Preface

This book investigates the stability and performance of control systems subject to actuator saturation. Actuator saturation is frequently encountered in practical control systems as practical actuators can only deliver signals of limited magnitudes and rates due to their physical constraints.

Actuator saturation degrades the performance of the control system and, in a severe case, even causes the loss of stability. A straightforward strategy to avoid such performance degradation is to prevent actuator saturation from occurring by operating the control system in the linear region of its actuator. The saturation avoidance approach ensures the stability and performance of the control system but leads to underutilization or oversizing of the actuator. Achieving the maximal closed-loop system performance with a given actuator or using the smallest actuator to achieve the specified closed-loop performance entails the actuator to operate in the saturation mode.

Over the past decades, stability and performance analysis of control systems in the presence of actuator saturation has attracted extensive attention in the research community. A large number of results can be found in the literature, and research on control systems with actuator saturation remains active. Among all the results available in the literature, two threads of thought are conspicuous. They are:

- Treatments of the saturation function
- Choices of the Lyapunov function

Indeed, it is the improved treatments of the saturation function and better choices of the Lyapunov function that have led to most of the new and stronger results on the stability and performance of control systems with actuator saturation.

One of the widely adopted approaches to treating the saturation function is the use of the global and regional sector conditions. Such an approach places the saturation function into a linear sector and leads to stability and performance characterizations in the form of matrix inequalities. The other popular approach to dealing with the saturation function that is less conservative than the sector condition approach is the convex hull representation of the saturation function. In this approach, a saturated linear feedback is placed inside a convex hull of a group of auxiliary linear functions. Both these treatments have been applied to formulate various stability and performance analysis/design problems into optimization problems with LMI or BMI constraints, which in turn can be solved numerically.

Quadratic Lyapunov functions, because of their simplicity, are the most commonly used Lyapunov functions in the analysis and design of control systems with actuator saturation. In an effort to reduce the conservatism associated with a quadratic Lyapunov function, more general forms of Lyapunov functions have also been well investigated. For example, a composite quadratic Lyapunov function is composed from a group of quadratic functions, and a saturation-dependent Lyapunov function takes into account the severity of the actuator saturation. Moreover, an integral of the saturation/deadzone function is added to a quadratic Lyapunov function to form a Lure-Postnikov-type Lyapunov function, which has been further generalized to form a piecewise quadratic Lyapunov function. All these Lyapunov functions are generalized from quadratic Lyapunov functions and lead to improved stability and performance results in the analysis and design of control systems with actuator saturation.

The objective of this book is to present our recent results on the analysis of and design for the stability and performance of control systems with actuator saturation. These new results are obtained by improving the treatment of the saturation function and by constructing new Lyapunov functions. In particular, two improved treatments of the saturation function are presented by exploiting the intricate structural properties of the traditional convex hull representation of the saturation function. We apply both of these two new treatments of the saturation function to the estimation of the domain of attraction and the finite-gain $\mathcal{L}_2$ performance by using both the quadratic Lyapunov function and the composite quadratic Lyapunov function. The estimates are obtained by solving optimization problems with matrix inequality constraints. Additionally, an algebraic computation method is presented for the exact determination of the maximal contractively invariant ellipsoid, a level set of a quadratic Lyapunov function. On the other hand, we construct a generalized piecewise quadratic Lyapunov function by embedding the information of the regional sector condition into the piecewise quadratic Lyapunov function. This generalized piecewise quadratic Lyapunov function can be used to more effectively carry out the stability and performance analysis of control systems with actuator saturation and an algebraic loop. By viewing the feedback gain as an additional free parameter in the optimization problems in the stability and performance analysis, such optimization problems can be readily adapted for control design.

Our presentation of this book is organized as follows. Chapter 1 includes preliminaries. We first introduce the concept of the null controllable region for linear systems with bounded controls. For a linear system subject to actuator saturation, results on the estimation of its domain of attraction, which is a subset of the null controllable region, are briefly summarized. This chapter also includes a brief introduction to a few other analysis and design problems for, for example, finite-gain $\mathcal{L}_2$ stability and anti-windup compensation.

Chapter 2 recalls the conventional representations of a saturated linear feedback and presents the improved convex hull representation by introducing multiple auxiliary matrices. This improvement enables the convex hull that represents the saturated linear feedback to have a flexible geometric "shape." In this chapter, we consider three different types of saturated feedbacks, single-layer saturated linear feedbacks, nestedly saturated linear feedbacks, and linear feedbacks subject to a piecewise linear function with multiple bends. The problems of estimating the domain of attraction for linear systems under these saturated linear feedbacks are then formulated as constrained optimization problems that maximize the size of an ellipsoid.

The largest contractively invariant ellipsoids obtained by solving the optimization problems formulated in Chapter 2 are not always the maximal contractively invariant ellipsoids. The situation becomes more complex when multiple inputs of the system are subject to saturation. Chapter 3 proposes an algebraic computation method for the exact determination of the maximal contractively invariant ellipsoid. This method does not incur any conservatism for a given positive definite matrix that characterizes the shape of the ellipsoid. Additionally, we present an LMI-based criterion to determine if the optimal ellipsoid obtained in Chapter 2 is the maximal contractively invariant ellipsoid.

Chapter 4 utilizes two composite quadratic Lyapunov functions, the convex hull quadratic Lyapunov functions and the max quadratic Lyapunov functions, for the estimation of the domain of attraction of control systems with actuator saturation. Both continuous-time and discrete-time settings are considered. The results of estimating the domain of attraction by using these two composite quadratic Lyapunov functions generalize those obtained in Chapter 2, which are based on the quadratic Lyapunov functions. Moreover, control systems with nestedly saturated feedbacks are also considered.

Chapter 5 discusses the finite-gain $\mathcal{L}_2$ stability of control systems with actuator saturation and exogenous disturbances. The problems of disturbance tolerance and rejection are addressed by using the quadratic Lyapunov functions and the composite quadratic Lyapunov functions. Both the $\mathcal{L}_2$ gain and the invariant sets characterize the ability of the closed-loop system to tolerate and/or reject disturbances. Systems subject to single-layer saturation, nested saturation, and an algebraic loop with saturation are studied. The treatment of saturation functions adopted in this chapter is the convex hull representation.

Chapter 6 provides another improvement of the conventional convex hull representation of saturated linear feedbacks by partitioning the convex hull into several convex sub-hulls. This improvement is based on the fact that, for a given system state at which at least one input saturates, not all the vertices of the conventional convex hull are necessary to form a convex hull where the saturated linear feedback resides. For a continuous-time saturated system, a separate static anti-windup gain is designed for each of convex sub-hulls and is implemented when the value of the saturated linear feedback falls into this convex sub-hull. Simulation results indicate that such a saturation-based switching anti-windup compensator has the ability to significantly enlarge the domain of attraction of the closed-loop system. This anti-windup design via partitioning the convex hull also applies to discrete-time systems.

The next two chapters concern with control systems with actuator saturation and an algebraic loop. Chapter 7 studies the stability and anti-windup synthesis of such systems. We partition the input space into several regions. This partitioning enables some special properties of a saturated feedback to emerge in different regions of the input space. This partitioning is combined with a piecewise quadratic Lyapunov function of an augmented state vector composing system states and saturation/deadzone functions to arrive a set of less conservative stability conditions, from which a larger estimate of the domain of attraction of a linear system under a saturated linear feedback can be obtained. Moreover, this approach also leads to tighter estimates of $\mathcal{L}_2$ gain from the exogenous disturbance to the performance output. On the other hand, we design a switching anti-windup compensator containing a group of anti-windup gains, each of which is associated with one region of the input space, to enlarge the domain of attraction and reduce the $\mathcal{L}_2$ gain of the resulting closed-loop systems.

Chapter 8 develops a new piecewise quadratic Lyapunov function, which results from adding a term that characterizes the regional sector condition of the saturation/deadzone function to the piecewise quadratic Lyapunov function adopted in Chapter 7. The matrix associated with this generalized piecewise quadratic Lyapunov function is not required to be positive definite, and thus less conservative conditions for the stability and performance analysis are established.

The saturation functions studied in Chapters 2–8 are all symmetric functions. Finally, in Chapter 9, the problem of estimating the domain of attraction of a linear system subject to asymmetric actuator saturation is considered. We propose an asymmetric Lyapunov function approach to estimating the domain of attraction. Two asymmetric piecewise Lyapunov functions are introduced. One is a piecewise quadratic Lyapunov function involving an asymmetric deadzone function, instead of a symmetric deadzone function, and is referred to as the asymmetric piecewise quadratic Lyapunov function. The other is a generalization of the asymmetric piecewise quadratic Lyapunov function, embedded with some special properties of the deadzone functions and a set of positive definite matrices.

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