Preface

Optimization is one of key ways of approaching and solving a huge variety of problems in virtually all fields of science and technology, and—even, more generally—human activities. The very essence of optimization is the selection of a best option (alternative, variant …), from the point of view of some criterion or multiple criteria, from some set of feasible options (alternatives, variants, …).

In the areas of our interest, that is, in broadly perceived decision making, control, systems analysis, operations research, etc., optimization is equated with mathematical programming, that is, it is formulated as the problem of finding an optimal (best) value (or values) of variables for which an objective (performance) function, usually assumed to be a real valued function, attains its minimum or maximum value subject to the satisfaction of some constraints which define the set of feasible values of the variables and are given as some equalities or inequalities. Of course, we can also have multiple objective (performance) functions which imply a necessity to use some specialized tools and techniques. For simplicity, we will use the term *optimization* for all kinds of such optimal choice problems, explaining more specifically what we mean by this while dealing with the particular problems in the consecutive chapters.

It is easy to see that due to universal importance of such a class of problems, optimization has been crucial, and has become a subject of intensive research, in both theoretical and computational directions. Moreover, it has become an important part of basic curricula for undergraduate and graduate students, in many areas of science and technology, at all universities all over the world.

The purpose of this book is to provide a comprehensive presentation of the very essence, characteristic features and development of optimization methods—from the case of optimization (i.e. maximization or minimization) of a scalar function of one variable through the case of a scalar function of several variables to the case of a scalar function of infinitely many variables, that is, the case of a functional (optimal control). Finally, we end up with the case of optimization with a vector valued objective (performance) function of several variables; this will be called in this book *polyoptimization*. Our analysis will be complemented with relevant properties, some algorithmic consideration, and then illustrations on many

examples. These examples will be somehow unusual and maybe unexpected yet interesting and challenging. We hope that they will help to better understand the essence and functioning of the methods presented.

The book is strongly based on a selection of my lectures that have been read for many years for Ph.D. students at the Faculty of Electrical Engineering, Automation, Computer Science and Biomedical Engineering, AGH University of Science and Technology in Kraków, Poland.

The book contains a total of 16 chapters, including 15 core chapters and an interesting example problem, the stabilization of a bike.

The book starts with Chap. 1 in which a classification of mathematical models of optimization is given, both from a general point of view and our needs.

Chapter 2 discusses basic concepts of formal logic with a focus on the essence of the necessary and sufficient conditions for the existence of an optimal solution.

Chapter 3 presents basic mathematical models illustrated with examples from electrical engineering and mechanics in the form of a long line and the inverted pendulum, and then from economics, mining and metallurgy, and space travels to the moon. It also gives an overview of methods for solving differential equations based on the Laplace transform and matrix theory.

Chapter 4 describes basic limitations in the analysis of dynamic systems such as how to ensure a robust stability, physical realizability and limited quality.

Chapter 5 is basically the beginning of our tour through optimization. First it discusses some basic issues like the finding of extrema (extreme values) of functions of one and several variables without constraints; throughout the book we will often use the term "extremum" for "minimum" or "maximum" when it will not be necessary to specify whether the maximum or minimum is concerned. The methods presented are illustrated with interesting examples in the L^1 and L^2 spaces.

Chapter 6 shows three different methods of searching for extremes of a function with equality constraints. The methods presented are illustrated with examples, notably on an interesting example of the synthesis of optimal hierarchical systems.

Chapter 7 contains a generalization of the methods of searching for extremes of functions of several variables in the presence of both equality and inequality constraints.

Chapter 8 is devoted to the parametric optimization of dynamic systems. It presents parametric optimization problems of finite and infinite dimensions, both with the continuous and discrete time. Some interesting and relevant examples from the field of mathematical models described by differential equations and functions are shown.

In Chap. 9 we show the transition from the case of searching for extremes of functions of several (finitely many) variables to the case of the searching for extremes of functions of infinitely many variables, that is, we proceed to the calculus of variations. We present some basic properties and theorems of the calculus of variations.

Chapter 10 is an introduction to the modern problems of dynamic optimization.

Chapter 11 shows modern methods of dynamic optimization, that is, the maximum principle in three variants, and control (over time) of various stationary linear systems, with some examples of applications and the synthesis of controllers for the linear and nonlinear systems, and also a discrete variant of the maximum principle.

In Chap. 12 we consider a very general method of dynamic programming, both in its discrete and continuous forms. There are given examples of applications to the formulation and solution of some combinatorial problems, and then the relationship of both methods, that is, the maximum principle and dynamic programming.

In Chap. 13 we show an application of dynamic programming to the optimization of linear non-stationary systems. At the end of this chapter we present an important approach to the synthesis of controllers via the Kalman equations.

Chapter 14 is dedicated to the optimization of discrete-continuous systems using the Kalman equations.

In the end of the book, in Chap. 15 we discuss problems of searching for extremes of vector functions of many variables, that is the polyoptimization problems. We also present a robust polyoptimization method, the so-called *skeleton method*.

In Chap. 16 we present a mathematical model of the bike and its stability analysis based on Keldysh's theory.

Though the issues considered in the book may seem basic, they constitute a foundation for the fields of science and technology considered, notably control theory, systems research, automatic control and automation, operations research, to name a few, in which the problem boils down to the selection of some best option or course of action. The familiarity of the problems considered, their analyses and solutions should be useful for virtually all graduate student, Ph.D. students, researchers and scholars in these areas, as well as many practitioners.

A unique characteristic feature of this book is that we mostly refer to older works in which the basic concepts, and tools and techniques have been first mentioned or proposed. This is, in our opinion, very illustrative and informative, providing information that is rarely included in other similar books and volumes in the area in which such classic problems are either considered without mentioning their roots and pioneering works or refer to newer literature sources which, again, usually do not mention that source and pioneering literature. A natural consequence is that we do not cite much of more recent information but this should by no means constitute a problem for any interested reader because, if needed, he or she can very easily find an additional modern literature online, from vast Internet resources, as it is practically presently done. Of course, when a person prefers more traditional sources of information, the hard copy literature, there should be no problem to find this either.

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