

Preface

One of the most remarkable physical phenomena is the *statistical stability (regularity) of mass phenomena* as revealed by the stability of statistics (functions of samples). There are two theories describing this phenomenon. The first is classical *probability theory*, which has a long history, and the second is *the theory of hyper-random phenomena* developed in recent decades.

Probability theory has established itself as the most powerful tool for solving various statistical tasks. It is even widely believed that any statistical problem can be effectively solved within the paradigm of probability theory. However, it turns out that this is not so.

Some conclusions of probability theory do not accord with experimental data. A typical example concerns the potential accuracy. According to probability theory, when we increase the number of measurement results of any physical quantity, the error in the averaged estimator tends to zero. But every engineer or physicist knows that the actual measurement accuracy is always limited and that it is not possible to overcome this limit by statistical averaging of the data.

Studies of the causes of discrepancies between theory and practice led to the understanding that the problem is related to an *unjustified idealization of the phenomenon of statistical stability*.

Probability theory is in fact a physical-mathematical discipline. The mathematical component is based on A.N. Kolmogorov's classical axioms, while the physical component is based on certain physical hypotheses, in particular the hypothesis of *perfect statistical stability of actual events, variables, processes, and fields* assuming the *convergence of statistics* when the sample size goes to infinity.

Experimental investigations of various processes of different physical kinds over broad observation intervals have shown that the hypothesis of perfect statistical stability is *not confirmed experimentally*. For relatively short temporal, spatial, or spatio-temporal observation intervals, an increase in data volume usually reduces the level of fluctuation in the statistics. However, when the volumes become very large, this tendency is no longer visible, and once a certain level is reached, the

fluctuations remain practically unchanged or even grow. This indicates *a lack of convergence for real statistics (their inconsistency)*.

If the volume of processing data is small, the violation of convergence has practically no influence on the results, but if this volume is large, the influence is very significant. The study of violations of statistical stability in physical phenomena and the development of an effective way to describe the actual world, one which accounts for such violations, has resulted in the construction of the new *physical-mathematical theory of hyper-random phenomena*.

The theory of hyper-random phenomena is also a physical-mathematical theory. Its mathematical component is based on the axioms and statements of the mathematical component of the probability theory, and its physical component is based on hypotheses that differ essentially from the physical hypotheses of probability theory, in particular the hypothesis of limited statistical stability assuming the absence of convergence in the actual statistics. Therefore, for mathematicians the theory of hyper-random phenomena is a *branch of probability theory*, and for physicists, it is a *new physical theory based on a new view of the world*.

There is much literature describing probability theory from various points of view (Kolmogorov 1929, 1956; Mises 1964; Bernshtein 1946, etc.) and oriented toward readerships with different mathematical knowledge (Feller 1968; Lo  ve 1977; Gnedenko 1988; Angot 1957; Devor 2012; Gorban 2003; Pugachev 1979; Peebles 1987; Tutubalin 1972; Rozhkov 1996; Ventsel 1962, etc.). Quite a few studies have also been published in the area of statistical stability violation and the theory of hyper-random phenomena. Among the latter, we can mention three monographs in Russian (Gorban 2007, 2011, 2014), two monographs in Ukrainian (Uvarov and Zinkovskiy 2011a, b), and a monograph in English (Gorban 2017) devoted to various mathematical, physical, and practical questions.

Probability theory and the theory of hyper-random phenomena give different descriptions of the phenomenon of statistical stability. Until recently, there were no books comparing these theories. However, this gap was closed in 2016 with the monograph in Russian (Gorban 2016). This book is an English version of that.

The aims of the current monograph, like those before it, are:

- *To acquaint the reader with the phenomenon of statistical stability*
- *To describe probability theory and the theory of hyper-random phenomena from a single standpoint*
- *To compare these theories*
- *To describe their physical and mathematical essence at the conceptual level*

This monograph consists of *five parts*. The *first* entitled *The Phenomenon of Statistical Stability* consists of the introductory chapter. Manifestations of this phenomenon and different approaches for its description are described in it. The *second part* entitled *Probability Theory* contains four chapters (Chaps. 2–5) and describes the foundations of probability theory. The *third part* entitled *Experimental Study of the Statistical Stability Phenomenon* contains only Chap. 6, dedicated to a description of the techniques developed to assess statistical stability violations and also the results of experimental investigations of statistical stability violations

in actual physical processes of various kinds. The title of the *fourth part* is *Theory of Hyper-random Phenomena*. It includes four chapters (Chaps. 7–10) presenting the foundations of the theory of hyper-random phenomena. The *fifth part* entitled *The Problem of an Adequate Description of the World* includes only Chap. 11, which discusses the concept of world building.

The book aims at a wide readership: from university students of a first course majoring in physics, engineering, and mathematics to engineers, postgraduate students, and scientists researching the statistical laws of natural physical phenomena and developing and using statistical methods for high-precision measurement, prediction, and signal processing over broad observation intervals. To understand the material in the book, it is sufficient to be familiar with a standard first university course on mathematics.

Kiev, Ukraine
25 June 2016

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Randomness and Hyper-randomness

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2018, XXXII, 216 p. 30 illus., 23 illus. in color., Hardcover

ISBN: 978-3-319-60779-5