

Chapter 2

Luenberger Observer Design for Uncertainty Nonlinear Systems

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Abstract Most of the nonlinear observers require the nonlinear systems to be known. If the systems are partly unknown, model-free observers such as high-gain observers, sliding mode observers, and neural observers, can be applied. However, the performances of these observers are not satisfactory, for example, they are sensitive to measurement noise and they can only estimate the derivative of the output. In this chapter, we use the structure of Luenberger observers for partially unknown nonlinear systems. Using a Riccati differential equation, we design a time-varying observer gain such that the observer error is robust with respect to bounded uncertainties. Compared with the other robust nonlinear observers, this observer is simple and effective with respect to the uncertainties in the nonlinear systems.

2.1 Introduction

The state observation problem is one of the essential points in modern control theory. If the systems are linear, the well-known observers can be the Kalman filters (stochastic noise cases) and Luenberger observers (deterministic cases) [11]. Since the early 80s, many implementations in theory and practice focus on the observers for nonlinear systems. Many nonlinear observers have been developed, such as Lie-algebra-based observers [24], Luenberger-like observers [12], high-gain observers [10], optimization-based observers [6], linearization approaches [25], stochastic systems [26], and reduced-order nonlinear observers [7], adaptive high-gain observers [2], etc. A basic requirement of the above observers is the nonlinear systems that are completely known [14]. The observers duplicate the nonlinear dynamics. There are no internal and external uncertainties.

If the uncertainties in the nonlinear systems can be parameterized, the nonlinear adaptive observers [20] can obtain the states. H_∞ and L_2 techniques are applied to

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construct the robust observers for linear systems in [3]. The results are extended to the Lipschitz nonlinear systems in [15]. These robust observers are very complex, and require the uncertainties to satisfy desired properties. If the uncertainties are structural, i.e., the uncertainties and disturbances are assumed to be bounded, nonlinear model-free observers are needed [13]. High-gain observers are the most popular model-free nonlinear observers, they are robust against model uncertainty and disturbances [17]. However, they can only estimate the derivative of the output and are sensitive to measurement noises. Sliding mode observers do not require estimate the derivative of the output [27]. However, the measurement noise and disturbances cause chattering in the nonlinear observers [18]. Another model-free nonlinear observer uses neural networks to approximate the unknown part and then to design an observer [4]. However, the observation error is large, because the output has to be used for neural approximation and observer estimation [16].

In this chapter, the nonlinear observer is also for unknown or partially unknown nonlinear system. We use a Luenberger-like structure. Compared with the other robust nonlinear observers, high-gain observer, sliding mode observer, and neural observer, our Luenberger-like nonlinear observer is more simple. It can estimate not only the derivative of the output. It is not sensitive to measurement noises. It does not necessarily model the nonlinear system. Compared with the other Luenberger-like nonlinear observers [1, 19], our observer do not need to copy the complete nonlinear system for the observer design. They assume the nonlinear systems do not have any uncertainties.

We assume both external disturbances (noises) and internal uncertainties (unmodeled dynamics) are bounded. The assumptions on the partly known nonlinear systems are standard, such as Lipschitz and uniformly observability. The gain of Luenberger observer is specially selected to guarantee the property of robustness using a Riccati differential equation. The stability of this robust nonlinear observer is proven. An example demonstrates the effectiveness of this observer for the system containing complex uncertain nonlinearities.

2.2 Robust Nonlinear Observer

We discuss a class of single input and single output nonlinear systems given by

$$\begin{aligned}\dot{x} &= f(x) + g(x)u_t + \xi_{1,t} \\ y_t &= Cx + \xi_{2,t}\end{aligned}\tag{2.1}$$

where $f(x) = \begin{pmatrix} x_2 \\ \vdots \\ x_n \\ \varphi_0(x) + \Delta\varphi(x) \end{pmatrix}$, $g(x, t) = \begin{pmatrix} g_1(x_1) + \Delta g_1(x_1) \\ g_2(x_1, x_2) + \Delta g_2(x_1, x_2) \\ \vdots \\ g_n(x_1 \cdots x_n) + \Delta g_n(x_1 \cdots x_n) \end{pmatrix}$, $x \in \mathfrak{N}^n$ is the state vector of the system at time $t \in R^+ := \{t : t \geq 0\}$, $u_t \in \mathfrak{U}$ is a

given control action, it is assumed to be bounded as $|u_t|^2 \leq \bar{u}$, $y_t \in \mathfrak{Y}$ is the output vector that is suggested to be measurable at each time t , $f(\cdot)$ and $g(\cdot) : \mathfrak{R}^{n+1} \rightarrow \mathfrak{R}^n$, are nonlinear functions describing the dynamic operator of the given system, $C = [1, 0 \cdots 0]$, $\xi_{1,t}$ and $\xi_{2,t}$ are the vector functions presenting the external state and output perturbations.

We assume that the nominal parts of nonlinear system $F(x)$ and $G(x)$ are known,

$$\begin{aligned} f(x) &= F(x) + \Delta f(x) \\ g(x) &= G(x) + \Delta g(x) \end{aligned} \quad (2.2)$$

where $F(x) = [x_2, \dots, x_n, \varphi_0(x)]$, $G(x) = [g_1(x_1), \dots, g_n(x_1 \cdots x_n)]$, $\Delta f(x) = [0, \dots, 0, \Delta\varphi(x)]$, and $\Delta g(x) = [\Delta g_1(x_1), \dots, \Delta g_n(x_1 \cdots x_n)]$.

In this chapter, we use the following four assumptions to design a robust nonlinear observer for the system (2.1).

A1. The perturbations $\xi_{1,t}$ and $\xi_{2,t}$ satisfy the following “bounded power” condition

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \int_0^T \|\xi_{i,t}\|_{\Lambda_{\xi_i}}^2 dt = \gamma_i < \infty, \quad i = 1, 2 \quad (2.3)$$

where $\Lambda_{\xi_i} = \Lambda_{\xi_i}^T > 0$, $\|\xi_{i,t}\|_{\Lambda_{\xi_i}}^2 = \xi_{i,t}^T \Lambda_{\xi_i} \xi_{i,t}$.

A2. The normal parts of the nonlinear system $F(x, t)$ and $G(x, t)$ are observable and Lipschitz in a compact set U

$$\begin{aligned} \|F^T(x_1) - F^T(x_2)\|_{\Lambda_f} &\leq L_f \|x_1 - x_2\|_{\Lambda_{fx}} \\ \|G^T(x_1) - G^T(x_2)\|_{\Lambda_g} &\leq L_g \|x_1 - x_2\|_{\Lambda_{gx}} \end{aligned} \quad (2.4)$$

where $\Lambda_f \in \mathfrak{R}^{n \times n}$, $\Lambda_{fx} \in \mathfrak{R}^{n \times n}$, $\Lambda_g \in \mathfrak{R}^{m \times m}$, $\Lambda_{gx} \in \mathfrak{R}^{n \times n}$ are strictly positive-definite matrices, or Lipschitz constant matrices. L_f and L_g are Lipschitz constants.

A3. The unmodeled dynamics Δf and Δg in (2.2) satisfy the “strip bounded” conditions

$$\begin{aligned} \Delta f^T(x) \Lambda_{\Delta f} \Delta f(x) &\leq C_{\Delta f} + x^T D_{\Delta f} x \\ \Delta g^T(x) \Lambda_{\Delta g} \Delta g(x) &\leq C_{\Delta g} + x^T D_{\Delta g} x \end{aligned} \quad (2.5)$$

where $0 < \Lambda_{\Delta f}^T = \Lambda_{\Delta f} \in \mathfrak{R}^{n \times n}$, $0 < D_{\Delta f}^T = D_{\Delta f} \in \mathfrak{R}^{n \times n}$, $0 < D_{\Delta g}^T = D_{\Delta g} \in \mathfrak{R}^{n \times n}$ are known constant matrices, $C_{\Delta f}$ and $C_{\Delta g}$ are known positive constants characterizing the behavior of the corresponding unmodeled dynamic mappings at the point $x = 0$.

A4. There exist a stable matrix A_0 , strictly positive-defined matrices Q_0 and R_t , such that the following matrix differential Riccati equation

$$\dot{P}_t + P_t A_0 + A_0^T P_t + P_t R_t P_t + Q_0 = 0 \quad (2.6)$$

has strictly positive solution P_t .

The assumption **A1** requires the noises or unknown disturbances are bounded. This is a basic assumption for the analysis of nonlinear systems [9]. Otherwise, the nonlinear systems are flooded with the noises.

The assumption **A2** comes from the fact that a nonlinear system is uniformly observable for any input if and only if it can be presented as the observer form (2.1), see Theorem 2 of [10]. The nominal parts are also Lipschitz, as

$$\begin{aligned} F(x + \Delta x) &= F(x) + \nabla^T F(x) \Delta x + v_f \\ G(x + \Delta x) &= G(x) + \nabla^T G(x) \Delta x + v_g \end{aligned} \quad (2.7)$$

where the vector v_g can be estimated as $\|v_f\| \leq 2L_f \|\Delta x\|$, $\|v_g\| \leq 2L_g \|\Delta x\|$.

The “strip bounded” condition **A3** is more general than the bounded unmodeled dynamic [10] and the sector condition [5]. If the unmodeled dynamics are bounded, $D_{\Delta f} = D_{\Delta g} = 0$. If the unmodeled dynamics satisfy sector conditions, $C_{\Delta f} = C_{\Delta g} = 0$.

A special case of **A4** is the following differential Riccati equation with constant A_0 , R_0 , and Q_0

$$\dot{P}'_t + P'_t A_0 + A_0^T P'_t + P'_t R_0 P'_t + Q_0 = 0 \quad (2.8)$$

Equation (2.8) has a positive solution P'_t , if the pair $(A_0, R_0^{1/2})$ is controllable and the pair $(Q_0^{1/2}, A_0)$ is observable. The detailed proof of it can be found in [22]. This is an extend of the matrix Riccati equation

$$A^T P + P A + P R P + Q = 0 \quad (2.9)$$

Equation (2.9) has a positive solution P , if the pair $(A, R^{1/2})$ is controllable, the pair $(Q^{1/2}, A)$ is observable. The controllability and observability conditions for A , R , and Q are equivalent to the following local frequency condition [22]

$$A^T R^{-1} A - Q \geq 0 \quad (2.10)$$

Lemma 2.1 in the next section shows that if the uncertainties are not very large, i.e., the condition (2.16) is satisfied, we can use condition (2.10) for (2.6). This means that it is not difficult to select a Hurwitz matrix A and Q such that (2.10) is established. So (2.10) and (2.16) are the conditions for **A4**.

The robust observer is selected as Luenberger's form [11],

$$\frac{d}{dt} \hat{x} = F(\hat{x}) + G(\hat{x}) u_t + K_t [y_t - C \hat{x}] \quad (2.11)$$

where $K_t \in \mathbb{R}^{n \times m}$ is the gain matrix. The observer uses only the available information: the nominal nonlinear mappings $F(\cdot)$, $G(\cdot)$, and C , and the online measurement y_t .

The observer error is defined as

$$\Delta_t = \hat{x} - x$$

The object of robust observer is to find a K_t such that following performance index

$$J(K_t) = \limsup_{T \rightarrow \infty} \frac{1}{T} \int_0^T \Delta_t^T Q \Delta_t dt \quad (2.12)$$

is minimized. $J(K_t)$ is the average of the observer error, $\|\Delta_t\|_Q^2 = \Delta_t^T Q \Delta_t$. Here the strictly positive constant matrix Q is a given to present different physical nature.

If K_t in the observer (2.11) is a constant matrix, (2.11) becomes the well-known high-gain observer [10]. It has the following property

$$\|\Delta_t\| \leq k(\theta) \exp\left(-\frac{\theta}{3}t\right) \|x_0 - \hat{x}_0\|, \quad (2.13)$$

where x_0 and \hat{x}_0 are initial states, θ is big enough positive constants, $k(\theta)$ is a positive function related to observer gain. It means that the observation error is asymptotically stable uniformly on initial conditions. This property requires the perturbations in **A1** be zero, and the unmodeled dynamics Δf and Δg in **A3** also be zero. In order to deal with these uncertainties, in this chapter, we use a time-variant gain K_t , which is designed in the next section.

2.3 Stability of the Robust Nonlinear Observer

The following lemma guarantees that the assumption **A4** is correct.

Lemma 2.1 *Consider the following two matrix differential Riccati equations: Riccati differential equation with time-varying parameter and algebraic Riccati equation*

$$\begin{aligned} \dot{P}_1(t) + A_1^T(t)P_1(t) + P_1(t)A_1(t) - P_1(t)R_1(t)P_1(t) + Q_1(t) &= 0 \\ A_2^T P_2 + P_2 A_2 - P_2 R_2 P_2 + Q_2 &= 0 \end{aligned} \quad (2.14)$$

with initial condition

$$P_1(0) > P_2 \quad (2.15)$$

the corresponding Hamiltonians are given by

$$H_{1,t} = \begin{bmatrix} Q_1(t) & A_1(t)^T \\ A_1(t) & -R_1(t) \end{bmatrix} \quad H_2 = \begin{bmatrix} Q_2 & A_2^T \\ A_2 & -R_2 \end{bmatrix}$$

If

$$H_2 \geq H_{1,t} \geq 0 \quad (2.16)$$

and the pair (A_2, R_2) are stable, then

$$P_1(t) \geq P_2 \geq 0, \forall t > 0 \quad (2.17)$$

Proof Let us define $e_t = P_1(t) - P_2$. Using condition (2.16) and the definition $H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$, the Riccati equation (2.14) in the Hamiltonian form is

$$\begin{aligned} -\dot{P}_1(t) &= \begin{bmatrix} I & P_1(t) \end{bmatrix} H_{1,t} \begin{bmatrix} I \\ P_1(t) \end{bmatrix} \\ 0 &= \begin{bmatrix} I & P_2 \end{bmatrix} H_2 \begin{bmatrix} I \\ P_2 \end{bmatrix} \end{aligned}$$

we derive:

$$\begin{aligned} -\dot{e}_t &= \begin{bmatrix} I & e_t + P_2 \end{bmatrix} H_{1,t} \begin{bmatrix} I \\ e_t + P_2 \end{bmatrix} \\ &\leq \begin{bmatrix} I & e_t + 0 & P_2(t) \end{bmatrix} H_2 \begin{bmatrix} I \\ e_t \\ P_2(t) \end{bmatrix} \\ &= (A_2^T + P_2 R_2) e_t + e_t (A_2 - R_2 P_2) - e_t R_2 e_t + Q_t - Q_t = L_t - Q_t \end{aligned}$$

where

$$L_t = (A_2^T + P_2 R_2) e_t + e_t (A_2 - R_2 P_2) - e_t R_2 e_t + Q_t$$

Based on Theorem 3 of [23], the term $(A_2^T - P_2 R_2)$ is stable when (A_2, R_2) is stable. From (2.15), $e_{t=0} > 0$. By Lemma 1 of [23]), there exists $Q_{t=0} > 0$ such that $L_{t=0} = 0$. This leads to

$$\dot{e}_t \geq Q_t > 0.$$

The solution of the Riccati differential equation is a continuous function, if its time-varying parameters are continuous. So we conclude that for time $t > 0$, there exists a $\varepsilon > 0$ such that

$$Q_\tau > 0 \quad \forall \tau \in [t, t + \varepsilon].$$

As a result,

$$e_{t+\varepsilon} = e_t + \int_t^{t+\varepsilon} \dot{e}_\tau d\tau \geq e_t + \int_t^{t+\varepsilon} Q_\tau d\tau \geq e_t + \inf_\tau [\lambda_{\min}(Q_t)] I \varepsilon > 0$$

that leads to

$$P_1(\tau) > P_2 \quad \forall \tau \in [0, 0 + \varepsilon]$$

Iterating this procedure for the next time interval $[\varepsilon, 2\varepsilon]$, we obtain the final result (2.17).

This lemma shows that if we let $A_0 = A_1(t)$, $Q_0 = Q_1(t)$, $R(t) = R_1(t)$, then (2.6) is (2.14). By Lemma 2.1, the solution of (2.6) $P(t)$ is not less than the solution of

$$A_2^T P + P A_2 + P R_2 P + Q_2 = 0 \quad (2.18)$$

providing that the initial condition of (2.6) is larger than that of (2.18). This means the existence condition for A4 is always satisfied.

Now, we define R_t and Q_0 in (2.6) as

$$\begin{aligned} R_t &= R_0 + \beta_t (C^+ \Lambda (C^+)^T)^{\frac{1}{2}} \times (I + \Pi^{-1}) (C^+ \Lambda (C^+)^T)^{\frac{1}{2}} \beta_t^T \\ Q_0 &= (2\Lambda_{fx} + K_1 \lambda_{\max}(\Lambda_{\partial f}) I) + (2\bar{u} \Lambda_{gx} + \bar{u} K_2 \lambda_{\max}(\Lambda_{\partial g}) I) + Q \end{aligned} \quad (2.19)$$

where $R_0 = \Lambda_f^{-1} + \Lambda_g^{-1} + \Lambda_{\xi_1}^{-1} + \Lambda_{\Delta f}^{-1} + \Lambda_{\Delta g}^{-1} + \Lambda_{\partial f}^{-1} + \Lambda_{\partial g}^{-1}$, $\Lambda = \Lambda_{\xi_2}^{-1}$, K_1 and K_2 are positive constants, \bar{u} is the upper bound of control u_t , the matrix $\beta_t \in \mathbb{R}^{n \times n}$ is defined as

$$\beta_t = \frac{\partial}{\partial x} F^T(\hat{x}) + \frac{\partial}{\partial x} G^T(\hat{x}) u_t - A_0 \quad (2.20)$$

C^+ is the pseudoinverse matrix of C in Moor–Penrose sense. A_0 is a Hurwitz matrix.

The following theorem formulates the main result of the chapter. It also provides the upper bound of the performance index (2.12).

Theorem 2.1 *Given a class of nonlinear system \mathcal{H} satisfies the assumptions A1–A4, for any gain matrix K_t , the following upper bound of the performance index (2.12) holds,*

$$J(K_t) \leq J^+(K_t) = \bar{C} + D + \gamma_1 + \gamma_2 + \phi(K_t) \quad (2.21)$$

where the constants Υ_1, Υ_2 are defined by $A\mathbf{I}, \bar{C} = C_{\Delta f} + C_{\Delta g}$,

$$\begin{aligned} D &= \limsup_{T \rightarrow \infty} \frac{1}{T} \int_0^T x^T (D_{\Delta f} + \bar{u} D_{\Delta g}) x dt \\ \phi(K_t) &= \limsup_{T \rightarrow \infty} \frac{1}{T} \int_0^T \Delta_t^T \left(X_t \widehat{\Sigma}_t^{\frac{1}{2}} + \widehat{\Sigma}_t^{-\frac{1}{2}} \right) \left(X_t \widehat{\Sigma}_t^{\frac{1}{2}} + \widehat{\Sigma}_t^{-\frac{1}{2}} \right)^T \Delta_t dt \end{aligned} \quad (2.22)$$

$$X_t = P_t (\beta_t - \widehat{K}_t C), K_t = \widehat{K}_t C C^+, \widehat{\Sigma}_t = C^+ \Lambda^{\frac{1}{2}} (I + \Pi) \Lambda^{\frac{1}{2}} (C^+)^T + \delta I, \delta > 0.$$

Proof To start the proof, we need to derive the error dynamic. Taking into account (2.1) and (2.11),

$$\begin{aligned} \dot{\Delta}_t &= \dot{\widehat{x}} - \dot{x} \\ &= F(\widehat{x}) + G(\widehat{x})u_t + K_t [y_t - C\widehat{x}] \\ &\quad - F(x) - G(x)u_t - \Delta f(x) - \Delta g(x)u_t - \xi_{1,t} \end{aligned} \quad (2.23)$$

Let us denote

$$\begin{aligned} F_t &= F(x, \Delta_t, u_t \mid K_t) \\ &= F(x + \Delta_t) - F(\widehat{x}) + G(x + \Delta_t)u_t - G(x)u_t - K_t C \Delta_t, \\ \Delta H_t &= \Delta H(\xi_{1,t}, \xi_{2,t}, \Delta f \mid K_t) = K_t \xi_{2,t} - \Delta f(\cdot) - \Delta g(\cdot)u_t - \xi_{1,t}. \end{aligned} \quad (2.24)$$

The vector function F_t describes the dynamic of a nominal model and the function ΔH_t corresponds to unmodeled dynamics and external disturbances. So

$$\dot{\Delta}_t = F_t + \Delta H_t$$

Define a Lyapunov like function as

$$V_t = \Delta_t^T P_t \Delta_t, \quad P_t^T = P_t > 0 \quad (2.25)$$

Along with the trajectories of the differential equation (2.23), we derive

$$\frac{dV_t}{dt} = \frac{\partial V_t}{\partial P_t} + \left(\frac{\partial V_t}{\partial \Delta_t}, \dot{\Delta}_t \right) = \Delta_t^T \dot{P}_t \Delta_t + 2\Delta_t^T P_t [F_t + \Delta H_t] \quad (2.26)$$

Using (2.4),

$$\begin{aligned} F(x + \Delta_t) - F(x) &= \frac{\partial}{\partial x} F^T(x) \Delta_t + v_f \\ G(x + \Delta_t) - G(x) &= \frac{\partial}{\partial x} G^T(x) \Delta_t + v_g \end{aligned} \quad (2.27)$$

According to (2.7),

$$\|v_f\|_{\Lambda_f} \leq 2 \|\Delta\|_{\Lambda_{fx}}, \quad \|v_g\|_{\Lambda_g} \leq 2 \|\Delta\|_{\Lambda_{gx}} \quad (2.28)$$

Substitute (2.27) into (2.24),

$$\begin{aligned} F_t &= \frac{\partial F^T(x)}{\partial x} \Delta_t + v_f + \frac{\partial G^T(x)}{\partial x} u_t \Delta_t + u_t v_g - K_t C \Delta_t \\ &= \left[\frac{\partial F^T(x)}{\partial x} + \frac{\partial G^T(x)}{\partial x} u_t - K_t C \right] \Delta_t + v_f + u_t v_g \end{aligned} \quad (2.29)$$

Because $\Delta_t^T P_t v_f$ is scalar, applying (2.28) and matrix inequality [16]

$$X^T Y + Y^T X \leq X^T \Lambda X + Y^T \Lambda^{-1} Y, \quad (2.30)$$

where $X \in \mathbb{R}^{n \times k}$, $Y \in \mathbb{R}^{n \times k}$, $\Lambda = \Lambda^T > 0$, $\Lambda \in \mathbb{R}^{n \times n}$ are any matrices. We obtain the inequalities:

$$\begin{aligned} 2\Delta_t^T P_t v_f &\leq \Delta_t^T \left(P_t \Lambda_f^{-1} P_t + 2\Lambda_{fx} \right) \Delta_t, \\ 2\Delta_t^T P_t v_g u_t &\leq \Delta_t^T \left(P_t \Lambda_g^{-1} P_t + 2\bar{u} \Lambda_{gx} \right) \Delta_t, \end{aligned} \quad (2.31)$$

Using the assumptions **A1**, we have

$$\begin{aligned} -2\Delta_t^T P_t \xi_{1,t} &\leq \Delta_t^T P_t \Lambda_{\xi_1}^{-1} P_t \Delta_t + \xi_{1,t}^T \Lambda_{\xi_1} \xi_{1,t}, \\ 2\Delta_t^T P_t K_t \xi_{2,t} &\leq \Delta_t^T P_t K_t \Lambda_{\xi_2}^{-1} K_t^T P_t \Delta_t + \xi_{2,t}^T \Lambda_{\xi_2} \xi_{2,t}, \end{aligned} \quad (2.32)$$

Using the assumptions **A3**

$$\begin{aligned} -2\Delta_t^T P_t \Delta f &\leq \Delta_t^T P_t \Lambda_{\Delta f}^{-1} P_t \Delta_t + C_{\Delta f} + x^T D_{\Delta f} x \\ -2\Delta_t^T P_t \Delta g u_t &\leq \Delta_t^T P_t \Lambda_{\Delta g}^{-1} P_t \Delta_t + \bar{u} C_{\Delta g} + \bar{u} x^T D_{\Delta g} x \end{aligned} \quad (2.33)$$

Let us denote

$$A_t = \frac{\partial F^T(x)}{\partial x} + \frac{\partial G^T(x)}{\partial x} u_t - K_t C. \quad (2.34)$$

Using the identity $2\Delta_t^T A_t \Delta_t = \Delta_t^T A_t \Delta_t + \Delta_t^T A_t^T \Delta_t$ and adding and subtracting the term $\Delta_t^T Q \Delta_t$ ($Q = Q^T > 0$) in the right hand side of (2.26), we obtain

$$\frac{dV_t}{dt} \leq \Delta_t^T L_t \Delta_t + (C_{\Delta f} + \bar{u} C_{\Delta g}) + \gamma_t + x^T (D_{\Delta f} + \bar{u} D_{\Delta g}) x - \Delta_t^T Q \Delta_t \quad (2.35)$$

where $\gamma_t = \xi_{1,t}^T \Lambda_{\xi_1} \xi_{1,t} + \xi_{2,t}^T \Lambda_{\xi_2} \xi_{2,t}$,

$$\begin{aligned} L_t &= P_t \left(\Lambda_f^{-1} + \Lambda_g^{-1} + \Lambda_{\xi_1}^{-1} + \Lambda_{\Delta g}^{-1} \right) P_t \\ &\quad + P_t + P_t A_t + A_t^T P_t + P_t K_t \Lambda_{\xi_2}^{-1} K_t^T P_t + 2(\Lambda_{fx} + \bar{u} \Lambda_{gx}) + Q, \end{aligned} \quad (2.36)$$

Choosing any Hurwitz matrix A_0 , we can rewrite (2.34) as follows:

$$\begin{aligned} A_t &= A_0 + \left(\frac{\partial}{\partial x} F^T(\hat{x}) + \frac{\partial}{\partial x} G^T(\hat{x}) u_t \right. \\ &\quad \left. - K_t C - A_0 \right) + \left(\frac{\partial}{\partial x} F^T(x) - \frac{\partial}{\partial x} F^T(\hat{x}) \right) + \left(\frac{\partial}{\partial x} G^T(x) - \frac{\partial}{\partial x} G^T(\hat{x}) \right) u_t \end{aligned} \quad (2.37)$$

Denote

$$\begin{aligned} \hat{A}_t &= \frac{\partial}{\partial x} F^T(\hat{x}) + \frac{\partial}{\partial x} G^T(\hat{x}) u_t - K_t C - A_0 \\ \partial f_t &= \frac{\partial}{\partial x} F^T(x) - \frac{\partial}{\partial x} F^T(\hat{x}) \\ \partial g_t &= \frac{\partial}{\partial x} G^T(x) u_t - \frac{\partial}{\partial x} G^T(\hat{x}) u_t. \end{aligned}$$

From (2.27), we know

$$\partial f_t = (F(x + \Delta_t) - F(x)) - (F(x + 2\Delta_t) - F(x + \Delta_t))$$

Similar to (2.32), we deduce

$$\begin{aligned} 2\Delta_t^T P_t \partial f_t &\leq \Delta_t^T \left(P_t \Lambda_{\partial f}^{-1} P_t + \partial f_t \Lambda_{\partial f} \partial f_t^T \right) \Delta_t \leq \Delta_t^T \left(P_t \Lambda_{\partial f}^{-1} P_t + K_1 \lambda_{\max}(\Lambda_{\partial f}) I \right) \Delta_t, \\ 2\Delta_t^T P_t \partial g_t &\leq \Delta_t^T \left(P_t \Lambda_{\partial g}^{-1} P_t + \bar{u} K_2 \lambda_{\max}(\Lambda_{\partial g}) I \right) \Delta_t, \end{aligned}$$

where K_1 and K_2 are positive constants. Substituting these inequalities into (2.36) and using (2.37), we obtain

$$L_t \leq \left(\dot{P}_t + P_t A_0 + A_0^T P_t + P_t R_0 P_t + Q_0 \right) + (P_t K_t \Lambda K_t^T P_t + P_t \hat{A}_t + \hat{A}_t^T P_t) \quad (2.38)$$

where the matrices R_0 , Q_0 , and Λ are defined in (2.19). If we select K_t have a special structure $K_t = \hat{K}_t C C^+$, using the pseudoinverse property $C C^+ C = C$, we can present

$$\begin{aligned} &P_t K_t \Lambda K_t^T P_t + P_t \hat{A}_t + \hat{A}_t^T P_t \\ &= P_t (\beta_t - \hat{K}_t C) + (\beta_t - \hat{K}_t C)^T P_t + P_t \hat{K}_t C G C^T \hat{K}_t P_t \end{aligned} \quad (2.39)$$

where

$$G = C^+ \Lambda (C^+)^T. \quad (2.40)$$

Using the definition of (2.22), the last term in (2.39) can be estimated as follows:

$$\begin{aligned}
 \Delta_t^T P_t \widehat{K}_t C G C^T \widehat{K}_t^T P_t \Delta_t &= \left\| G_t^{\frac{1}{2}} (\widehat{K}_t C)^T P_t \Delta_t \right\|^2 \\
 &= \left\| G_t^{\frac{1}{2}} x \Delta_t - G_t^{\frac{1}{2}} \beta_t P_t \Delta_t \right\|^2 \\
 &\leq \Delta_t^T x G_t^{\frac{1}{2}} (I + \Pi) G_t^{\frac{1}{2}} x^T \Delta_t + \Delta_t^T P_t \beta_t G_t^{\frac{1}{2}} (I + \Pi^{-1}) G_t^{\frac{1}{2}} \beta_t^T P_t \Delta_t
 \end{aligned}$$

where Π is any positive matrix. Define

$$\Phi_t = \beta_t G_t^{\frac{1}{2}} (I + \Pi^{-1}) G_t^{\frac{1}{2}} \beta_t^T,$$

The first term of (2.39) can be rewritten in the following equivalent form:

$$P_t K_t \Lambda K_t^T P_t \leq x \Omega_t x^T + P_t \Phi_t P_t$$

Since $\Omega_t \in \mathbb{R}^{n \times n}$ and $\text{rank}(\Omega_t) = \min\{n, m\}$, when $m < n$ the inverse matrix of Ω_t does not exist. We introduce the matrix

$$\widehat{\Omega}_t = \Omega_t + \delta I, \quad \delta > 0$$

Equation (2.39) can be written as

$$\begin{aligned}
 P_t K_t \Lambda K_t^T P_t + P_t \widehat{A}_t + \widehat{A}_t^T P_t &\leq X_t + X_t^T + X_t \Omega_t X_t^T + P_t \Phi_t P_t \\
 &\leq \left(x \widehat{\Omega}_t^{\frac{1}{2}} + \widehat{\Omega}_t^{-\frac{1}{2}} \right) \left(x \widehat{\Omega}_t^{\frac{1}{2}} + \widehat{\Omega}_t^{-\frac{1}{2}} \right)^T - \widehat{\Omega}_t^{-1} - \delta X_t X_t^T + P_t \Phi_t P_t \\
 &\leq \left(x \widehat{\Omega}_t^{\frac{1}{2}} + \widehat{\Omega}_t^{-\frac{1}{2}} \right) \left(x \widehat{\Omega}_t^{\frac{1}{2}} + \widehat{\Omega}_t^{-\frac{1}{2}} \right)^T + P_t \Phi_t P_t
 \end{aligned}$$

If we define R_t as in (2.19), using the definition (2.22), we transform (2.35) as follows:

$$\frac{dV_t}{dt} \leq \Delta_t^T L'_t \Delta_t + \overline{C} + \gamma_t + D_t - \Delta_t^T Q \Delta_t + \Delta_t^T \widehat{\gamma}_t \Delta_t \quad (2.41)$$

where

$$\begin{aligned}
 L'_t &= \dot{P}_t + P_t A_0 + A_0^T P_t + P_t R_t P_t + Q_0 \\
 \widehat{\gamma}_t &= \left(x \widehat{\Omega}_t^{\frac{1}{2}} + \widehat{\Omega}_t^{-\frac{1}{2}} \right) \left(x \widehat{\Omega}_t^{\frac{1}{2}} + \widehat{\Omega}_t^{-\frac{1}{2}} \right)^T
 \end{aligned}$$

According to the assumption **A4**, we conclude that $\widehat{L}_t = 0$. Integrating (2.41) within the interval $t \in [0, T]$ and dividing both sides on T , we obtain:

$$\begin{aligned}
\frac{1}{T} \int_0^T \Delta_t^T Q \Delta_t dt &\leq \bar{C} + \frac{1}{T} \int_0^T \Upsilon_t dt + \frac{1}{T} \int_0^T D_t dt + \frac{1}{T} \int_0^T \Delta_t^T \widehat{\Upsilon}_t \Delta_t - \frac{1}{T} (V_T - V_0) \\
&\leq \bar{C} + \frac{1}{T} \int_0^T \Upsilon_t dt + \frac{1}{T} \int_0^T D_t dt + \frac{1}{T} \int_0^T \Delta_t^T \widehat{\Upsilon}_t \Delta_t - \frac{1}{T} V_0.
\end{aligned}$$

Taking the limit of $T \rightarrow \infty$, we finally obtain (2.21), and

$$\phi(K_t) = \limsup_{T \rightarrow \infty} \frac{1}{T} \int_0^T \Delta_t^T \widehat{\Upsilon}_t \Delta_t dt$$

The following theorem provides the design process of the robust observer.

Theorem 2.2 *If \widehat{K}_t satisfies*

$$K_t = \left\{ P_t^{-1} \left[C^+ \Lambda^{\frac{1}{2}} (I + \Pi) \Lambda^{\frac{1}{2}} (C^+)^T + \delta I \right]^{-1} + \beta_t \right\} C^+ \quad (2.42)$$

the upper bound (2.12) is minimum

$$\min_{K_t} J = \bar{C} + D + \Upsilon_1 + \Upsilon_2$$

Proof Since \bar{C} , D , Υ_1 , Υ_2 , and $\phi(\widehat{K}_t)$ are positive, to minimize (2.21), we must choose

$$\phi(K_t) = 0, \quad X_t = -\widehat{\Omega}_t^{-1} \quad (2.43)$$

that leads to

$$P_t(\beta_t - \widehat{K}_t C) = -(\Omega_t + \delta I)^{-1}$$

In equivalent form,

$$\widehat{K}_t C = \beta_t + P_t^{-1} (\Omega_t + \delta I)^{-1} \quad (2.44)$$

Using $K_t = \widehat{K}_t C C^+$, (2.42) is established.

If there are no any unmodeled dynamics ($\bar{C} = D = 0$) and no any external disturbances ($\Upsilon_1 = \Upsilon_2 = 0$), the robust observer (2.11) with the optimal matrix gain given by (2.42) guarantees “the stability in average”,

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \int_0^T \Delta_t^T Q \Delta_t dt = 0.$$

It is equivalent to the fact that

$$\lim_{t \rightarrow \infty} \Delta_t = 0$$

The robust nonlinear observer (2.11) has Luenberger form. It is very simple. The time-varying gain K_t is calculated by (2.42). Here C , Π , and Λ are constant matrices, β_t is calculated by the normal parts (2.20), P_t is obtained from the Riccati differential equation (RDE) (2.6). This RDE has a time-varying matrix R_t . It is not easy to discuss the existence conditions for $P(t)$. Lemma 2.1 shows how to use a time-invariant algebraic Riccati equation to decide the solution of a RDE with time-varying parameters.

Now, we compare our observer (2.11) with the others. If the nonlinear system (2.1) is known completely, and the perturbations $\xi_{1,t}$ and $\xi_{2,t}$ are zero, the model-based nonlinear observer in Luenberger form is

$$\frac{d}{dt}\hat{x} = f(x) + g(x)u_t + K_t(y_t - C\hat{x})$$

If $f(x)$ and $g(x)$ are unknown or partly known as $F(\hat{x})$ and $G(\hat{x})$, the model-based methods [2, 6, 7, 10, 12, 24–26] cannot be used.

The high-gain observer require the nonlinear system (2.1) can be transformed to

$$\begin{aligned}\dot{x}_i &= x_{i-1}, \quad i = 1 \cdots n-1 \\ \dot{x}_n &= F(x) + G(x)u \\ y &= x_1\end{aligned}\tag{2.45}$$

by a local diffeomorphism, here $F(x)$ and $G(x)$ are unknown. The high-gain observer is [13, 17]

$$\begin{aligned}\frac{d}{dt}\hat{x}_i &= \hat{x}_{i-1} + \frac{\alpha_i}{\varepsilon^i}(y - \hat{y}) \quad i = 1 \cdots n-1 \\ \frac{d}{dt}\hat{x}_n &= \frac{\alpha_n}{\varepsilon^n}(y - \hat{y}) \\ \hat{y} &= \hat{x}_1\end{aligned}\tag{2.46}$$

where $\alpha_i > 0$ are constant design parameters, ε is a small positive constant. Obviously, the observed states \hat{x}_i are derivative of the output in different degrees. From (2.1), we can see that the high gain $\frac{\alpha_n}{\varepsilon^n}$ enlarges the measurement noise $\xi_{2,t}$ directly. However, our Luenberger observer (2.11) uses time-varying gain K_t which only use the upper bound $\Lambda = \Lambda_{\xi_2}^{-1}$ of the measurement noise in (2.19). The measurement noise $\xi_{2,t}$ is filtered in the Luenberger observer (2.11).

If the nonlinear system (2.1) cannot be transformed into (2.45), or we do not want to estimate the derivative of the output, the sliding mode observer can be applied [8, 18]

$$\frac{d}{dt}\hat{x} = F(\hat{x}) + G(\hat{x})u_t - KC^T \text{sign}(y_t - C\hat{x})\tag{2.47}$$

where $K > 0$, is a big gain matrix. The measurement noises are included inside the sign function, it causes chattering.

A smooth version model-free nonlinear observer is the neural observer [16]

$$\frac{d}{dt}\hat{x} = F(\hat{x}) + G(\hat{x})u_t + W_{1,t}\sigma(V_{1,t}\hat{x}) + W_{2,t}\phi(V_{2,t}\hat{x})u_t + K(y_t - C\hat{x}) \quad (2.48)$$

where $W_{1,t}\sigma(V_{1,t}\hat{x})$ and $W_{2,t}\phi(V_{2,t}\hat{x})$ are neural networks to approximate the unknown functions $f(x)$ and $g(x)$. The output error $(y_t - C\hat{x})$ has to be used for the observer (2.48) and the weights training as

$$\dot{W}_{1,t} = K_1 C^+ (y_t - C\hat{x}) \sigma^T$$

So the observer accuracy is low, compared with the other model-free observers.

2.4 Simulations

We consider a single-link robot rotating in a horizon plane. It is described as

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} x_2 \\ -\sin(x_1) + u \end{bmatrix} - 0.05 \begin{bmatrix} x_1 \cos(x_1) \\ x_2 \sin(x_2) \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\ y_t &= x_1 + w_3 \end{aligned} \quad (2.49)$$

where w_1, w_2 are external state perturbations modeling square wave and saw-tooth functions; w_3 is an output perturbation with the “white noise” nature. We use a PD control to force the system following the reference signal $x_{1,d} = \sin t, x_{2,d} = \cos t, u = -5(x_1 - x_{1,d}) - 10(x_2 - x_{2,d})$. The unmodeled dynamics are given by

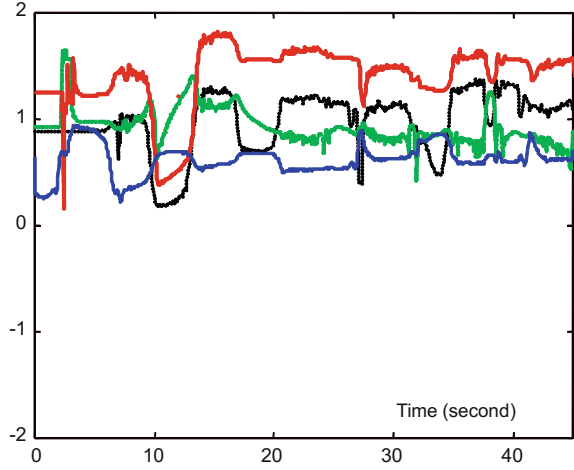
$$\Delta f(\cdot)^T = -0.05 [x_1 \cos(x_1); x_2 \sin(x_2)]$$

This example is similar to [21], but we consider a general case, i.e., there are bounded unmodeled dynamics and external disturbances. Obviously the assumptions **A1–A3** are satisfied. (2.49) is already the observer form (2.1). We construct the following robust observer

$$\begin{aligned} \begin{bmatrix} \frac{d}{dt}\hat{x}_1 \\ \frac{d}{dt}\hat{x}_2 \end{bmatrix} &= \begin{bmatrix} \hat{x}_2 \\ -\sin(\hat{x}_1) + u \end{bmatrix} + K_t (x_1 - \hat{x}_1) \\ K_t &= \left\{ P_t^{-1} \left[C^+ \Lambda^{\frac{1}{2}} (I + \Pi) \Lambda^{\frac{1}{2}} (C^+)^T + \delta I \right]^{-1} + \beta_t \right\} C^+ \end{aligned} \quad (2.50)$$

where the gain matrix K_t is computed according to (2.42), $C = [1, 0]$, $\beta_t = \begin{bmatrix} 0 & 1 \\ -\cos(\hat{x}_1) & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} u_t - A_0$. In order to satisfy **A4**, we select $A_0 = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$,

Fig. 2.1 Time evolution of P_t



$Q = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$, $\Lambda = \Pi = I$. So $Q_0 = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.2 \end{bmatrix}$, $R_0 = \begin{bmatrix} 2.5 & 0 \\ 0 & 2.5 \end{bmatrix}$. To obtain the solution of the differential Riccati Equation (2.6), we start from the initial conditions $P_0 = \begin{bmatrix} 0.3 & -0.01 \\ -0.01 & 0.3 \end{bmatrix}$ which is the solution of the corresponding algebraic

Riccati equation ($\dot{P}_t = 0$). The time evolution for the elements P_t is shown in Fig. 2.1.

To illustrate the effectiveness of our observer, we compare the observer (RO) (2.50) with the other four nonlinear observers: (1) mode-based nonlinear observer in Luenberger form (LO) [12]; (2) model-free high-gain observer (HO) [17]; (3) model-free sliding mode observer (SO) [8]; (4) model-free neural observer (NO) [16]. They are

$$\begin{aligned}
 \text{LO: } \begin{bmatrix} \frac{d}{dt} \hat{x}_1 \\ \frac{d}{dt} \hat{x}_2 \end{bmatrix} &= \begin{bmatrix} \hat{x}_2 \\ -\sin(\hat{x}_1) + u \end{bmatrix} + 20(x_1 - \hat{x}_1) \\
 \text{HO: } \begin{bmatrix} \frac{d}{dt} \hat{x}_1 \\ \frac{d}{dt} \hat{x}_2 \end{bmatrix} &= \begin{bmatrix} \hat{x}_2 \\ \frac{-1}{0.001}(x_1 - \hat{x}_1) \end{bmatrix} \\
 \text{SO: } \begin{bmatrix} \frac{d}{dt} \hat{x}_1 \\ \frac{d}{dt} \hat{x}_2 \end{bmatrix} &= \begin{bmatrix} \hat{x}_2 \\ -\sin(\hat{x}_1) + u \end{bmatrix} + \begin{bmatrix} 0 \\ -100\text{sign}(x_1 - \hat{x}_1) \end{bmatrix} \\
 \text{NO: } \begin{bmatrix} \frac{d}{dt} \hat{x}_1 \\ \frac{d}{dt} \hat{x}_2 \end{bmatrix} &= \begin{bmatrix} \hat{x}_2 \\ -\sin(\hat{x}_1) + u \end{bmatrix} + W_{1,t}\sigma(\hat{x}) + W_{2,t}\phi(\hat{x})u_t + 15(x_1 - \hat{x}_1) \\
 \dot{W}_{1,t} &= 10C^+(x_1 - \hat{x}_1)\sigma^T
 \end{aligned}$$

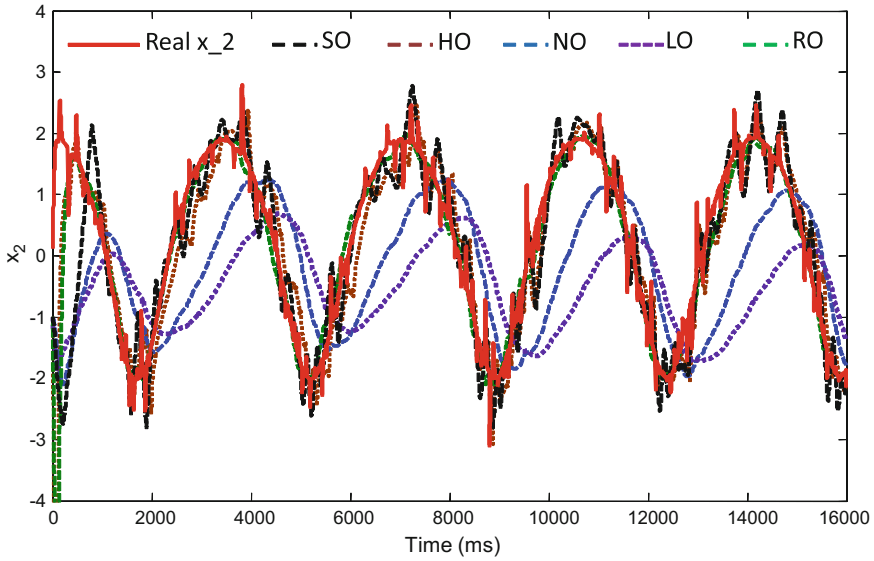
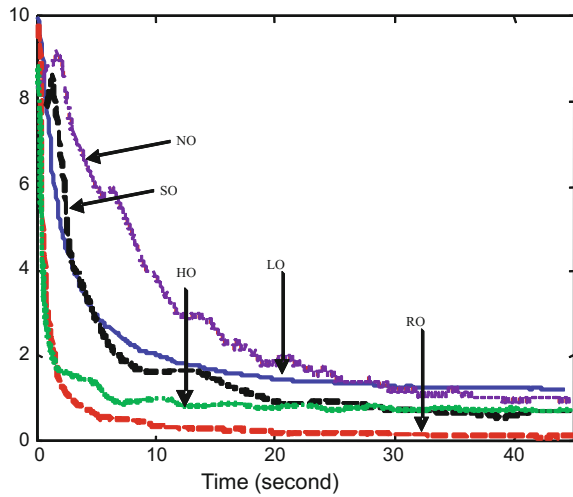


Fig. 2.2 The comparison results for x_2

Fig. 2.3 Average of observation errors



The initial conditions are $x_1(0) = 2$, $x_2(0) = 1$, $\hat{x}_1(0) = 1$, $\hat{x}_2(0) = 1$. The comparison results are shown in Fig. 2.2. Defining performance indexes $J_T = \frac{1}{T} \int_0^T (x_2 - \hat{x}_2) dt$, the average observation errors are shown in Fig. 2.3.

From the simulations of these five nonlinear observers, we obtain the following conclusions:

(1) In Fig. 2.2, our robust observers based on the Riccati differential equation (RO) has similar instant observation error as the high-gain observer (HO) [17] and the sliding mode observer (SO) [8]. These three observers are better than the mode-based nonlinear Luenberger observer (LO) [12] and the neural observer (NO) [16], although LO and NO are more smooth. The reason is LO and NO do not consider uncertainties.

(2) In Fig. 2.3, for the average of observation error after a long time, RO is better than HO and SO, because HO and SO are sensitive to the measurement noises w_1 and w_2 .

(3) The structure of our observer RO is more simple than NO, and more complex than HO and SO. However, our observer is much better than NO, HO, and SO.

2.5 Conclusion

In this chapter, we propose a novel robust observer for nonlinear uncertainty systems with unmodeled dynamics and external perturbations. This observer has a simple structure, being a Luenberger-like observer. Using a Riccati differential equation and a time-varying observer gain, the stability of the observer is proven. The observer proposed in this chapter can be considered as an alternative approach to the nonlinear robust feedback control. Future work is to extend the results to multi-input and multi-output unknown nonlinear systems.

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