEE Systems Conference* (2016)
8. A. Kumar, A. Abdelhadi, T.C. Clancy, A delay optimal MAC and packet scheduler for heterogeneous M2M uplink. CoRR, abs/1606.06692 (2016)
9. A. Kumar, A. Abdelhadi, T.C. Clancy, A delay-optimal packet scheduler for M2M uplink, in *IEEE MILCOM* (2016)
10. A. Abdelhadi, C. Clancy, A utility proportional fairness approach for resource allocation in 4G-LTE, in *IEEE International Conference on Computing, Networking, and Communications (ICNC), CNC Workshop* (2014)

11. A. Abdelhadi, C. Clancy, A robust optimal rate allocation algorithm and pricing policy for hybrid traffic in 4G-LTE, in *IEEE International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)* (2013)
12. Y. Wang, A. Abdelhadi, A QoS-based power allocation for cellular users with different modulations, in *2016 International Conference on Computing, Networking and Communications (ICNC)*, February 2016, pp. 1–5
13. A. Abdelhadi, H. Shajaiah, Optimal resource allocation for smart phones with multiple applications with MATLAB instructions. Technical Reports (2016)
14. A. Abdelhadi, H. Shajaiah, Application-aware resource allocation with carrier aggregation using MATLAB. Technical Reports (2016)
15. Z. Kbah, A. Abdelhadi, Resource allocation in cellular systems for applications with random parameters, in *2016 International Conference on Computing, Networking and Communications (ICNC)*, February 2016, pp. 1–5
16. A. Abdelhadi, H. Shajaiah, Optimal resource allocation for cellular networks with MATLAB instructions. CoRR, abs/1612.07862 (2016)
17. M. Ghorbanzadeh, A. Abdelhadi, C. Clancy, Utility functions and radio resource allocation, in *Cellular Communications Systems in Congested Environments* (Springer, Berlin, 2017), pp. 21–36
18. M. Ghorbanzadeh, A. Abdelhadi, C. Clancy, Centralized resource allocation, in *Cellular Communications Systems in Congested Environments* (Springer, Berlin, 2017), pp. 37–60
19. M. Ghorbanzadeh, A. Abdelhadi, C. Clancy, Radio resource block allocation, in *Cellular Communications Systems in Congested Environments* (Springer, Berlin, 2017), pp. 117–146
20. M. Ghorbanzadeh, A. Abdelhadi, C. Clancy, Spectrum-shared resource allocation, in *Cellular Communications Systems in Congested Environments* (Springer, Berlin, 2017), pp. 147–178
21. F. Wilson, I. Wakeman, W. Smith, Quality of service parameters for commercial application of video telephony, in *Proceedings of the Human Factors in Telecommunications Symposium* (Darmstadt, 1993), pp. 139–148
22. M. Ghorbanzadeh, A. Abdelhadi, A. Amanna, J. Dwyer, T. Clancy, Implementing an optimal rate allocation tuned to the user quality of experience, in *Computing, Networking and Communications (ICNC), 2015 International Conference on*, February 2015, pp. 292–297
23. F. Kelly, A. Maulloo, D. Tan, Rate control in communication networks: shadow prices, proportional fairness and stability. J. Oper. Res. Soc. **49**, 237–252 (1998)
24. R. Madan, S. Boyd, S. Lall, Fast algorithms for resource allocation in wireless cellular networks. IEEE/ACM Trans. Netw. **18**, 973–984 (2010)
25. J.B. Taylor, *Principles of Microeconomics*. Microeconomics Series (Houghton Mifflin, Boston, 1998)
26. S. Boyd, L. Vandenberghe, *Introduction to Convex Optimization with Engineering Applications*. Convex Optimization, Course Reader for EE364 (1999)
27. M. Iwamura, K. Etemad, M.-H. Fong, R. Nory, R. Love, Carrier aggregation framework in 3GPP LTE-advanced [WiMAX/LTE Update]. IEEE Commun. Mag. **48**(8), 60–67 (2010)
28. Y. Wang, K.I. Pedersen, T.B. Sørensen, P.E. Mogensen, Utility maximization in LTE-advanced systems with carrier aggregation, in *VTC Spring* (2011), pp. 1–5
29. G. Yuan, X. Zhang, W. Wang, Y. Yang, Carrier aggregation for LTE-advanced mobile communication systems. IEEE Commun. Mag. **48**, 88–93 (2010)
30. Z. Shen, A. Papasakellariou, J. Montojo, D. Gerstenberger, F. Xu, Overview of 3GPP LTE-advanced carrier aggregation for 4G wireless communications. IEEE Commun. Mag. **50**, 122–130 (2012)
31. *Frequency Spectrum Wall Chart* (Commerce Dept., National Telecommunications and Information Administration, Office of Spectrum Management, 2016)

Springer

http://www.springer.com/978-3-319-60539-5

Resource Allocation with Carrier Aggregation in Cellular Networks
Optimality and Spectrum Sharing using C++ and MATLAB
Shajaiah, H.; Abdelhadi, A.; Clancy, T.C.
2018, XVI, 201 p. 65 illus., 62 illus. in color., Hardcover
ISBN: 978-3-319-60539-5