Chapter 2
Basic Electromagnetic Theory

Electromagnetism

The man who does not read good books has no advantage over the man who cannot read them.

Mark Twain (1835–1910)

Abstract Physics of plasmas is described by the electromagnetic theory. Elementary principles of electromagnetism are reviewed in this chapter. Electric fields, magnetic fields, and Maxwell equations, which self-consistently relate these fields to each other, are introduced. Charged particles are sources of electric fields and moving charged particles can generate magnetic fields. The most obvious manifestation of electromagnetic fields is the light. The elecromagnetic spectrum is composed of a broad range of wavelengths from low-frequency radio waves ($\sim$10 MHz) to very high-frequency gamma rays ($\sim 10^{27}$ Hz). Finally the concept of phase space and collisions between particles of different species are discussed.

Keywords Electric field · Magnetic field · Electromagnetism · Magnetic flux · Electric currents · Maxwell equations · Poynting flux · Phase space · Collisions · Summation convention

2.1 Electric Field

Around a region of some electric charge distribution σ, there exists an electrostatic field and thus an electric field **E**. The simplest scenarios of a positively and a negatively charged particles are illustrated in Fig. 2.1. The term "electric" is of Greek origin, "elektron", and means amber. If there are any other charged particles within this field, that is in the space around σ, they experience a force, which is called the electric force. Strictly speaking, this force is said to be of electric origin if and only if it exists when a charged particle is inserted into the field. The resulting electric force on the inserted test particle at a point is proportional to the strength of the electric field and the charge intensity:

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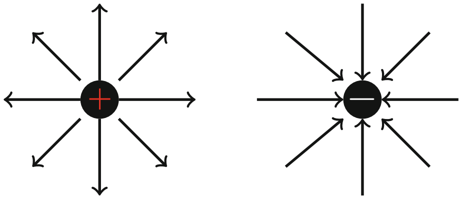

Fig. 2.1 Illustration of the electric field around a positive (*left*) and a negative charge (*right*). For a positive point charge, the field lines diverge, while they converge for a negative point charge

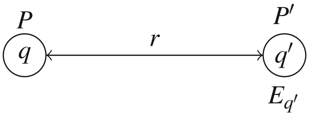

Fig. 2.2 The electric field $|\mathbf{E}| = E_{q'}$ at point P' experienced by a test charge q', distant r away from a point charge q located at point P

$$|\mathbf{F}_E| = |\mathbf{E}|q. \tag{2.1}$$

The electric field $\mathbf{E} = (E_x, E_y, E_z)$ is a vector field and the direction of the force is given by the direction of the electric force on a positive charge. The unit of the electric field is Newton per Coulomb (N C^{-1}). It is instructive to ask how one can determine the electric field distribution around a charged particle. To find the electric field at point P, distant $r = r_{qq'}$ away from a point charge q, imagine a test charge q' at P', as illustrated in Fig. 2.2. The force between the two charges is given by the *Coulomb's law*:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}, \tag{2.2}$$

where

$$\varepsilon_0 = \frac{1}{(\mu_0 c^2)} = 8.854 \times 10^{-12}\ \mathrm{F\,m^{-1}} \tag{2.3}$$

is the permittivity of free space and

$$\frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9\ \mathrm{m\,F^{-1}}. \tag{2.4}$$

Then, the electric field $E_{q'}$ at the position of the test charge q' is

$$E_{q'} = \frac{F}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}. \tag{2.5}$$

This formalism can be extended to multiple charges, $q_1, q_2, q_3, \ldots$, with distances $r_{12}, r_{13}, \ldots$, etc. In general, if there are N number of point charges, the resulting electric field at the location P' of the test charge is a superposition of all the individual ambient electric fields associated with other point charges:

$$E = \frac{1}{4\pi\varepsilon_0} \sum_{i}^{N} \frac{q_i}{r_i^2}. \tag{2.6}$$

2.2 Elecric Potential Energy

As an electric field exerts a force on a charged particle in the vicinity, work is performed, given by

$$W_{1\to 2} = \int_1^2 F_E \, ds \tag{2.7}$$

in moving the charge from location 1 to location 2. This work is independent of the path taken between the initial and the final locations and is thus said to be "conservative", just like the force performed in a gravitational field. This work can be represented by a potential energy E_p

$$E_p = \int_1^2 E \, q \, ds. \tag{2.8}$$

Generalizing this concept to a static charge distribution yields:

$$E_p = \frac{1}{4\pi\varepsilon_0} \sum_{i<j} \frac{q_i q_j}{r_{ij}}. \tag{2.9}$$

Using the potential energy, an electric potential V at any point in an electric field can be defined as the potential energy per unit charge

$$V = \frac{E_p}{q}. \tag{2.10}$$

Electric field and potential are closely related. The potential difference between two points "a" and "b" is the line integral of the electric field strength.

$$V_a - V_b = \int_a^b \mathbf{E} \cdot \mathbf{ds}. \tag{2.11}$$

This potential difference can be interpreted as the energy required to move a unit charge from a to b within the ambient electric field distribution $\mathbf{E}$ as illustrated in Fig. 2.3. This electrostatic field is conservative, thus

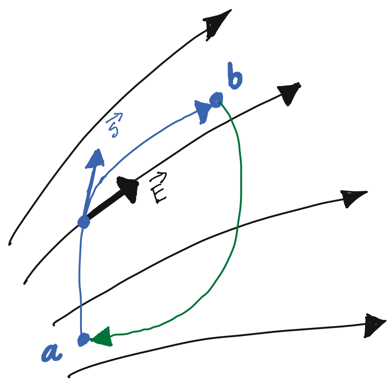

Fig. 2.3 Illustration of the work done in a representative electric field **E**, shown with *black field lines*, by moving a test charge from point *a* to *b* (*blue path*) and then from *b* to *a* (*green path*), which constitutes a closed path. The orientations of the path vector and the local electric field at a representative point are shown with *thick blue* and *black arrows*, respectively

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{s} = 0, \tag{2.12}$$

indicating again that work done in the electric field is independent of the path. So, in a closed path, such as the one illustrated in Fig. 2.3, no net work is performed.

Equivalently, the electric field can be deduced from an electric potential as

$$\mathbf{E} = -\boldsymbol{\nabla} V, \tag{2.13}$$

where the electric field points in the direction opposite from the direction of maximum rate of potential increase. So, as a charge is a sorce of an electric field, it also generates an electric potential field in its vicinity whose strength decreases with increasing distance (i.e., $V \propto q/r$). Note that the electric field strength is inversely proportional to the square of the distance ($V \propto q/r^2$).

2.3 Magnetic Field

The origin of magnetic fields can be understood by considering moving charges. A moving charge sets up in the space around it a magnetic field, which exerts a force on another charge moving through it. So, the magnetic field **B** is said to exist at a point if a force is exerted on a moving charge at that point. The associated magnetic force is

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}, \tag{2.14}$$

or equivalently

$$F_i = q\ \varepsilon_{ijk} v_j B_k, \tag{2.15}$$

where $\mathbf{v} = (v_x, v_y, v_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$ are the three-dimensional charged particle velocity and the magnetic field vector, respectively, and ε_{ijk} is the Levi-Civita

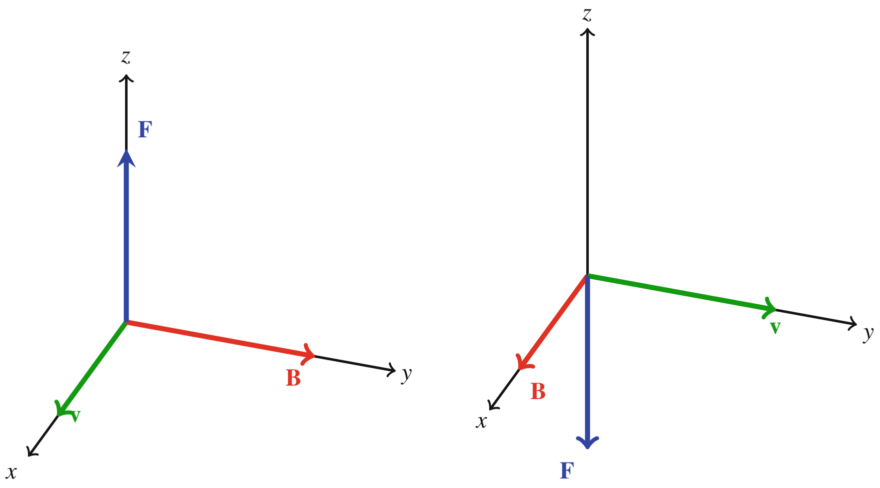

Fig. 2.4 Illustration of the vectors $\mathbf{v} \times \mathbf{B}$ in the rectangular Cartesian coordinate system for two scenarios: charged particle motion is the x-axis and the magnetic field is along the y-axis (*left*); motion is along the y-axis and the field is directed along the x-axis (*right*)

symbol (appendix B.1). The resulting force is perpendicular to both the motion and the field as shown in Fig. 2.4 for two different cases. The cross-product (or vector product) operator "×" implies a counterclockwise rotation described by the rotation angle $\alpha_{\mathbf{vB}}$

$$\mathbf{v} \times \mathbf{B} = v\ B\ \sin \alpha_{\mathbf{vB}} \tag{2.16}$$

where $v = |\mathbf{v}|$, $B = |\mathbf{B}|$, and $\mathbf{v} \times \mathbf{B} = -\mathbf{B} \times \mathbf{v}$, which explains why the force is pointing in the direction of the $-z$-axis in the figure on the right. In practical terms, this cross-product can be represented by the "right-hand-rule", where, e.g., the pointing finger is aligned with the velocity vector, the middle finger shows, with an angle $\alpha_{\mathbf{vB}}$ with respect to the pointing, in the direction of the magnetic field, then the resulting magnetic force is along a line perpendicular to the **v**-**B** plane, shown by the direction of the thumb. The unit of the magnetic field is the tesla (in recognition of Nikola Tesla) and is given by the unit of $F/(qv)$, that is:

$$1 \text{ tesla} = 1 \text{ T} = \frac{\text{N}}{\text{C m s}^{-1}} = \frac{\text{N}}{\text{A m}}. \tag{2.17}$$

Another practical unit of the magnetic field is the gauss G, where 1 G = 10^{-4} T. The gauss is a useful unit because Earth's magnetic field strength on the surface is in the order of 10 G. In comparison, in a laboratory environment, the strongest field that can be produced is around 5×10^5 G = 50 T.

More precisely, the vector field **B** is called the *magnetic induction field* and the lines of induction describes the magnetic field. The components of the magnetic force field in Cartesian coordinates are given by

$$\mathbf{F}_B = q \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = q\,[\,\hat{\mathbf{i}}(v_y B_z - v_z B_y) - \hat{\mathbf{j}}(v_x B_z - v_z B_x) + \hat{\mathbf{k}}(v_x B_y - v_y B_x)], \tag{2.18}$$

where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the Cartesian coordinate unit vectors with the properties $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$, $\hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$, and $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}$.

The field at a point can be characterized by the number of lines per unit area. The line of induction is expressed in the units of Weber (Wb) and the magnetic field is quantified by the unit Wb m^{-2}, which is equal to 10^4 Gauss.

The total number of lines through a surface A defines the *magnetic flux* Φ_m:

$$\Phi_m = \mathbf{B} \cdot \mathbf{n}\, A = B\, A \cos\alpha, \tag{2.19}$$

where $\mathbf{n}$ is the surface normal vector, α is the angle between the surface normal and the magnetic field vector. For a uniform magnetic field that is normal to the surface A, the magnetic flux is $\Phi_m = B\,A$. The magnetic induction is equivalent to the magnetic flux density.

2.4 Electromagnetic Field

The electromagnetic field consists of an electric and magnetic field component. Then, in the presence of an ambient electric and magnetic field, a point charge q moving with velocity $\mathbf{v}$ is subject to an electric force $\mathbf{F}_E$ and a magnetic force $\mathbf{F}_B$:

$$\mathbf{F}_L = m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{F}_E + \mathbf{F}_B$$

$$= q\mathbf{E} + q\mathbf{v} \times \mathbf{B}, \tag{2.20}$$

$$= q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{2.21}$$

where $\mathbf{F}_L$ is called the Lorentz force; electric field and magnetic field in the Cartesian coordinate system are described by the Cartesian unit vectors. In our discussions, the charge speed $v = |\mathbf{v}|$ is assumed to be much smaller than the speed of light

$$c \approx 3 \times 10^8 \text{ m s}^{-1}, \tag{2.22}$$

i.e., ($|\mathbf{v}| \ll c$), so that $\gamma = (1 - v^2/c^2)^{-1/2} \approx 1$ and relativistic effects can therefore be neglected.

The electric field causes an acceleration $(q/m)\mathbf{E}$ that is parallel to the electric field, in the direction of the field for positively charged particles and in the opposite direction of the field for the electrons. The magnetic field causes a centripetal acceleration $(q/m)\mathbf{v} \times \mathbf{B}$ that causes the particle to gyrate in a circular orbit around the magnetic field.

2.5 Electric Currents

Electric currents are caused by the motion of charged particles. The current density **j** at a given point in space expresses the amount of electric current per unit area (or simply charge flux). For free electron motion, the associated current density is

$$\mathbf{j}_e = \rho_e \mathbf{v}_e, \tag{2.23}$$

where $\rho_e = |e|n_e$ is the space charge density of electrons. In general, in conducting fluids, the current density is proportional to the electric field:

$$\mathbf{j} = \sigma \mathbf{E}, \tag{2.24}$$

where σ [in Siemens per meter] is the electric conductivity. The relation (2.24) is known as Ohm's law.

Metals are very good conductors ($\sigma \sim 10^7$ S m^{-1}), while fluids have relatively smaller conductivity (sea water: $\sigma \sim 5$ S m^{-1}).

For moving conductors, Ohm's law can be generalized as

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{2.25}$$

where all variables are measured with respect to the same fixed frame of reference. The magnetic field is a net field, which includes any internal fields (or seed fields) and the field due to the current density (thus the charge flow). One important manifestation of this principle occurs in sunspots, which are essentially self-excited dynamos (Lorrain and Koutchmy 1998). In sunspots due to the motion of the plasma, a current density $\mathbf{j} = \sigma(\mathbf{v} \times \mathbf{B})$ is induced within the seed field $\mathbf{B}$. The associated magnetic field of the induced current has the same sign as the seed field, thus provides a positive feedback. The resulting magnetic force density [N m^{-3}] is perpendicular to both the current density and the magnetic field:

$$\mathscr{F}_M = \mathbf{j} \times \mathbf{B}. \tag{2.26}$$

In a plasma also ions are mobile and the associated ion current density is

$$\mathbf{j}_i = \rho_i \mathbf{v}_i. \tag{2.27}$$

Therefore, one key plasma property is the net current due to the combined electron and ion motion. Presence of charged particles does not necessarily mean that there exist a net current. If electrons and positive ions move together than no net current is generated, but if they have a relative drift then the resulting current density is

$$\mathbf{j} = e(Zn_i\mathbf{v}_i - n_e\mathbf{v}_e), \tag{2.28}$$

where Z is the number of charges per ion. For example, for O^+ $Z = 1$. In the case of charge neutrality in a plasma $n_e = n_i$, the current density is

$$\mathbf{j} = en_e(\mathbf{v}_i - \mathbf{v}_e). \tag{2.29}$$

2.6 An Overview of Maxwell Equations of Electromagnetic Field in Vacuum

The unification of the electric and the magnetic fields have been proposed for the first time in 1873 by Maxwell as the electromagnetic (EM) theory. In general, sources of electromagnetic fields are charges, magnetized bodies, and currents. The EM field can be discrete or continuous; and stationary or time-dependent. Maxwell formulated four fundamental equations of the EM field in vacuum:

1. The **Gauss' law** (in differential form) for the electric field states that the electric field is generated by a charge density ρ_c

$$\nabla \cdot \mathbf{E} = \frac{\rho_c}{\varepsilon_0}, \tag{2.30}$$

   which can be written in terms of the Poisson's equation as

$$\nabla^2 V = -\rho_c/\varepsilon_0. \tag{2.31}$$

2. **No magnetic monopoles** (charges) or no magnetic currents exist, described by

$$\nabla \cdot \mathbf{B} = 0 \tag{2.32}$$

3. **Faraday's law** states that an electric field is generated by a changing magnetic field:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2.33}$$

4. The **Ampere's law** says that a (rotational) magnetic field is generated by a time-dependent electric field and a current density $\mathbf{j}$:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}, \tag{2.34}$$

   where

$$\mu_0 = \frac{1}{(\varepsilon_0 c^2)} = 4\pi \times 10^{-7}\ \mathrm{H\ m^{-1}} \tag{2.35}$$

   is the permeability of free space and the current density is defined as the current I per unit area.

2.7 Electromagnetic Energy Flow: Poynting Flux

The electromagnetic field carries energy. We need an appropriate equation that describes the energy balance in an EM-field. Starting with the Faraday Law described by Eq. (2.33) we can obtain the so-called electromagnetic field energy equation. First multiply the left-hand side of the Faraday's Law with the magnetic field and integrate over volume

$$-\int_V \mathbf{B} \cdot (\nabla \times \mathbf{E})\, \mathrm{d}^3 r = \int_V \mathbf{B} \frac{\partial \mathbf{B}}{\partial t}\, \mathrm{d}^3 r, \tag{2.36}$$

where the divergence of a curl is given by the identity

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b}). \tag{2.37}$$

So, the divergence of the vector product of the electric field with the magnetic field can be written as

$$\nabla \cdot (\mathbf{E} \times \mathbf{B}) = \underbrace{\mathbf{B} \cdot (\nabla \times \mathbf{E})}_{=1} - \underbrace{\mathbf{E} \cdot (\nabla \times \mathbf{B})}_{=2}. \tag{2.38}$$

If Ampere's Law (2.34) for the expression 2 is used, then Eq. (2.36) becomes

$$\mathbf{B} \cdot (\nabla \times \mathbf{E}) = \nabla \cdot (\mathbf{E} \times \mathbf{B}) + \mu_0\, \mathbf{E} \cdot \mathbf{j} + \mu_0 \varepsilon_0\, \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} \tag{2.39}$$

Using the divergence theorem

$$\int_V \nabla \cdot \mathbf{a}\, \mathrm{d}V = \oint_S \mathbf{a} \cdot \mathrm{d}\mathbf{S} \tag{2.40}$$

and the property

$$\mathbf{a} \cdot \frac{\partial \mathbf{a}}{\partial t} = \frac{1}{2} \frac{\partial (\mathbf{a} \cdot \mathbf{a})}{\partial t} \tag{2.41}$$

Eq. (2.36) becomes

$$\int_V \left(\mathbf{B} \frac{\partial \mathbf{B}}{\partial t} + \varepsilon_0 \mu_0\, \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} \right) \mathrm{d}^3 r = -\int_S \mathbf{E} \times \mathbf{B} \cdot \mathrm{d}\mathbf{S} - \int \mu_0\, \mathbf{E} \cdot \mathbf{j}\, \mathrm{d}^3 \mathbf{r} \tag{2.42}$$

$$\Rightarrow \frac{\partial}{\partial t} \int_V \left(\underbrace{\frac{B^2}{2\mu_0}}_{\varepsilon_B} + \underbrace{\varepsilon_0 \frac{E^2}{2}}_{\varepsilon_E} \right) \mathrm{d}^3 \mathbf{r} = -\int_S \underbrace{\frac{\mathbf{E} \times \mathbf{B}}{\mu_0}}_{\mathbf{S}_p} \cdot \mathrm{d}\mathbf{S} - \int_V \mathbf{E} \cdot \mathbf{j}\, \mathrm{d}^3 \mathbf{r}, \tag{2.43}$$

where $\varepsilon_B = B^2/2\mu_0$ is the magnetic energy density, $\varepsilon_E = \varepsilon_0 E^2/2$ is the electric field energy density,

$$\mathbf{S}_p = \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \tag{2.44}$$

is the energy flux vector or the Poynting flux vector, and $\mathbf{E} \cdot \mathbf{j}$ is the Ohmic losses. $\mathbf{E} \cdot \mathbf{j} > 0$ represents loss and $\mathbf{E} \cdot \mathbf{j} < 0$ is a generator. Additionally, work is done by matter as $\mathbf{j} \times \mathbf{B}$, meaning that a field exerts a force on particles.

The Poynting flux can be interpreted as a measure of electromagnetic energy transfer to a system. For space science applications, this flux is formulated in terms of a convection electric field $\mathbf{E}_c$ and the perturbation of the geomagnetic field $\mathbf{B}'$ as $\mathbf{S}_p = \mathbf{E}_c \times \mathbf{B}'/\mu_0$. The solar UV/EUV portion of this energy flux impacts the upper atmosphere while passing through the magnetosphere and entering the thermosphere. The Earth-directed Poynting flux component can be retrieved from satellites such as the DMSP (Defense Meteorological Satellite Program) satellite (Knipp et al. 2011). The estimated values are up to 15 mW m^{-2}, being highly dependent on the Interplanetary Magnetic Field (IMF), and can reach 100 mW m^{-2} during magnetically disturbed conditions.

The most obvious manifestation of the electromagnetic energy is the light, which itself is an electromagnetic wave composed of a spectrum of frequencies, which I shall discuss next.

2.8 Electromagnetic Spectrum and Photons

Light is an electromagnetic wave. Its interpretation as an electromagnetic wave has not been established until Maxwell who proposed that the time-varying magnetic fields generate electric fields ($\partial\mathbf{B}/\partial t \rightarrow \mathbf{E}$) and time-varying electric fields are a source of magnetic fields ($\partial\mathbf{E}/\partial t \rightarrow \mathbf{B}$). The electric and magnetic fields sustain each other, forming the basis for an electromagnetic wave, for example, the visible light. An electromagnetic wave carries energy and momentum and do not require a medium, while, for example mechanical waves do. Though, electromagnetic waves and mechanical waves can be described using similar mathematical techniques.

Figure 2.5 schematizes the electromagnetic spectrum, including the wavelength, frequency [Hz], and energy [eV] range, which covers a broad range of waves from the low-frequency radio waves ($\lambda \sim 10$ m) to the high-frequency γ-rays ($\lambda \sim 10^{-6}$ nm), demonstrating an amazing 15 orders of magnitude scale variation within the spectrum.

The visible spectrum, that is, the portion of the electromagnetic spectrum that can be directly seen by the human eye, extends from the familiar colors blue to red, corresponding to wavelengths from $\sim$380 to 750 nm. The frequency and wavelength are inversely related as

$$c = \frac{\lambda}{T} = \lambda f. \tag{2.45}$$

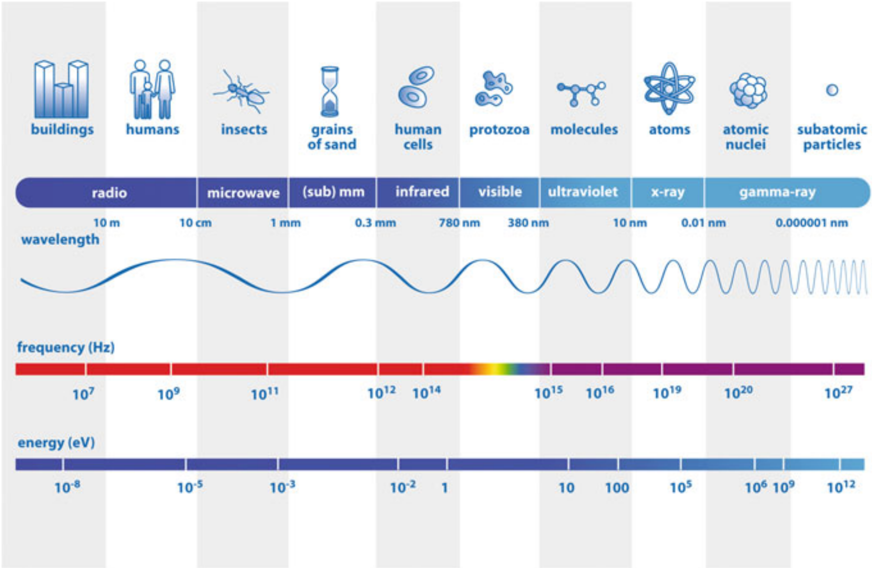

Fig. 2.5 A scheme of the electromagnetic spectrum with indication of wavelengths [nm], frequencies [Hz], and energies [eV] covering a spectrum of radio waves to gamma rays. The visible range illustrated with different color scales span wavelengths from 380 to 750 nm. Copyright: ESA / AOES Medialab

Thus the visible spectrum have frequencies ~7.9–4 $\times 10^{14}$ Hz. This implies that a photon, which is a part of the visible spectrum completes 10^{14} oscillations per second, which the human eye perceives as a blue to red color, although we are not capable of localizing individual photons. The energy of an electromagnetic wave with a given frequency (or wavelength) can be determined from

$$E = hf, \tag{2.46}$$

where $h = 6.626 \times 10^{-34}$ J s is the Planck constant. This relation is a consequence of Planck's postulate, with regard to the cavity radiation problem, that an oscillator cannot have continuous energies but only discrete energies specified by a number called the quantum number, n. His second postulate was that the oscillators do not radiate energy continuously, but only in certain steps or levels. When a change of state from one quantized level to another takes place, then energy is radiated that is given by the quantum number changes $\Delta E = \Delta nhf$, Following his postulates, Planck has derived a function that expresses the blackbody intensity as a function of temperature and frequency (wavelength) as

$$B_f = \frac{2hf^3}{c^2}\left[\exp\left(\frac{hf}{kT}\right) - 1\right]^{-1}. \tag{2.47}$$

Light as an electromagnetic wave is quantized. Einstein postulated that a beam of light consists of small packages of energy called *photons* or *quanta*. Equation (2.46) thus represents the energy of a single photon. One has to be cautious about the particle interpretation of light because photons are not particles in the usual sense. They travel in vacuum at the speed of light, have zero rest mass, possess momentum $p = E/c$, and demonstrate wave-like characteristics. The phenomenon of photons demonstrating both particle and wave characteristics is known as the *wave-particle duality*, which has caused an extensive amount of controversy in the development of quantum mechanics before the Second World War. However, the *complementarity principle* has been introduced, which was originally adopted from the German expression "Komplementaritätsprinzip", has allowed the application of both models without a contradiction.

According to (2.46), high-frequency waves, such as ultraviolet waves, have larger energy than low-frequency waves, such as the infrared waves. The energy emitted or radiated by electromagnetic waves is called the *electromagnetic radiation*.

So, how much is the energy of a single photon? In fact, the energy of a single photon is extremely small. For example, a photon belonging to the visible spectrum has an energy range of ~2.65–5.24 $\times 10^{-19}$ J. Given that 1 eV is 1.602×10^{-19} J we get a range of 1.65–3.37 eV. The solar radiation is the most obvious example of electromagnetic radiation although maybe we do not appreciate it on a day-to-day basis, as we are used the natural day-night cycle. Sun is the source of natural light.

The solar spectral irradiance (SSI) or the solar spectrum is shown in Fig. 2.6 as observed by NASA's SORCE spacecraft launched in 2003. The primary goal of the SORCE mission is to precisely measure solar radiation. It provides spectral irradiance measurements in the range 1–2000 nm, which accounts for $\sim 95\%$ of the spectral contribution to the total solar irradiance (TSI). The SSI plot presents the solar irradiance expressed in W m^{-2} nm^{-1} as a function of wavelength in nm. It is seen that the spectral energy density peaks around 500 nm, i.e., within the visible range of the spectrum, with about 2.1 W m^{-2} nm^{-1}. These values have also been reported in earlier texts (Ratcliffe 1972). Overall, the greatest amount of energy is in the range between 200 and 1200 nm. In this wavelength range, the average spectral energy density is ~ 1.11 W m^{-2} nm^{-1} and the total energy flux amounts to ~ 1110 W m^{-2}. Below 200 nm, the spectral density is very small (order of 10^{-5} W m^{-2} nm^{-1}), however, the short-wavelength part of the solar spectrum is the range that concerns the ionosphere the most, therefore its variability substantially impacts the ionosphere. The total energy flux of ~ 1110 W m^{-2} calculated from the SORCE data is smaller than the *solar constant*

$$S_c = 1368 \text{ W m}^{-2}, \tag{2.48}$$

which is the average amount of radiative solar energy reaching the top of Earth's atmosphere at a mean distance of 1 AU ($= 149.6 \times 10^6$ km). The total energy output of Sun is by no means constant. It demonstrates variations over various time scales. The Total Irradiance Monitor (TIM) on board the SORCE spacecraft measures the radiant power across the entire solar spectrum from X-ray to far infrared scales. The

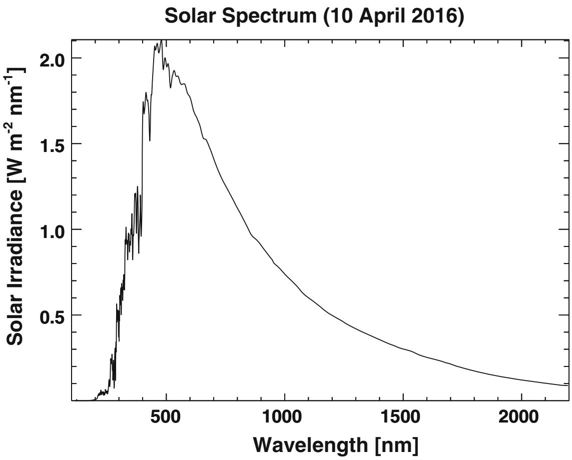

Fig. 2.6 Solar irradiance spectrum measured by the SORCE (Solar Radiation and Climate Experiment) spacecraft on 10 April 2016. The irradiance is plotted in W m^{-2} nm^{-1} as a function of wavelengths [nm]. The data is obtained from http://lasp.colorado.edu/lisird/sorce/sorce_ssi/

derived TSI was around 1361 W m^{-2} (Kopp et al. 2005), which was several W m^{-2} smaller than previous measurements.

The portions of the electromagnetic wave spectrum that are in particular relevant to the context of planetary atmospheres and ionospheres are the infrared (IR, $\lambda \sim 750$ nm–300 μm), ultraviolet ($\lambda \sim 10$–380 nm), and X-rays ($\lambda < 10$ nm) range, where the wavelength from 10–100 nm constitute the extreme ultraviolet portion (EUV). UV and EUV photodissociates O_2 in the terrestrial thermosphere. In particular, CO_2 and O_2 are strongly dissociated by solar radiation in the Schumann-Runge continuum (130–175 nm).

Figure 2.7 presents the penetration depth of solar radiation of wavelengths up to 300 nm in the terrestrial atmosphere as a function of altitude. While both UV and EUV contribute to energy absorption in the thermosphere, UV is the main contributor to heating in the middle atmosphere as it is primarily absorbed by ozone.

Figure 2.8 shows the Earth's atmospheric spectra in the visible and near-infrared range of the electromagnetic spectrum as observed by the OMEGA instrument (Table 1.2) onboard Mars Express spacecraft on 3 July 2003, as the spacecraft is on its long journey to Mars. The observed spectra is influenced in particular by the Pacific Ocean for this rotational configuration. It is seen that the spectra is dominated by the molecular species, such as molecular oxygen, water, carbon dioxide, ozone, and methane in the visible and infrared range.

2.9 Single Particle Motion in Electromagnetic Fields

When charged particles are in electric and/or magnetic fields they experience an external force as discussed earlier. We typically make some assumptions in order to simplify the charged particle motion in electromagnetic fields. The plasma density

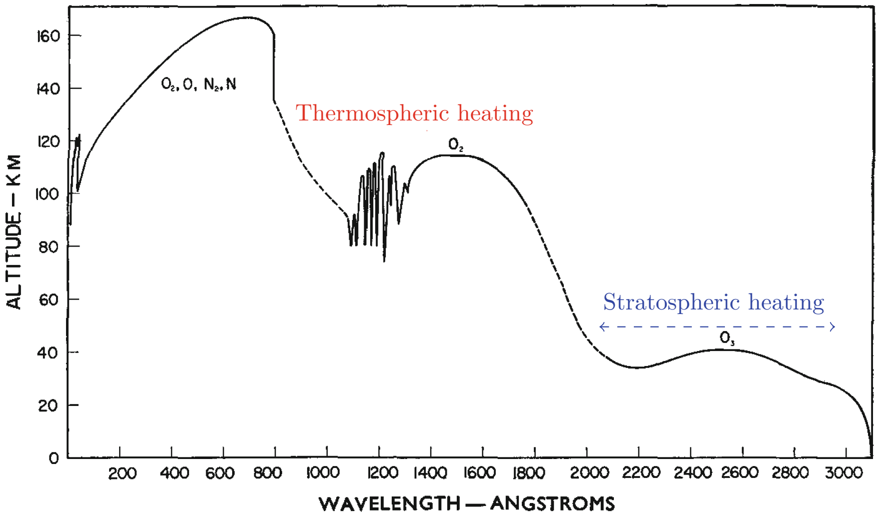

Fig. 2.7 Penetration depth of solar radiation in the terrestrial atmosphere. Photoabsorption by molecular oxygen above 100 km contributes toward thermospheric heating while photoabsoption by ozone between 30 and 50 km is the main source of stratospheric heating. Adopted from Friedman (1960)

is assumed to be small. Collisions between particles are ignored. The motion of the particles are determined only by the external fields and the energy density of the particles are small so that the external fields are not modified by charged particles.

One also has to consider the nature of the fields. Essentially, what one refers to as a field is a spatial distribution of a physical property. Typically, fields are two- or three-dimensional. For example, in a two-dimensional temperature field, the value may vary in two dimensions in space described by two coordinates, such as $T = T(\mathbf{r}) = T(x, y)$. Here temperature is a scalar field. The electric field $\mathbf{E} = \mathbf{E}(\mathbf{r})$ is a vector field. Fields can be time-dependent. In planetary atmospheres and ionosphere, we deal with time-dependent three-dimensional fields, e.g., $T = T(\mathbf{r}, t)$, $\mathbf{v} = \mathbf{v}(\mathbf{r}, t)$, etc. Thus, the most sophisticated flavor of atmospheric models are three-dimensional time-dependent nonlinear general circulation models (GCMs). Such models calculate the radiative, chemical, and dynamical properties of planetary atmospheres. From the perspective of variability, one can overall distinguish between smooth and turbulent fields. Smooth fields vary weakly on the spatiotemporal scales of gyration, while stochastic processes have to be taken into account in a turbulent field because the associated variations have time scales comparable to or less than the time scales of gyration. Particle ensembles have to be considered instead of individual particles and transport equations are solved instead of the equation of motion. For example, transport processes in the interplanetary medium are typically turbulent.

One simple case of particle motion is in the case of uniform B field and zero electric field. If the magnetic field is uniform in space in the absence of an electric field $\mathbf{E} = 0$ then a given charged particle will experience a force due to the magnetic force only. The equation of motion is given by

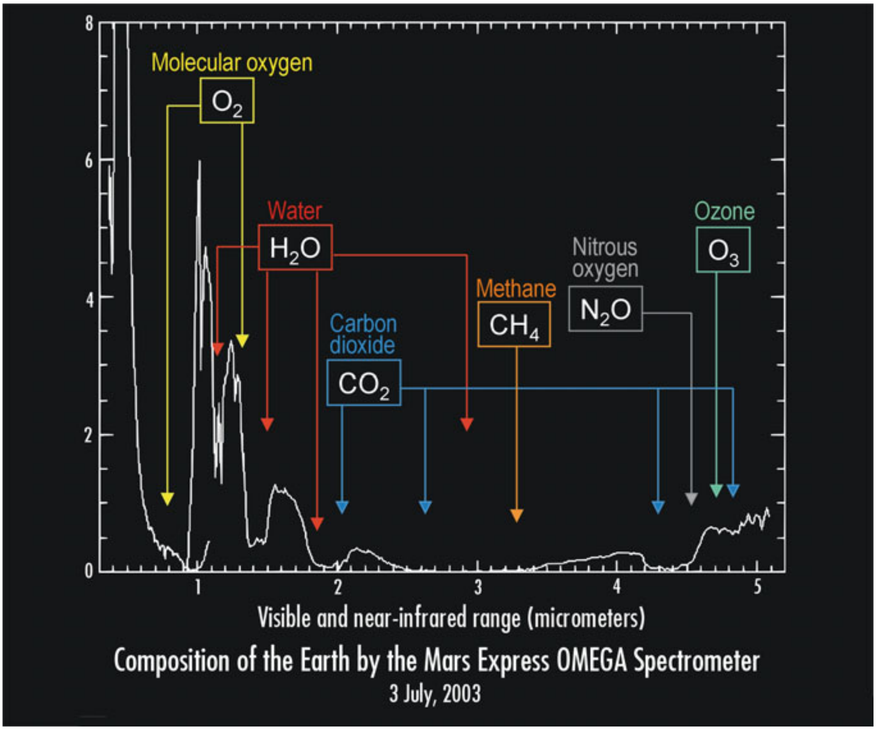

Fig. 2.8 Composition of Earth's atmosphere as detected by the Mars Express spacecraft in the visible and near-infrared range on 3 July 2003 on the way to Mars. The measurement was conducted by the OMEGA instrument facing Earth. The observed spectra is dominated by the Pacific Ocean and indicated the presence of molecular species, such as molecular oxygen, carbon dioxide, water, and ozone, and some other trace species like methane Credits: ESA/Institut d'Astrophysique Spatiale (Orsay, France)

$$\begin{aligned}\mathbf{F}_L &= \mathbf{F}_B \\ m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} &= q\mathbf{v}\times\mathbf{B}\end{aligned} \tag{2.49}$$

Let us assume that the magnetic field is directed along the z-axis as illustrated in Fig. 2.9, therefore $\mathbf{B} = (0, 0, B_z) = (0, 0, B)$ and $|\mathbf{B}| = B$. Then, the magnetic force is perpendicular to $\mathbf{B}$ and the components of the equation of motion are:

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = q(v_y B, -v_x B, 0) \tag{2.50}$$

$$m\dot{v}_x = q v_y B \tag{2.51}$$

$$m\dot{v}_y = -q v_x B \tag{2.52}$$

$$m\dot{v}_z = 0 \tag{2.53}$$

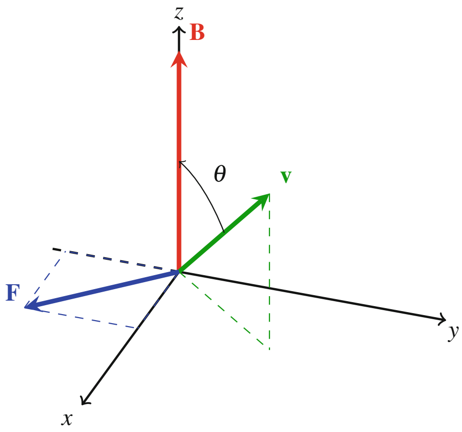

Fig. 2.9 Illustration of the magnetic field vector **B** which is aligned along the z-axis in this example, and a representative particle velocity vector **v**, which has components in all three coordinate directions, is shown in *green*. The angle between the magnetic field and the velocity is denoted by θ and the rotation of the velocity vector, $\mathbf{v} \times \mathbf{B}$, is shown by the *arrow*. The direction of the force vector **F** is marked in *blue* and is in the x-y-plane

The resulting force vector is on the xy-plane and is oriented in the positive x-direction and negative y-direction. This property can be deduced qualitatively from the components (2.51) and (2.52) of the equation of motion. Along the magnetic field no force acts on the particle and thus no acceleration occurs along the z-direction. Overall, the component of the velocity along the magnetic field, ($\mathbf{v}_\parallel$), leads to a drift along the magnetic field, while gyrating around it, owing to the perpendicular component of the velocity ($\mathbf{v}_\perp$), which experiences a magnetic force perpendicular to both the magnetic field and itself.

More complicated cases of magnetic field and electric field distribution are possible. For example, the electric field may be nonzero and the magnetic field may be nonuniform. In the presence of electric and magnetic fields, plasma drift manifests itself as a $\mathbf{E} \times \mathbf{B}$ drift. The exact nature of the electric field, whether it is time-dependent or uniform, will determine the $\mathbf{E} \times \mathbf{B}$ drift. This drift is given by

$$\mathbf{v}_{E\times B} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}, \tag{2.54}$$

which is independent of the sign of the charge. Thus electrons and ions drift in the same direction, with electrons and ions being accelerated when moving antiparallel and parallel to the electric field, respectively.

2.10 Concept of Phase Space—Collection of Particles

The single particle motion treatment (Sect. 2.9) can be expanded to multiple particles by introducing the concept of *phase space* in which a collection of particles can be described by an associated distribution function that provides detailed information on the position and velocity of a given particle at time t. This description is more appropriate because plasmas in nature consist of a collection of particles. In particular, large numbers of plasma particles have a spectrum of velocities.

The phase space consists of the configuration space $\mathbf{r}$ and the velocity space $\mathbf{v}$. It is a six-dimensional space in time. That is, every particle (atom or molecule) is described by six coordinates in space: three coordinates denoted by x, y, and z, which are the positions of its center of gravity, and additional three coordinates of velocity components v_x, v_y, and v_z. The phase space is expressed by the generic distibution function

$$f(\mathbf{r}, \mathbf{v}, t), \tag{2.55}$$

which is the *phase space distribution function* for a single particle species. This distribution function can be used to describe the properties of plasma consisting of a large number of particles by stating how many particles are contained in the six-dimensional space (or volume element). The associated *phase space differential volume* $\mathrm{d}V_p$ is given by

$$\mathrm{d}V_p = \mathrm{d}|\mathbf{r}|\,\mathrm{d}|\mathbf{v}| = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}v_x\,\mathrm{d}v_y\,\mathrm{d}v_z \tag{2.56}$$

Multiplication of the phase space distribution function with the differential volume element represents the number of particles $\mathrm{d}N_p$ in the phase space differential volume

$$\mathrm{d}N_p = f(\mathbf{r}, \mathbf{v}, t)\,\mathrm{d}|\mathbf{r}|\,\mathrm{d}|\mathbf{v}|. \tag{2.57}$$

The total number of particles is then given by the integration of the phase space distribution function over all infinitesimally small phase space volume elements:

$$N_p = \int_{-\infty}^{+\infty} f(\mathbf{r}, \mathbf{v}, t)\,\mathrm{d}V_p. \tag{2.58}$$

2.11 Collisions

Collisions describe the process of two or more particles coming in contact with each other and exchanging various dynamical and thermodynamical properties. There are, in general, two basic simplified types of collisions in nature: *elastic* and *inelastic* collisions. Linear momentum $\mathbf{p} = m\mathbf{u}$ and total energy, kinetic plus potential ($E_k + E_p$), are conserved for both, but for inelastic collisions kinetic energy is transformed to other forms of energy, e.g., heat, ionizations energy, etc., while for elastic collisions dissipative loss processes are negligible. In the discussion of collision, I will not go into the details of collision integrals, as these are beyond the scope of this book. Some rigorous treatment of collision integrals can be found, for example, in the books by Chapman and Cowling (1970) and Schunk and Nagy (2009). Thus, practical aspects of collisions will be discussed in our context. The force density (force per unit volume) experienced by the particles of species "i", $\mathbf{F}_{ij}$ due to collisions with particles of species “j” is equal to the negative of the force density experienced by particles of species “j”, $\mathbf{F}_{ji}$, due to collisions with particles of species “i”

$$\mathbf{F}_{ij} = -\mathbf{F}_{ji}. \tag{2.59}$$

This mutual force arises because of the relative velocity of the different particles with respect to each other. The force densities can then be conveniently expressed as

$$\mathbf{F}_{ij} = n_i\, m_i\, \nu_{ij}(\mathbf{u}_j - \mathbf{u}_i) = -n_i\, m_i\, \nu_{ij}(\mathbf{u}_i - \mathbf{u}_j), \tag{2.60}$$

$$\mathbf{F}_{ji} = n_j\, m_j\, \nu_{ji}(\mathbf{u}_i - \mathbf{u}_j) = -n_j\, m_j\, \nu_{ji}(\mathbf{u}_j - \mathbf{u}_i), \tag{2.61}$$

where n_i and n_j are the number densities; m_i and m_j are the masses of the particles; $\mathbf{u}_i$ and $\mathbf{u}_j$ are the drift velocities of the particle species; and ν_{ij} is the collision frequency for the collisions of the “i” particles with “j” particles, and ν_{ji} is the collision frequency for the collisions of the “j” particles with “i” particles. Often the collision frequency is interpreted as an effective momentum transfer coefficient and it is important to note that it is in general not a symmetric quantity, i.e., the frequency of the collisions of the “i” particles with “j” particles is not necessarily equal to that of the “j” particles with “i” particles, i.e., $\nu_{ij} \neq \nu_{ji}$. However, ν_{ij} and ν_{ji} are related to each other as

$$\rho_i \nu_{ij} = \rho_j \nu_{ji}, \tag{2.62}$$

where mass densities of the species are given by

$$\rho_i = m_i n_i, \qquad \rho_j = m_j n_j. \tag{2.63}$$

Considering collisions among various particles as a source/sink term in the momentum balance of a given particle species, we can write the associated time rate of change of velocity, resulting from collisional interactions. The acceleration of the “i” particles resulting from their collisions with the “j” particles is then

$$\left(\frac{\partial \mathbf{u}_i}{\partial t}\right)_{ij} = -\nu_{ij}(\mathbf{u}_i - \mathbf{u}_j). \tag{2.64}$$

Depending on the sign of the differential velocity, the collision force can lead to an acceleration or deceleration. We can solve the relation (2.62) for one of the collision frequencies

$$\nu_{ij} = \frac{\rho_j}{\rho_i}\nu_{ji}. \tag{2.65}$$

Therefore, in the case of the collisions of two particle species, the associated collision frequencies are equal to each other provided that $\rho_i = \rho_j$. In general, in planetary atmospheres there are multiple particle species, such as electrons, ions, and neutrals. Besides, there can be further species of neutrals and ions. Thus, a given particle species, let us say a specific ion species can have collision with all other existing ion species, neutral compositional species, and electrons. One simplification that can be made in planetary atmospheres and ionospheres is that the particles of a particular species move with the same speed (see fluid model, Sect. 3.1), which implies that the members of the same species do not experience any collisions with each other, so in summary

$$\mathbf{F}_{ij} = \begin{cases} |\mathbf{F}_{ij}| = 0 \text{ if } i = j \\ |\mathbf{F}_{ij}| > 0 \text{ if } i \neq j \end{cases} \tag{2.66}$$

Such a simplification has great computational advantages in modeling of planetary atmospheres.

2.12 A Useful Mathematical Note: Summation and Product Notations

There is a large number of notations in physics. The summation notation "$\sum$" is one of the most common notations that is used, for example, to conveniently represent calculations with matrices and tensor. Given a variable $a_r (r = 1, 2, 3)$ and $x^r (r = 1, 2, 3)$. For example, a homogeneous linear function of the variables can be written in the form

$$\sum_{m=1}^{3} a_m x^m = a_1 x^1 + a_2 x^2 + a_3 x^3, \tag{2.67}$$

which depends only on one index "m", which runs from 1 to 3. This expression describes a system of first order (or simple systems) and the terms a_m and x^m are called the elements (or components) of the system. A second order system is defined by two indices, "m" and "n":

$$\begin{aligned} \sum_{m,n=1}^{3} a_{mn}\, x^m x^n &= \sum_{m=1}^{3}\sum_{n=1}^{3} a_{mn}\, x^m x^n \\ &= \left(\sum_{m=1}^{3} a_{m1}\, x^m x^1\right) + \left(\sum_{m=1}^{3} a_{m2}\, x^m x^2\right) + \left(\sum_{m=1}^{3} a_{m3}\, x^m x^3\right) \end{aligned} \tag{2.68}$$

which is a set of homogeneous quadratic function. Often, summations can be conveniently represented using the *summation convention* as

$$a_m x^m = a_1 x^1 + a_2 x^2 + a_3 x^3, \tag{2.69}$$

the left hand side of which is more compact than explicitly using the summation notation in (2.67). In the summation convention, a repeated small index is summed from 1 to 3. The repeated index is often referred to as the dummy index. For example, using the summation convention, the full system of linear equations can be conveniently represented by $a_{mn}\, x^m x^n = b_{mn}$.

The rules for the product notation are similar to the summation notation. For example,

$$\prod_{i=1}^{I} a_i = a_1 \cdot a_2 \cdot \cdots \cdot a_I. \tag{2.70}$$

For example, $n!$ (n factorial) can be expressed by the product notation

$$\prod_{i}^{n} i = n!. \tag{2.71}$$

2.13 Concluding Remarks

In this chapter I have summarized some basic aspects of electromagnetic theory, focusing on the electromagnetic waves and forces, currents, and collisions between particles. The next chapter is going to introduce transport equations, which are the basis of many fundamental equations relevant to planetary atmospheres and ionospheres.

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