Structural Dynamic Characteristics of an Ancient Egyptian Obelisk

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Abstract. Ancient Egyptian obelisks have existed for several millennia and have withstood numerous natural disasters among which is the Hatshepsut Obelisk. This study investigates the structural dynamic characteristics of the Obelisk using a two-staged approach. The first, involved a series of tests to study the mechanical properties of the Red Aswan granite of which the obelisk was made while the second incorporates the test results in modeling the Obelisk pyramid using finite element software. An Eigen-value modal analysis was performed in order to generate the natural periods and corresponding modes of vibration of the structure. These natural periods were compared to the dominant periods of earthquakes in order to analyze the obelisk's response to loading and its degree of resonance. Following, a time-history analysis was performed in which the structure was subjected to an earthquake signal. The results show that the Hatshepsut obelisk could safely withstand seismic loads.

Keywords: Structural dynamics · Earthquake engineering · Obelisk · Modal analysis · Dynamic analysis · Granite

1 Introduction

The ancient Egyptians used numerous materials in building different structures for religious and sacral purposes such as pyramids, temples, obelisks and various forms of tomb structures. While buildings used for the dwelling of royalty and nobility were built using bricks made from the mud available by the annual flooding of the Nile, funeral and religious structures were built using more resistant material to last for eternity (Klemm and Klemm 2001).

Obelisks are regarded as one of the key attractions of the ancient Egyptian civilization that are still present today. There are several obelisks today decorating major international cities like Paris, London and New York, after being moved from Egypt during the last two centuries, in addition to the Hatshepsut obelisk in the Karnak temple in Luxor, Egypt. According to (Engelbach 1923), the number of obelisks built throughout various dynasties of ancient Egypt must have been numerous. He reported that there must have been more than 50 obelisks with a length exceeding 10 m. He attributed the decrease in number of obelisks present today to earthquakes and soil-subsidence problems.

Most obelisks studied by historians and archeologists are reported to be made of a red granite from quarries located to the south of Aswan in Southern Egypt. The

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discovery of an "unfinished obelisk" on site in that area was a clear evidence that ancient Egyptian obelisks were extracted from that quarrying area (Engelbach 1923; Klemm and Klemm 2008; Kelany et al. 2009). This type of granite is generally named "Red Aswan Granite". This type of granite consists of large reddish feldspar crystals in a fabric that also contains quartz, plagioclase and biotite (Klemm and Klemm 2008; Serra et al. 2010). The practical reasons for selecting this type of granite to build the obelisks is because its natural joints are far enough to enable the extraction of large structures like obelisks as a single piece of stone mass with no fissures or cracks (Engelbach 1923).

While extensive research was conducted on the red Aswan granite from the archeological, geological and chemical perspective (Kelany et al. 2009; Klemm and Klemm 2001; Serra et al. 2010), little information is available on some of the mechanical properties of this material such as its modulus of elasticity. Moreover, there is a scarcity in the information regarding the seismic behavior of monumental structures built using this material such as obelisks and tombs. Hence, the objective of this work is to conduct experimental testing to determine the mechanical properties of red Aswan granite such as the unit weight, ultimate compressive strength and modulus of elasticity. Consequently, these parameters are used as input parameters to study the structural dynamic characteristics of the Hatshepsut obelisk using a finite element model. Finally, a time-history analysis is conducted in which a real earthquake signal is applied on the structure to check its ability to withstand the earthquake vibrations.

2 Material Testing

2.1 Mechanical Tests on Red Aswan Granite

Samples of Red Aswan granite were acquired from a quarry operating in close proximity to the "unfinished obelisk" site and sawn into eight 50 mm × 50 mm × 50 mm cubes. The unit weight was determined according to the procedures outlined in ASTM C97 (American Society for Testing Materials 2015). Moreover, the specimens were tested according to the experimental program outlined in ASTM C 170-16 for testing the compressive strength of dimension stone (American Society for Testing Materials 2016). The test was carried out using MTS 810 Material Test System equipment as shown in Fig. 1. The load and the resulting displacement were instantaneously measured through an electronic data acquisition system until the specimen failed and the ultimate load at failure is recorded. The output data were used to calculate the stress and strain for each sample and to plot the stress-strain curve. Consequently, the elastic modulus was calculated for each specimen from its stress-strain curve. The unit weight and the elastic modulus value were used as input parameters for red Aswan granite in numerical models to study the behavior of the Hatshepsut obelisk under dynamic loading.

Fig. 1. The experimental setup.

2.2 Results of Mechanical Tests

The results of the experimental work performed were used to represent the material properties of the red Aswan granite of which the obelisk was constructed as shown in Table 1. As the standard deviations of the material properties were not large when compared to the average values, the average values were considered to be fairly representing the general behavior of this material and hence the density was set to be 2541 kg/m^3 while the modulus of elasticity was considered to be 5425 MPa and the ultimate compressive strength was considered to be 140.6 MPa.

Table 1. Mechanical properties of red Aswan granite.

Specimen #	Unit Weight (Kg/m^3)	Ultimate compressive strength (MPa)	Elastic modulus (MPa)
1	2580.39	143.15	5687
2	2525.49	121.14	5202
3	2517.64	146.04	5608
4	2556.86	153.02	5508
5	2529.79	139.1	5073
6	2256.86	143.76	5137
7	2517.64	142.68	5679
8	2544	135.62	5509
Average	**2541.09**	**140.56**	**5425.38**
Standard dev	**22.43**	**9.34**	**249.91**

3 Numerical Model

3.1 Finite Element Model

The Hatshepsut obelisk was modeled on the finite element software SAP2000 (Computers and Sturctures Inc. 2016) in which three-dimensional eight-node solid elements were used as shown in Fig. 2. This choice of solid elements targeted representing the mass distribution and stiffness within the obelisk structure in the most accurate way as if such a structure was modeled using one dimensional or two dimensional elements there would have been a high loss of accuracy within at least one dimension however the three dimensional elements guarantee the highest possible accuracy in results.

The modeled obelisk had a total height of 29.57 m, a squared cross-section of 2.41 m × 2.41 m and tapered until having a 1.77 m × 1.77 m cross-section at the base of the pyramidal located at the top of the obelisk at a point 26.61 m above the obelisk base (Engelbach 1923).

3.2 Modal Analysis

The modal analysis was performed in order to determine the modes of vibration of the obelisk and the corresponding natural period of each. The results of the modal analysis shown in Table 2 are ordered in a descending order with the first mode corresponding to the largest natural period and the largest modal participation factor of 57% as a translation in the y direction and 0% in the other directions as shown in Table 2. This first mode is the principal translational mode in the y-direction as shown in Fig. 3. Also it could be noticed that the second mode of vibration has exactly the same natural period of vibration however the mode itself is changed as the modal participation factor is 57% as a translation in the x direction and 0% as a translation in the other directions. This is attributed to the fact that the cross-section is squared hence the first two translational modes are identical however they are acting perpendicular to each other while having the same period as the stiffness and mass in the x direction are exactly

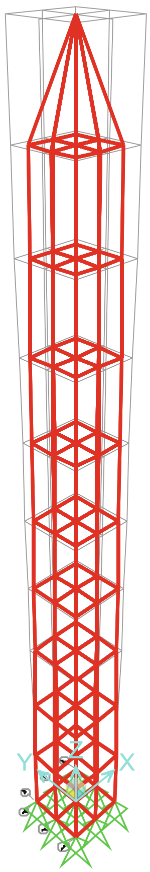

Fig. 2. The three dimensional finite element model of the Hatshepsut obelisk.

Table 2. The natural periods and the modal participation factors.

Mode #	Period (s)	Modal participation factors		
		Translation in X	Translation in Y	Translation in Z
1	1.166	0%	57%	0%
2	1.166	57%	0%	0%
3	0.230	1%	21%	0%
4	0.230	21%	1%	0%
5	0.115	0%	0%	0%
6	0.089	9%	0%	0%
7	0.089	0%	9%	0%
8	0.066	0%	0%	80%
9	0.048	1%	4%	0%
10	0.048	4%	1%	0%
11	0.047	0%	0%	0%
12	0.030	2%	1%	0%

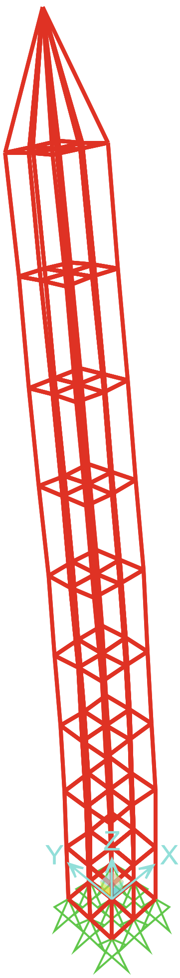

Fig. 3. The first mode of vibration

equal to those in the y direction hence it is expected to have the principal translational modes in each direction occurring at the same natural period. This could also be seen occurring for modes three and four, six and seven, nine and ten as each of these modes has exactly the same natural period of vibration with another mode with the same values of modal participation factors in the x direction in one mode equal to those in the y direction in the other mode.

On the other hand, it could be noticed that the fifth and eleventh modes have no translational components as they are both twisting modes as shown in Fig. 4. However, such modes are not considered to be of a concern as the structure has a center of rigidity which is coinciding with its center of mass causing no eccentricity and hence no expected excitation of these twisting modes.

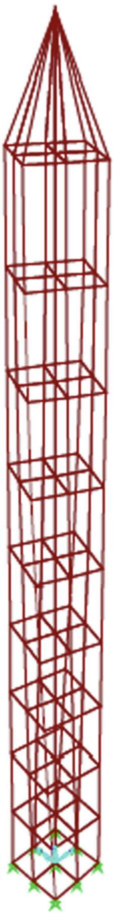

Fig. 4. Fifth mode of vibration

It could be also noticed that for eleven of the twelve modes, the vertical modal participation is 0% while it is 80% for mode number eight which has modal participation factors of 0% in the two horizontal directions as it is the principle mode in the vertical direction as shown in Fig. 5.

As shown in Table 2, the period for the first two modes of vibration was 1.166 s which is within the range of the dominant periods of typical earthquakes and typical boundary winds which exceed 0.5 s (Tedesco et al. 1999). That implies that the Hatshepsut obelisk is expected to resonantly respond due to earthquakes and winds and will mainly behave in a dynamic manner when subjected to such dynamic loads. Hence, it is necessary to perform a dynamic analysis when structurally analyzing this obelisk as a static analysis is not expected to produce accurate results.

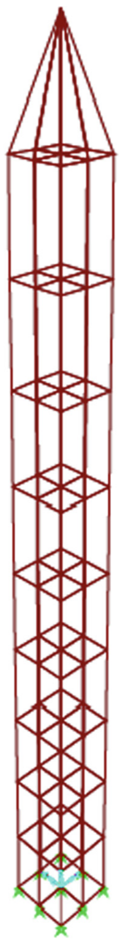

Fig. 5. Eighth mode of vibration

3.3 Time-History Analysis

Following on the modal analysis, a time-history analysis was performed in order to determine the time-varying response of the structure while subjected to a real earthquake using the Newmark direct integration method. The earthquake chosen was the 1940 El Centro earthquake that happened in California, USA with perceived intensity of X on the Mercalli intensity scale (Wikimedia Foundation, Inc. 2016). This earthquake is significantly stronger than any recorded earthquake that has ever been recorded in Egypt in which the obelisk is located. Hence, the signal of the earthquake event was scaled down to match the peak ground acceleration of Luxor, Egypt (where the Hatshepsut obelisk is located in the Karnak temple), which is 0.125 g according to the Egyptian loading code (Housing and Building National Research Center 2008).

The Newmark direct integration method used applies the concept of proportional damping in which the coefficients α and β are multiplied by the stiffness and mass matrices. These two coefficients were calculated based on a conservatively assumed damping of 1%. This percentage of damping is based on conservatively assuming that the granite has a similar performance as concrete which is typically considered to have a damping even larger than this percentage (Tedesco et al. 1999). Another factor taken into account is the time step size as according to (Bathe 2006) the solution will not converge if the time step exceeds T/π and it could only produce accurate results if it is less than or equal to T/10 where T is the smaller of the natural period of the highest mode of interest and the dominant period of loading.

3.4 Results of Time-History Analysis

The results of the time-history dynamic analysis were plotted twice; once in the time domain in which the horizontal displacement of the uppermost point was plotted versus time as shown in Fig. 6 and the other time as a response spectrum in which the spectral horizontal displacement is drawn versus the period of vibration as shown in Fig. 7. The maximum horizontal displacement shown in Fig. 6 was 99.5 mm which is 1/300 of the height of the obelisk showing that this obelisk could safely withstand such an earthquake in terms of the deflection criteria. Additionally, the maximum stress due to the seismic load was 0.52 MPa which is significantly less than the ultimate strength of the red Aswan granite which was 140.1 MPa showing that the structure passes the strength criteria. On the other hand, when examining the response spectrum shown in Fig. 7 it could be obviously seen that the maximum horizontal spectral displacement occurs at a period that is matching the principal natural period of 1.166 s reported in Table 2 and

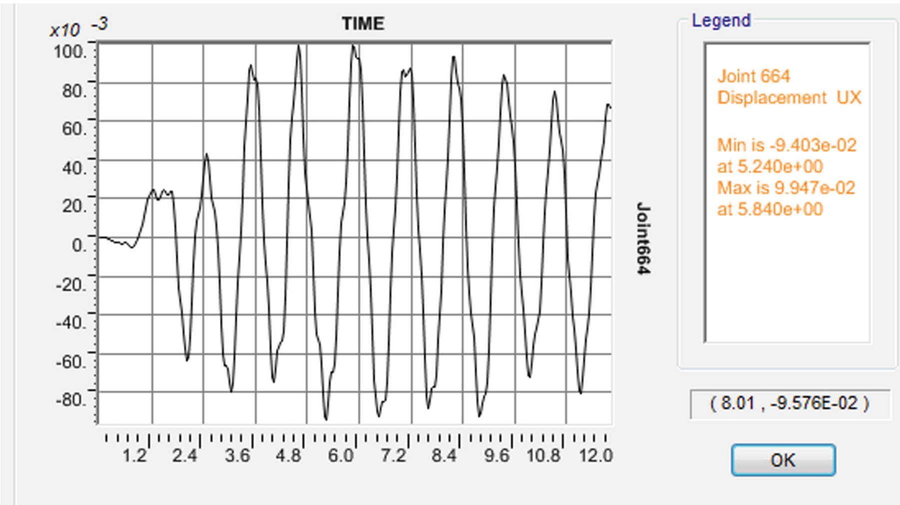

Fig. 6. Variation of the horizontal displacement of the uppermost point with time.

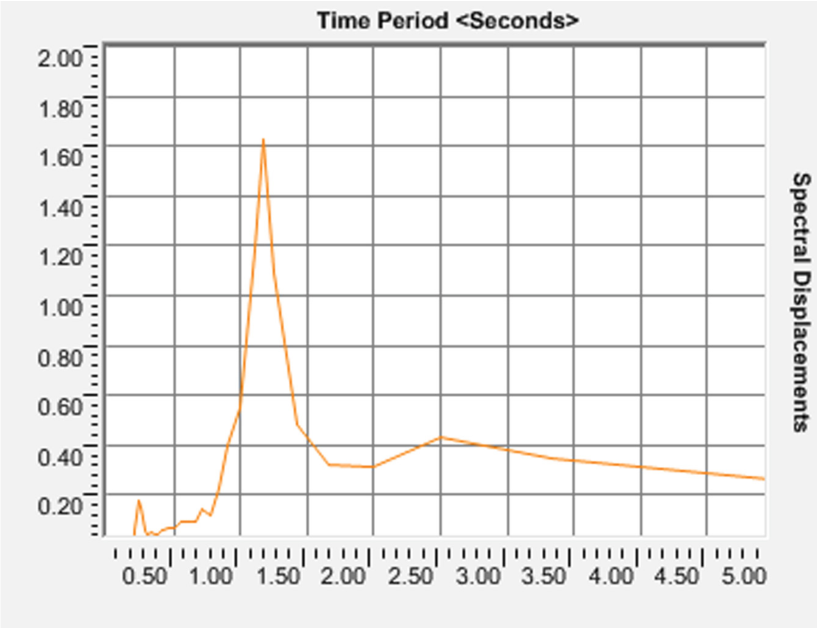

Fig. 7. Response spectrum of the spectral horizontal displacement of the uppermost point with the period of vibration.

the area beneath this region is significantly large indicating a significantly large resonant component of vibration. Hence, the 99.5 mm drift is mainly a result of the first mode of vibration that was surprisingly matching the deflection criteria explaining how this obelisk existed to date and raising a question mark on how did the ancient Egyptians design such a structure thousands of years before the invention of structural dynamics?

4 Conclusions

The following conclusions could be drawn from the performed study:

- The natural periods for the twelve modes of vibrations of the studied obelisk ranged between 1.166 s and 0.03 s.
- The principal natural periods of vibration for the various modes were within the range of the dominant periods of the earthquake ground motions implying that the resonant component of vibration is expected to be significant.
- According to the results produced by the time-history analysis, the maximum spectral displacement occurred at a period equal to the principal natural period of the structure confirming the occurrence of resonance due to the earthquake signal.
- The maximum stresses are significantly less than the ultimate strength of the red Aswan granite proving that the structure could safely withstand the earthquake load.

- According to the results produced by the time-history analysis, the horizontal drift at the top of the obelisk was 1/300 of the obelisk height which is within the safe limits of vibrations explaining how that obelisk existed for thousands of years.

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