

A New Approach to Tune the Vold-Kalman Estimator for Order Tracking

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Abstract In the purpose to diagnose rotating machines using vibration signal, engineers use order tracking method to process non-stationary signals. We deal here with order tracking when the vibration signal is represented in a state space model. Such a methodology leads to the Kalman estimator that requires knowledge about the noise statistics affecting the state and the measurement equation. These noise statistics are usually unknown and need to be estimated from operating data for the use of the Kalman estimation algorithm. Several methods to tune these parameters have been developed for time-invariant model. In this paper, we introduce a technique to estimate the noise covariances for a linear time-variant system using the innovation process. The efficiency of this new approach is evaluated using a synthetic non-stationary vibration signal. The advantage of this approach is that it converges quickly and provides a small estimation error compared to those used for the linear time-invariant model.

Keywords Order tracking · Kalman estimator · Non-stationary signal · Covariance matrix estimation

1 Introduction

Nowadays, the tools dedicated to the condition monitoring deal more and more with non-stationary signal from mechanical systems. Non-stationary events occur during machine run-up or shut-down. The non-stationary phenomena can also be observed during the variation of wind speed in wind turbines or the variation of load in a crushing machine. Over the last two decades, several approaches have been proposed to extract information from non-stationary signals. All these approaches consist in tracking orders of a vibration signal under non-stationary conditions. The first category is based on the time-frequency representation. A well-known technique in this

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category is the short time Fourier transform which supposes that the signal is stationary in a short time interval. And there is unanimity that the short time Fourier transform suffers from its time-frequency resolution limitation [1, 2]. The second one is the computed order tracking. This latter consists of re-sampling the non-stationary raw signal from time domain to angle domain in order to avoid the effects of speed variation. Computed order tracking is suitable for low speed variations [3]. For high speed variations, the computed order tracking method leads to a bad estimation of the envelope [4, 5]. It also has an impact on the re-sampled signal [6]. The last category uses adaptive estimators. The Kalman estimator is one of these kind of estimators which can be used for order tracking. This approach is based on the representation of the vibration signal in a state space model. For this purpose, Vold et al. proposed the Vold-Kalman estimator [7, 8]. This approach, from the first to the second generation of the Vold Kalman estimator, has known some important improvements.

However, the drawback of this tool is still the difficulty in correctly choosing the two parameters that influence the accuracy of the estimation. These parameters are: the covariance matrix of the measurement noise and the covariance matrix of the state noise. In the 1970, for the time invariant-model, Mehra [9] introduced a correlation method that produces unbiased and consistent estimates of these covariances. In [10], a new auto-covariance least-squares method is proposed which improves that of Mehra [9]. Few works have been devoted to the time variant-model. In this instance, Mohamed and Schwarz [11] have been one of the first authors to propose a tuning method. It is based on the innovation process and the residual error estimation. This method is modified by Almagbile et al. [12] for a problem related to the inertial navigation systems/global positioning systems. These approaches estimate the covariances of the disturbance signals using a sliding window. For application to the vibration signal, we remark that the positiveness of the covariance matrix of the state noise is not always guaranteed.

In the context above, we propose a new approach to tune the covariance matrix of the state noise. This one is based on the residual error estimation. The outline of the paper is as follows. The Sect. 2 presents the basics of the Vold-Kalman estimator. In the Sect. 3, we expose our new approach and the Sect. 4 presents simulations and comparative study with that of [12]. Conclusions are given in the Sect. 5.

2 The Vold-Kalman Estimator

The gearbox vibration signal can be modeled by the following equation

$$y(t) = \sum_{i=1}^M A_i(t) \cos(\theta_i(t) + \phi_i(t)) + v(t) \quad (1)$$

where A_i is the amplitude of the i th order, ϕ_i is the phase of the i th order, v is the measurement noise assumed to be centered, white and Gaussian and θ_i is the instantaneous angular displacement. It is calculated using the following equation

$$\theta_i(t) = 2\pi O_i \int_0^t f_r(u) du \quad (2)$$

where O_i is the value of the i th order, f_r is the instantaneous rotating frequency. Thus, the discrete form of the equation can be expressed as

$$\theta_i(k) = 2\pi O_i \sum_{j=1}^k \frac{f_r(j)}{f_s} \quad (3)$$

where f_s is the sampling frequency.

The Vold-Kalman estimator is a well known tool to process the vibration signal. And our goal is to evaluate the unknown amplitude and phase of some specific orders of interest using this tool. For this purpose, we construct the state equation and the measurement equation. These latter represent the basic equation of the Vold-Kalman estimator [8].

2.1 The Measurement Equation

The Eq. (1), in discrete domain, leads to

$$y(k) = \sum_{i=1}^M [\cos(\theta_i(k)) - \sin(\theta_i(k))] \begin{bmatrix} a_{i,c}(k) \\ a_{i,s}(k) \end{bmatrix} + v(k) \quad (4)$$

where $a_{i,c} = A_i \cos(\phi_i)$, $a_{i,s} = A_i \sin(\phi_i)$ and $v(k)$ is the discrete form of the measurement noise. Its covariance matrix is R_k .

Let put $a_i(k) = \begin{bmatrix} a_{i,c} \\ a_{i,s} \end{bmatrix}$ and $B_i(k) = [\cos(\theta_i(k)) - \sin(\theta_i(k))]$. Therefore, the Eq. (4) can be rewritten as

$$y(k) = \begin{bmatrix} \overbrace{0 \dots 0}^{M\text{-times}} & B_1(k) & \dots & B_M(k) \end{bmatrix} \begin{bmatrix} a_1(k-1) \\ \vdots \\ a_M(k-1) \\ a_1(k) \\ \vdots \\ a_M(k) \end{bmatrix} + v(k) \quad (5)$$

and symbolized by

$$y_k = H_k x_k + v_k \quad (6)$$

where $H_k = \begin{bmatrix} \overbrace{0 \cdots 0}^{M-\text{times}} & B_1(k) & \cdots & B_M(k) \end{bmatrix}$ is the measurement matrix and

$$x_k = \begin{bmatrix} a_1(k-1) \\ \vdots \\ a_M(k-1) \\ a_1(k) \\ \vdots \\ a_M(k) \end{bmatrix}$$

is the state variable.

The Eq. (6) represents the measurement equation of the Vold-Kalman estimator.

2.2 The State Equation

To estimate the components of the vector x_k , we use a Vold-Kalman constraint. It consists to model each component of the state variable by a smooth polynomial [7]. In general, this polynomial is of degree two and 2 – *times* differentiable. This approximation in time domain is

$$\frac{d^2 a_i(t)}{dt^2} = w_i(t) \quad (7)$$

where w_i is the i th state noise. In discrete time domain, the last equation becomes

$$a_i(k+1) - 2a_i(k) + a_i(k-1) = w_i(k) \quad (8)$$

The matrix form of the Eq. (8) yields to

$$\begin{bmatrix} a_i(k) \\ a_i(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} a_i(k-1) \\ a_i(k) \end{bmatrix} + \begin{bmatrix} 0 \\ w_i(k) \end{bmatrix} \quad (9)$$

To track all the M orders components, we generalize the previous equation and we obtain this one

$$\begin{bmatrix} a_1(k) \\ \vdots \\ a_M(k) \\ a_1(k+1) \\ \vdots \\ a_M(k+1) \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1 \\ -1 & \cdots & 0 & 2 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -1 & 0 & \cdots & 2 \end{bmatrix} \begin{bmatrix} a_1(k-1) \\ \vdots \\ a_M(k-1) \\ a_1(k) \\ \vdots \\ a_M(k) \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ w_1(k) \\ \vdots \\ w_M(k) \end{bmatrix} \quad (10)$$

The symbolic form of the state equation is

$$x_{k+1} = Fx_k + w_k \quad (11)$$

where w is the state noise with an unknown covariance matrix Q_k . Having a priori knowledge of R_k and Q_k , the algorithm to estimate the state variable can be implemented as follows

$$P_{k+1|k} = FP_kF^T + Q_k \quad (12)$$

$$K_{k+1} = P_{k+1|k}H_{k+1}^T[H_{k+1}P_{k+1|k}H_{k+1}^T + R_k]^{-1} \quad (13)$$

$$\hat{x}_{k+1} = F\hat{x}_k + K_{k+1}[y_{k+1} - H_{k+1}F\hat{x}_k] \quad (14)$$

$$P_{k+1} = (I - K_{k+1}H_{k+1})P_{k+1|k} \quad (15)$$

where $\hat{x}_k$ is the estimation of x_k , K_k is the Kalman gain, $P_k = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T]$ is the updated covariance matrix of estimation error and $P_{k+1|k}$ is the predicted covariance matrix of estimation error using only the information available at time k . $[\bullet]^T$ stands for transpose symbol. To compute this algorithm we need also the initial value of the state estimation $\hat{x}_1$ and the initial updated covariance matrix of estimation error P_1 .

3 The Tuning of the Vold-Kalman Estimator

The proper choice of covariance matrices R_k and Q_k highly determines the accuracy of the estimation. In general, these parameters are arbitrary fixed. The estimation of the parameter R_k is done using an usual adaptive approach presented in [11, 12]. The main practical issue is the setting of Q_k . The new adaptive approach to estimate Q_k is established below.

From the Eq. (15), we remark that we can estimate the state noise at instant k as followed

$$\hat{w}_k = K_k[y_k - H_kF\hat{x}_k] \quad (16)$$

We show that the covariance matrix for each sample can be written

$$\hat{Q}_k = K_kH_k\hat{P}_kH_k^TK_k^T + K_k\hat{R}_kK_k^T \quad (17)$$

where $\hat{P}_k = E[(\hat{x}_k - F\hat{x}_{k-1})(\hat{x}_k - F\hat{x}_{k-1})^T]$ and $\hat{R}_k = [\hat{v}_k\hat{v}_k^T]$ with $\hat{v}_k = y_k - H_k\hat{x}_k$. We assume that the residual $(\hat{x}_k - F\hat{x}_{k-1})$ is uncorrelated from the estimated of the measurement noise $\hat{v}_k$.

According to Mohamed and Schwarz [11] and Ali et al. [12], R_k can be estimated using innovation process

$$I_k = y_k - H_kF\hat{x}_k \quad (18)$$

and then $\hat{R}_k$ is computed as follows

$$\hat{R}_k = C_I + H_k P_{k|k-1} H_k^T \quad (19)$$

where C_I is the variance of innovation process. It is computed using a window of length L and equal to

$$C_I = \frac{1}{L} \sum_{i=1}^L I_{k-i} I_{k-i}^T \quad (20)$$

4 Numerical Example

To evaluate the accuracy of the new tuning approach for the Vold-Kalman estimator, we use the synthetic signal (see Fig. 1) described by the following equation

$$y(t) = \sum_{i=1}^3 A_i(t) \cos(2\pi O_i \int_0^t f_r(u) du) + v(t) \quad (21)$$

where f_r is the instantaneous frequency linearly increasing from 0 to 50 Hz in 5 *seconds*, O_i contains the value of orders and v is a centered, white and Gaussian noise.

The signal is composed of three orders presented in the Table 1. Figure 2 displays the rpm-frequency spectrum using the conventional windowing Fourier transform that characterizes three orders.

The initialization of the parameters of the estimator is as follows:

- The initial modeling error is $Q = 10^{-6}I$.
- The initial covariance matrix of the measurement noise is $R = 1$.

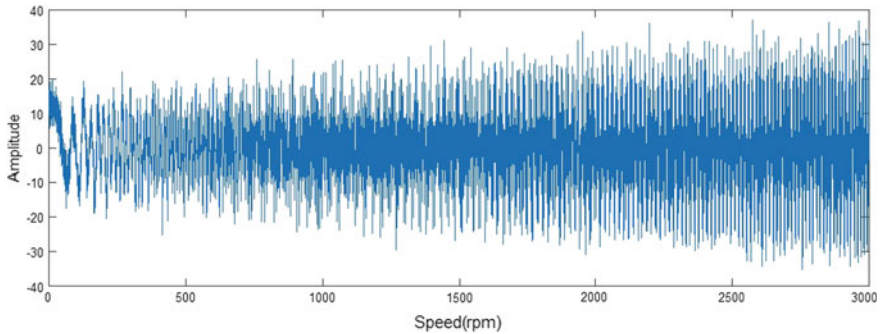
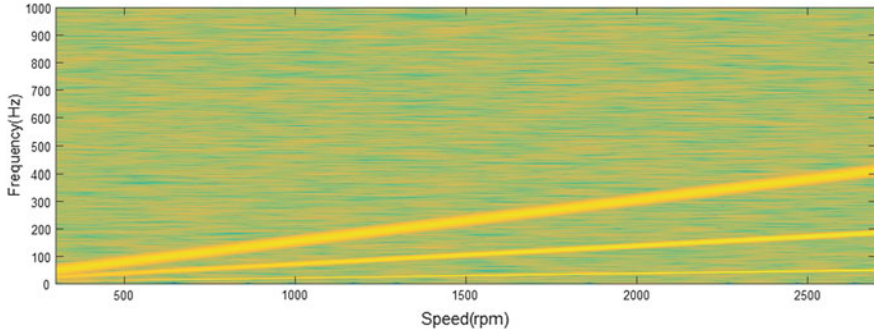
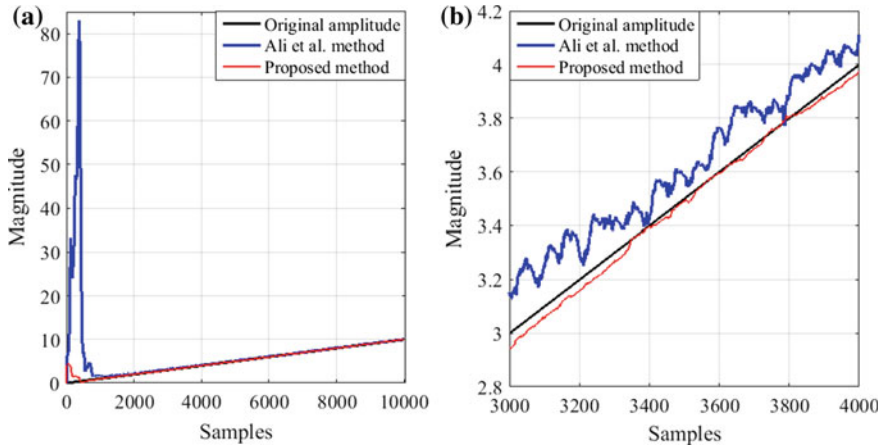


Fig. 1 Synthetic signal

Table 1 The synthetic signal's amplitude of orders

Order number	1	4	9
Amplitude	Linearly increasing from 0 to 10	Linearly increasing from 3 to 13	Fixed at 10

**Fig. 2** Illustration of the rpm-frequency spectrum**Fig. 3** **a** Estimation of the amplitude of the first order **b** Zoom on the estimation

- The initial value of the estimation is $\hat{x}_1 = [0, \dots, 0]$
 $\underbrace{\hspace{1.5cm}}_{12\text{-times}}$
- The initial covariance matrix of estimation error is $P_1 = 10^{-3}I$.

Here, the amplitudes of three orders are estimated by using the adaptive tuning approach of [12] and the one presented in this paper. On the Figs. 3, 4 and 5, we remark that we can quickly track the true amplitude of orders. The amplitude in red line (proposed method) is closer to the original amplitude than the amplitude in blue

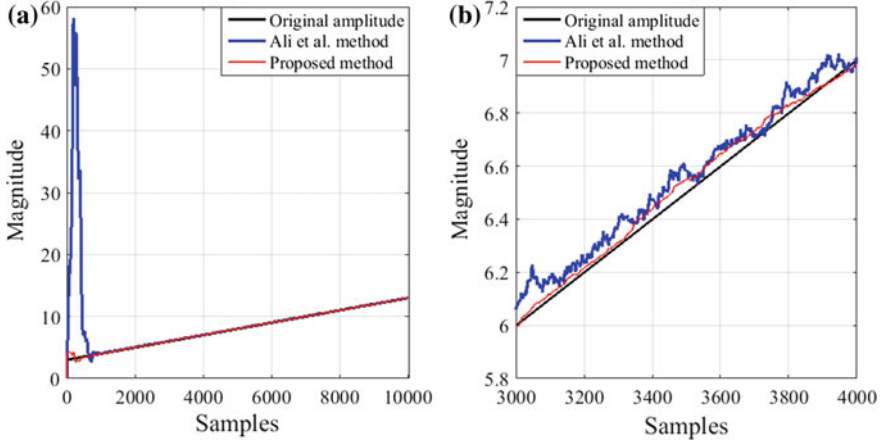


Fig. 4 **a** Estimation of the amplitude of the second order **b** Zoom on the estimation

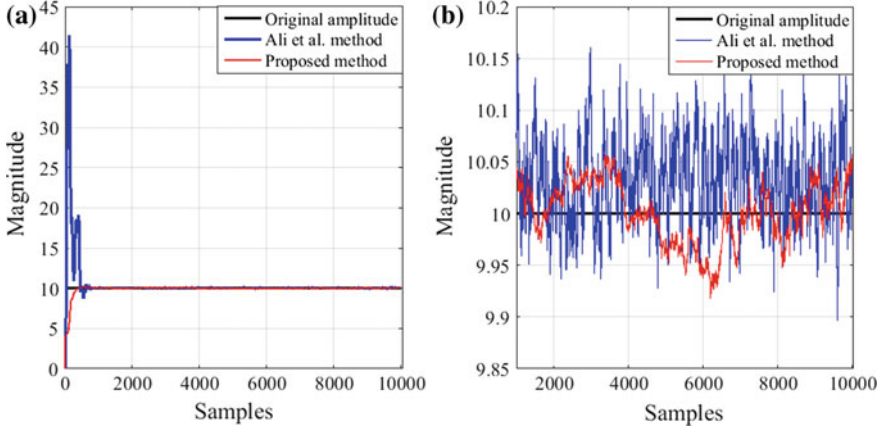


Fig. 5 **a** Estimation of the amplitude of the third order **b** Zoom on the estimation

line (Ali et al. method). For the constant amplitude, the estimation provides by the method of Ali et al. is very noisy and biased. It confirms the effectiveness of the proposed method when dealing with a non-stationary signal.

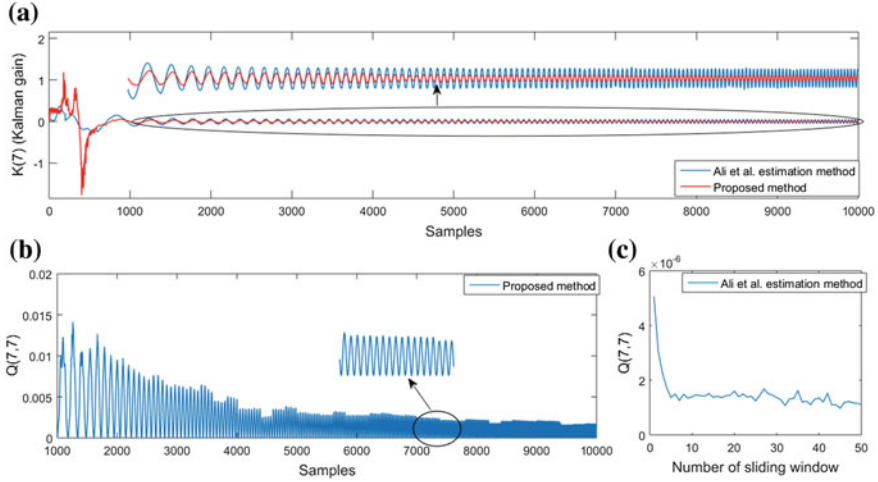
To accurately evaluate the performance of each method, we use the criterion based on the output signal to noise ratio given by

$$SNR_{out} = 10 \log_{10} \frac{\sum_{k=1}^N (H_k x_k)^2}{\sum_{k=1}^N [H_k (x_k - \hat{x}_k)]^2} \quad (22)$$

where N is the number of samples. In the Table 2, we observe that the estimation is improved at least by 5 dB when the input $SNR = 5$ dB and by 11.71 dB when the input $SNR = 15$ dB using the proposed method.

Table 2 Performance comparison of Ali et al. and the proposed method

$SNR_{in}(dB)$	SNR_{out}	
	Ali et al. method	Proposed method
15	28.8776	40.4928
10	22.2278	33.4668
5	19.2005	24.8501

**Fig. 6** **a** Kalman gain using the two methods of estimation, **b** Estimation of $Q(7,7)$ using the proposed method and **c** Estimation of $Q(7,7)$ using Ali et al. method [12]

The Fig. 6c represents the estimation of $Q_k(7,7)$ using the method in [12]. We remark that the estimation is almost constant. In the other side, on the Fig. 6b, the $Q_k(7,7)$ values decrease and change with the same frequency as that of the vibration signal. We observe the same behavior in the Kalman gain estimation (Fig. 6a). The Kalman gain has a constant amplitude by using the method of [12] (blue line) whereas its amplitude is more small and converging toward zero (red line). This allows the estimator to adapt itself and to provide a smallest estimation error.

5 Conclusion

In this paper, a new method has been introduced to estimate recursively the covariance matrices for the Vold-Kalman estimator. It is based on the residual estimation error. A new formula to calculate the covariance matrix of the state noise has been established. This approach allows the estimator to adapt itself to the signal variations. It also ensures the positiveness of the covariance matrices. And finally, a numerical implementation has been made to prove the effectiveness of the method over the previous method reported here.

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