A New Approach for Solving Fuzzy Supplier Selection Problems Under Volume Discount

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Abstract. In order to achieve a compromised solution for a multi-objective linear programming with fuzzy right hand sides, Tchebycheff norm and a new approach based on α-cut is suggested to minimize the distance from the current estimate of the objective values from the ideal point. Since the obtained solutions by the Tchebycheff approach are weakly efficient for multi-objective problems. Hence, an augmented weighted Tchebycheff norm has been proposed. Here, the satisficing tradeoff algorithm is used to solve the augmented weighted Tchebycheff problems. Since the supplier selection problem is usually a multi-objective problem, the augmented weighted Tchebycheff method is applied for obtaining its solutions.

Keywords: Fuzzy multi-objective linear programming · Augmented weighted Tchebycheff norm · α-cut approach · Satisficing tradeoff algorithm

1 Introduction

Companies have to work with several suppliers in order to supply their raw material. More than 70% of product's final price is related to raw material's cost. Because of this reason buying management is one of the most important parts in supply chain. In such circumstances the purchasing department can play a key role in cost reduction, and supplier selection is one of the most important functions of purchasing management [1]. Several factors may affect a supplier's performance. Dickson [2] identified 23 different criteria for vendor selection including quality, delivery, performance history, warranties, price, technical capability and financial position. Selecting the best suppliers and quota allocations to them reduces purchasing costs, improves competitiveness, and improving quality and flexibility to meet the requirements of the end consumer [3].

Basically there are two kinds of supplier selection problem based on the number of suppliers:

(1) Single sourcing,
(2) Multiple sourcing,

In the first kind of supplier selection, one supplier can satisfy all the buyer's needs. The management needs to make only one decision: which supplier is the best? In the second type, no supplier can satisfy all the buyer's requirements. That means, the buyers makes balance between suppliers and its overall demand is bought from several

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suppliers. The decision maker should make two decision: which suppliers are the best? And how much should be purchased from each selected supplier?

Based on the number of objective functions, the supplier selection programming is going to be divided into two clusters:

(1) Single objective;
(2) Multi objectives.

First cluster is consists of problems that have one objective function. This objective function can be considered as cost, quality or delivery on time, etc. In second cluster, several objective functions are supposed as objectives of decision making.

In reality most input data is not accurate. In the way that, most of these data can be mention as verbal variables such as high, low, tall and so on. Crisp models can't consider this inaccurate data. Fuzzy logic is one of the strong ways to manage this inaccurate data [4].

Here, we introduce some existed methods and criteria for supplier selection problem. We can point to these criteria as the most important ones: coast, quality of products, service aspects, delivery time, risk factors and trade restrictions. Some of the most important criteria are used for supplier selection problem from 1966 until now are summarized in Table 1.

Table 1. Literature review for supplier selection criteria

Author	Criteria				
	Cost	Quality	Delivery	Capacity	Warranty period
Lin [6]	✓	✓	✓		
Chen [7]		✓			
Chan [8]	✓	✓	✓	✓	
Ghodsypour [9]	✓	✓	✓	✓	
Stavropolous [10]	✓				
Min [11]	✓	✓	✓		
Weber [12]	✓	✓	✓	✓	
Abratt [13]	✓				
Lehmann [14]	✓		✓		✓
Dickson [2]	✓	✓	✓	✓	✓

In the previous works, several methods applied to solve supplier selection and order allocation program. Here, we introduce some of them.

Gaballa [15] is the first author who applied mathematical programming to supplier selection in a real case. He used mixed integer programming to minimize the total discounted price of allocated items to the suppliers. He also formulated a single-objective, mixed-integer programming to minimize the sum of purchasing, transportation and inventory costs by considering multiple items, multiple time periods, vendors' quality, delivery and capacity. Weber and Current [16] used a multi objective approach to systematically analyze the trade-offs between conflicting criteria in supplier selection problems. Ghodsypour and O'Brien [17] developed a Decision Support

System (DSS) for reducing the number of suppliers according to supply based optimization strategy. They used an integrated Analytical Hierarchy Process (AHP) with mixed-integer programming and considered suppliers' capacity constraint and the buyers' limitations on budget and quality etc. Ghodsypour and O'Brien [1] developed an integrated AHP and linear programming model to consider both qualitative and quantitative factors in purchasing activity. Wang et al. [18] provided an AHP method to choose from agile/lean supply chain strategies and then used Pre-emptive Goal Programming (PGP) to obtain the optimal order quantity from their suppliers. Xia and Wu [5] introduced rough sets theory to improve AHP and integrated multi-objective mixed integer programming to determine which suppliers should be selected and the quantity that should be allocated to them while considering volume discount policy.

Zadeh [19] initiated the fuzzy set theory. Bellman and Zadeh [20] presented some applications of fuzzy theories to the various decision-making processes in a fuzzy environment. Zimmerman [21, 22] presented a fuzzy optimization technique to linear programming problem with single and multiple objectives. Since then the fuzzy set theory has been applied to formulate and solve the problems in various areas such as artificial intelligence, image processing, robotics, pattern recognition, etc. Narsimhan [23] proposed a Fuzzy Goal Programming (FGP) technique to specify imprecise aspiration levels of the fuzzy goals. Yang, Ignizio and Kim [24] formulated the FGP with nonlinear membership functions.

This article is divided into the following sections: In Sect. 2, we introduce a multi objective linear programming model, and then we consider right hand side values as fuzzy term. In Sect. 3, we use α-cut approach to change fuzzy model into crisp type. After that, we use a method to solve crisp model. In Sect. 4, we use from the model that represented by Xia [5] and based on the data set adopted from a case company, and then formulate the supplier selection and order allocation model. Numerical findings are applied to show the usage of the suggested method. In Sect. 5, the conclusions are presented.

2 Fuzzy Multi Objective Linear Programming

Many of the decision problems in real life are multi objective. That mean, there are several objectives that each of them should be optimal at the same time.

Generally, a multi objective programming with p objective and is as follow:

$$\begin{array}{ll} \min & F(x) = [f_1(x), f_2(x), \ldots, f_p(x)] \\ \text{S.t.} & g(x) \leq b, \\ & x \geq 0, \end{array} \tag{1}$$

where $g(x)$ is linear function. In a real-life situation for a supplier selection problem, many input information related to the various supplier are not known with certainty such as capacity, quality, delivery time, etc. Such vagueness in the critical information cannot be captured in a deterministic problem and therefore the optimal results of these deterministic formulations may not serve the real purpose of modeling the problem.

Due to this, we have considered the model as a fuzzy model. Fuzzy mathematical programming has the capability to handle both multi objective problems and vagueness.

A multi objective programming with fuzzy resource can be formulated as:

$$\begin{aligned} \min \quad & F(x) = [f_1(x), f_2(x), \ldots, f_p(x)] \\ \text{S.t.} \quad & g(x) \leq \widetilde{b}, \\ & x \geq 0, \end{aligned} \tag{2}$$

Where the fuzzy number $\widetilde{b}$ is in the fuzzy region of $[b, b+u]$ with given fuzzy tolerance u, Assume that the fuzzy tolerance u for the fuzzy constraint is known. Then, $\widetilde{b}$ is equivalent to $(b+\theta u)$, where θ is in $[0, 1]$. In this case, a fuzzy constraint problem is transformed to be a crisp parametric programming problem. The following section, we apply Verdegay α-cut approach to transform fuzzy constraint to crisp constraint.

3 Propose Method

In this section, we first introduce Verdegay α-cut approach for transforming fuzzy constraint to crisp constraint. Then, we present a method in order to solve crisp equivalent multi objective programming.

3.1 Verdegay α-cut Approach [25]

For dealing problem (2), Verdegay considered if the membership function of the fuzzy constraint (shown in Fig. 1) has the following form:

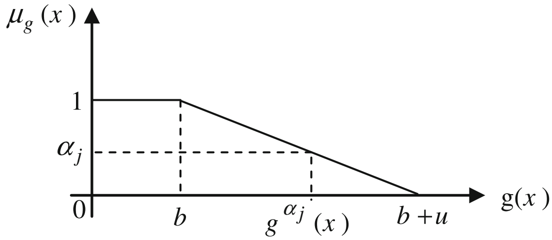

Fig. 1. Membership function of $\mu_g(x)$, with level α_j-cut

then,

$$\mu_g(x) = \begin{cases} 1, & g(x) \leq b, \\ 1 - \frac{g(x)-b}{u}, & b \leq g(x) \leq b+u, \\ 0, & g(x) > b+u, \end{cases} \tag{3}$$

Simultaneously, the membership functions of $\mu_g(x)$, is continuous and monotonic function and trade-off between this fuzzy constraint is allowed; then, problem (2) is equivalent to the following:

$$\begin{array}{ll} \min & F(x) = [f_1(x), f_2(x), \ldots, f_p(x)] \\ \text{S.t.} & x \in X_\alpha, \end{array} \tag{4}$$

where $X_\alpha = \{x \mid \mu_g(x) \le \alpha, x \ge 0\}$, for each $\alpha \in [0,1]$.

This is the fundamental concepts of α-cuts method of fuzzy mathematical programming. One can then substitute (3) into (4) and obtain the following formulation:

$$\begin{array}{ll} \min & F(x) = [f_1(x), f_2(x), \ldots, f_p(x)] \\ \text{S.t.} & \quad g(x) \le b + (1-\alpha)u, \\ & x \ge 0, \end{array} \tag{5}$$

where $\alpha \in [0,1]$. Thus the problem given in (5) is equivalent to a crisp parametric programming formulation. For each α, one will have an optimal solution. In the following we represent a method to solve problem (5).

Now, we explain augmented weighted Tchebycheff approach to solve multi-objective linear programming that obtained from above.

3.2 Augmented Weighted Tchebycheff Approach [26]

A common method for solving multi objective problems is augmented weighted Tchebycheff approach. Next, we introduce this approach and an algorithm to solve multi objective programming.

A multi objective programming consider as follow [26]:

$$\begin{array}{ll} \min & F(x) = [f_1(x), f_2(x), \ldots, f_p(x)] \\ \text{S.t.} & x \in X, \end{array} \tag{6}$$

where X is feasible region. Suppose that $v_k \ge 0, k = 1, 2, \ldots, p$, are nonnegative weights such that $\sum_{k=1}^{p} v_k = 1$. So, augmented weighted Tchebycheff norm related to $F(x) \in \mathbb{R}^p$ define as follow:

$$\|F(x)\|_\rho^v = \|F(x)\|_\infty^v + \rho \|F(x)\|_1, \tag{7}$$

where,

$$\|F(x)\|_\infty^v = \max_{k=1,2,\ldots,p} \{v_k |f_k(x)|\} \tag{8}$$

and,

$$\|F(x)\|_1 = |f_1(x)| + |f_2(x)| + \ldots + |f_p(x)| \tag{9}$$

and, ρ is a nonnegative scalar that usually is a small number between 0.01 and 0.0001.

We apply augmented weighted Tchebycheff norm to find minimum distance between objective functions and vector ideal solutions. So, we have to solve an optimization problem as fellow:

$$\begin{array}{ll} \min & \|F(x) - F^I\|_\infty^v \\ S.t. & x \in X. \end{array} \tag{10}$$

The equivalent program is obtaining from (7), (8) and (9). Therefore, we have:

$$\begin{aligned} &\min\{\max_{k=1,2,\ldots,p} v_k(f_k(x) - f_k^I)\} + \rho \sum_{k=1}^{p} (f_k - f_k^I) \\ &S.t. \quad x \in X, \end{aligned} \tag{11}$$

In order to solve (11) by using to linear programming techniques, we reformulate it into a linear programming as follow:

$$\begin{aligned} &\min\{\beta + \rho \sum_{k=1}^{p} (f_k - f_k^I)\} \\ &S.t. \quad \beta \geq v_k(f_k(x) - f_k^I), \quad k = 1, 2, \ldots, p, \\ &\qquad x \in X. \end{aligned} \tag{12}$$

One of the iterative algorithms to solve (12) is Satisficing Trade of Method (STOM). Next, we introduce steps of STOM algorithm.

STOM algorithm [27]:

Step 1. Obtain ideal solution f_k^I for each f_k objective function as follow:

$$\begin{array}{ll} \min & f_k \\ S.t & x \in X, \end{array} \tag{13}$$

f_k^I is equals to optimal value form (13).

Step 2. The decision maker has to specify aspiration levels for each function (the aspiration levels are determined by decision maker such that $f_k^+ > f_k^I$, where f_k^+ is aspiration level for f_k objective function).

Step 3. The relative weights determine as follow:

$$v_k = \frac{1}{f_k^+ - f_k^I}. \tag{14}$$

In fact, v_k is reverse of distance between ideal value and aspiration levels for f_k objective function.

Then, solve (12).

Step 4. The solutions whose obtain in step 3 offer to decision maker. The decision maker is asked to classify the objective functions into three classes:

i. The unacceptable objective functions whose values should be improved.
ii. The objective functions whose values may weakly.
iii. The acceptable objective functions whose values are acceptable as they are.

If no objective function is in the group (i) then, STOP. This solution is optimal. Otherwise, the decision maker has to specify new aspiration levels for functions in group (i) and (ii) then, go to step 3.

If the new program is infeasible then, the decision maker has determines more weakly aspiration levels. This process continues until the new program being feasible.

The optimal points whose obtain from STOM algorithm is a Pareto solution of (11).

In order to illustrate the performance of the propose approach we apply it on a case study from a drilling company.

4 Case Study

The proposed approach to solve supplier selection model was implemented in a drilling company. There are three sources and three raw materials for purchasing. Decision maker has to select the best sources and decide how many material buy from them. Four objectives are considered by decision maker in this company for select suppliers and order allocation. These objectives are cost, quality, on time delivery, suppliers score. There are three kinds of commodity, Pipe (P), Gravel (G), Bentonite powder (B) for purchasing.

We applied proposed method to solve case study model. In order to determine distance between ideal solution and current solution, we use metric function that represent by Steuer in [27] as follow:

$$D_K(\lambda, p) = [\sum_{i=1}^{p} \lambda_i^K (1 - d_i)^K]^{\frac{1}{K}}, \tag{15}$$

Where d_i indicates the degree of closeness between obtained solution $Z_i(x^*)$ and their ideal solution $Z_i^I(x)$ and obtain as follow:

When the i-th objective is maximized as:

$$\frac{Z_i(x^*)}{Z_i^I(x)}. \tag{16}$$

Otherwise,

$$\frac{Z_i^I(x)}{Z_i(x^*)}. \tag{17}$$

Also, λ is unit vector of aspiration levels for objective function. K is distance parameter such that, $1 \leq K \leq \infty$.

Here, we use distance function (19) for obtain distance between obtained solution and ideal solution for $K = 2$.

We compare the obtained compromise solution by proposed method with the weighted additive approach. These results are shown in Fig. 2.

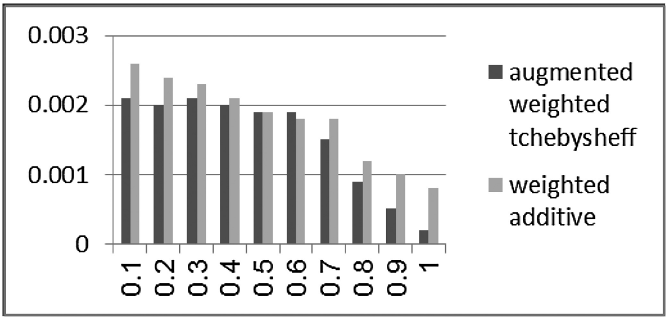

Fig. 2. Comparison of distance between obtained solution and ideal solution

As seen in Fig. 2, the compromise solution that obtain by proposed method has less distance from ideal solution related to weighted additive method for each α-cut, $\alpha = 0.1, 0.2, \ldots, 1$.

So, the result obtained from proposed method is better than weighted additive approach for each α-cut, $\alpha = 0.1, 0.2, \ldots, 1$.

5 Conclusion

Supplier selection is a complex multi objective decision-making problem. Since each supplier has its own advantages and disadvantages in terms of cost, quality, delivery and the technology, a flexibility model is required. In this paper, we use Xia_s model for formulating our case study. Since, many information of firm is not precise in real life so, we consider fuzzy number for show this information. We proposed an interactive approach by using - cut method, augmented weighted Tchebycheff norm and STOM algorithm. We compare proposed method with weighted additive approach by using a distance function. According to Eq. (10) proposed method from ideal solution has less distance related to weighted additive approach. Future studies may like to use stochastic variable instead of fuzzy variable. Moreover, using different norm to minimize the distance between the obtain solution of the objectives and the ideal solution.

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