Chapter 2
Determining of Asphalt Stiffness Modulus

Abstract This chapter deals with the development of equation that expresses the bitumen stiffness modulus as a function of time, temperature and the simple properties of bitumen such as penetration (or penetration index) and softening temperature. As mathematical model for describing the stiffness of bitumen, Christensen and Anderson (CA) equation was used. The parameters of CA model were related to the bitumen properties based on Van der Poel's experimental data. The instantaneous value for longitudinal modulus was obtained by extrapolation of values for stiffness modulus according to Van der Poel at low temperatures and small load durations. With the purpose of extrapolation, the model developed to describe the viscoelastic properties of amorphous glass forming polymers was applied. To express the zero-shear viscosity of bitumen as a function of temperature based on Van der Poel's data, the parameters of time-temperature superposition function were related with the penetration index of binder. The approximate formula was obtained for the exponent of power in CA model as a function of penetration index. Using developed equation, stiffness modulus of bitumen can be easily calculated in a wide range of temperatures and loading time.

In this section, we obtain empiric formula, showing asphalt stiffness modulus as a function of temperature and load duration. As the basic information regarding asphalt, we use only its penetration (i.e., the depth of needle penetration under GOST 11501-78 or ASTMD 5) and softening point (calculated by "ring-and-ball" test under GOST 11506-73 or ASTMD 36)—indexes for asphalt, which are always calculated in all road laboratories of Russia, Ukraine, Kazakhstan, Belorussia and western European countries.

2.1 Test Data on Asphalt Stiffness Modulus

As it was mentioned before, Van der Poel (1954a, b) called asphalt stiffness modulus a ratio of constant tensile stress σ_c to the strain $\varepsilon(t)$

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$$S(t) = \sigma_c / \varepsilon(t) \tag{2.1}$$

To determine $S(t)$, Van der Poel (1954a, b) used two types of loading experiments, namely constant stress experiment—static creep test—and the dynamic test with the alternating stress of constant amplitude and frequency.

Creep test was carried out by steady loading conditions of tension or bending, as well as shear (twisting in rotation viscometer), and $S(t)$ was calculated, which Van der Poel called static asphalt stiffness modulus. During such a test, the load duration exceeded 1 s and reached 10,000 s. Stiffness modulus, obtained by torsion test, was transformed into stiffness modulus under tension by multiplying into 3, as the Poisson's ratio of asphalt was roughly assumed as equal to 0.5 without regard to temperature.

Van der Poel carried out tests by cyclic loading in conditions of bending of the beam on two piers or consoles with frequency ω from 1 to 1000 rad/s. Relation of amplitude of applied stress to amplitude of measured deformation, i.e., absolute value of longitudinal complex modulus $E_d(\omega) = |E^*(\omega)| = \sigma_0/\varepsilon_0(\omega)$ he called dynamic stiffness modulus. Van der Poel mentioned that inaccuracy for repeated test results with cyclic loading was 6%—much less than with steady load, especially for asphalt of low temperature susceptibility.

Having compared test results for steady and cyclic load, Van der Poel made a conclusion that if to replace a cyclic frequency with the value, reciprocal to time of loading, then the dynamic modulus is roughly equal to static stiffness modulus:

$$S(t) \approx E_d(\omega)|_{\omega=1/t} \tag{2.2}$$

He verified transformation (2.2) from tests with cyclic loading to the creep test, using a simple model of Maxwell's visco-elastic liquid Eq. (1.9). Therefore, Van der Poel showed relation of asphalt stiffness modulus with load duration in the diagram $\log S(t) - \log t$, laying off $S(t)$ for small durations of loading as the modulus E_d with similar scale for time t and for value, reciprocal to cyclic frequency $1/\omega$. One curve $S(t)$ slipped into continuation of another at t around one second. It gave the chance to use jointly the test results of constant stress and oscillating loading covering large period of load duration within 10^{-3} до 10^4c on one diagram $\log S(t) - \log t$ for the given asphalt with certain temperature.

Van der Poel joined into single variable impact of temperature and load duration on asphalt stiffness modulus, having used principle of time-temperature analogy, which we described in Sect. 1.5. As the work was performed before publication of the mentioned article of M. Williams, R. Landel and J. Ferry (WLF), he used only the formula like S. Arrhenius (1.32) to determine time–temperature shift function, accepting activation energy for asphalt E_a = 50 kcal/mol = 2.09 × 10^5 J/mol.

Van der Poel tested 47 types of asphalt, obtained from various crude oils by different methods (distillation, temperature cracking, oxidation, precipitation, mixing). These types of asphalt were essentially differed from each other regarding response of stiffness modulus $S(t)$ to temperature variation. To identify them, Van der Poel used penetration index PI, which was introduced by Pfeifer and Van Doormal in 1936 (Pfeiffer and Van Doormal 1936) as a parameter of the temperature susceptibility of asphalt:

$$PI = \frac{20 - 500A}{1 + 50A} \tag{2.3}$$

where A is temperature susceptibility, characterizing slope of the line log (P) versus T, where P is a penetration of needle into asphalt:

$$A = \frac{\lg(800/P)}{T_{rb} - 25} \tag{2.4}$$

Here P is a penetration depth at 25 °C, T_{rb}—softening point for asphalt, measured by “ring and ball” method (ring and ball temperature), whereas it is assumed that penetration depth for all types of asphalt at softening point is equal to 800, i.e., 800 dmm (decimillimeters).

The cause, why Pfeifer and Van Doormal chose this Eq. (2.3) and proposed to use PI but not A as the characteristics of temperature response, was their aim to characterize Mexican asphalt, which had the penetration 200 at 25 °C, as zero index PI = 0. In accordance with the formula (2.3), for asphalt with PI = 20, penetration does not depend on temperature at all (A = 0), and with PI = −10 asphalt, to the contrary, is infinitely susceptible to the temperature variation.

As a rule, penetration index for asphalt does not exceed PI from −3 to +5. In the vast majority of cases, the paving asphalt is characterized by PI from +2 to −2. Asphalt with PI less than −2 is so responsive to temperature variation that the mixes, prepared with it, are subject to forming of cracks during fast cooling in winter. Particularly, GOST 22245-90 for paving asphalt oil restrains penetration index for asphalt of types BND 40/60 −BND 200/300 within −1 < PI < +1.

Therefore, Van der Poel used penetration at 25 °C P (according to ASTM D5) and softening point T_{rb} (according to ASTM D36) and penetration index PI, which was calculated on their base to characterize asphalt.

Asphalts, tested by Van der Poel, were of penetration index from −2.6 to +6.3, i.e., performed tests included as the asphalt, very susceptible to temperature variation, as well as asphalts of low temperature susceptibility. Testing temperature was chosen with consideration of temperature susceptibility for asphalt and its softening point. For example, asphalt with PI = −2.3 and T_{rb} = 66 °C was tested at T = 5, 15, 25, 35, and 45 °C, and asphalt with low temperature susceptibility with PI = + 5.3 and T_{rb} = 116 °C—at T = −20, 0, 20, 40 and 60 °C. Experimentally obtained values for stiffness modulus for the given T with different t during tests by constant and cyclic load were included into diagram of relation for stiffness modulus—time in the scale $\log S(t) - \log t$ and smooth curve $S(t)$ was drawn through the experimental points.

Based on experimental data, Van der Poel developed a nomograph for determination of stiffness modulus (Van der Poel 1954a, b). With this nomograph for a given penetration of asphalt P and its softening point T_{rb}, one can determine the stiffness of asphalt cement as a function of T and load duration t or cyclic frequency ω. Depending on penetration index, P difference of temperatures (T_{rb} − T), and load duration t within 10^{-6} and 10^{10} s, Van der Poel's nomograph covers the range of values for bitumen stiffness modulus S from 10^{-4} to 2.5×10^{9} Pa.

Having determined a bitumen stiffness modulus from the nomograph, it is possible to predict the stiffness modulus for asphalt concrete depending on characteristics of bitumen, its content in the mix and porosity of the mix (Heukelom and Klomp 1964; Bonnaure et al. 1977).

Van der Poel's nomograph was included into reference books (The SHELL Bitumen Handbook 2003), manuals for Civil Engineers (Roberts et al. 1996 etc.), monographs for pavement design (Huang 1993) etc.), as well as in a number of technical papers. Van der Poel did not publish empirical equations, which he used in construction of the nomograph, and therefore, stiffness modulus can be found only graphically, and the procedure is time consuming and inaccurate.

According to Van der Pole's evaluation, maximum inaccuracy for determining of stiffness modulus via the nomograph is up to 50% compared with experimental data, on which it is based. It is due to construction the curves that fit to a series of data points on logarithmic scale, variety of experimental points and replacement of wavy sections for the curves by smooth ones. If to have in mind, that a range for moduli, which are determined using the nomograph, covers 13 decades of time (from 10^{-4} to 2.5×10^{9} Pa), then maximum inaccuracy of 50% is quite reasonable. Average inaccuracy is much less. In addition, as Van der Poel mentioned (Van der Poel 1954a), double difference of asphalt moduli complies roughly with the difference of temperatures about 2 °C, and it is difficult to expect practically that information regarding actual temperature conditions of pavement will be much more accurate.

Later the staff of SHELL lab enlarged the Van der Poel's nomograph, scanned and digitized. Then Ullidtz and Peattie developed Fortran-program PONOS for determination of stiffness modulus by programmable calculators (Bats 1973; Ullidtz and Peattie 1980). G.M. Rowe and M.J. Sharrock (ABATECH, Inc.) developed a program BitProps for personal computers in 2000, based, like PONOS, on interpolation between the points of digitized Van der Poel's nomograph. It allows finding the stiffness of asphalt modulus of bitumen without graphical procedure.

2.2 Empirical Formulas Earlier Suggested for Stiffness Modulus

For practical and research purposes, mathematical equations are needed for the important viscoelastic characteristics of bitumen such as a stiffness modulus $S(t)$, the shear and longitudinal creep compliance $J(t)$ и $D(t)$, the shear and longitudinal relaxation moduli $G(t)$ and $E(t)$, and for the complex modulus $|G^{*}(\omega)|$ or $|E^{*}(\omega)|$.

Those equations should allow obtaining these characteristics depending on standard properties of asphalt (penetration P at 25 °C and the softening point T_{rb}), load duration and temperature.

Several researchers tried to develop mathematical equation for stiffness modulus of bitumen $S(t)$ depending on P, T_{rb} and T.

Saal (1955) proposed such a formula, based on the graphs in Figs. 11 and 12 of Van der Poel's article (1954a). It relates the asphalt stiffness modulus with load duration t = 0.4 s and its penetration without regard to the temperature susceptibility of asphalt:

$$S_{0.4} = \frac{10^{9.4}}{P^{1.9}} \tag{2.5}$$

where stiffness modulus is expressed in Pa.

This formula has a number of drawbacks. First, stiffness modulus can be determined only at one load duration (0.4 s). Second, to determine it at some temperature you should know the depth of needle penetration at this temperature, but it is usually known only for T = 25 °C. Third, according to formula (2.5), asphalt stiffness modulus with t = 0.4 s does not depend on softening point, i.e., penetration index. For example, with $P = 100$ dmm under formula (2.5) $S_{0.4}$ = 0.4 MPa. Actually, for P = 100 dmm and softening points T_{rb} = 38.8, 47.5, and 59.5 °C (penetration indexes PI = −3, 0, and +3), the Van der Poel's nomograph (or program BitProps) shows S = 0.26, 0.40 and 0.52 MPa respectively. It turns out that only asphalt stiffness modulus for PI = 0 agrees with the formula, and moduli with penetration indexes PI = −3 and +3 differ in two times, although according to formula (2.5) stiffness does not depend on PI.

Ullidtz and Larsen (1984) proposed the equation

$$S(t) = 1.157 \cdot 10^{-7} t^{-0.368} e^{-PI} \cdot (T_{rb} - T)^5 \tag{2.6}$$

Drawbacks of this simple formula are obvious. With load duration, tending to zero, stiffness modulus under formula (2.6) tends to infinity, and therefore it is doubtful that this formula is applicable for very small t. When $T = T_{rb}$, according to formula (2.6), stiffness modulus equals to zero, which does not comply with reality. For $T > T_{rb}$, it becomes negative, which does not have sense. Therefore, the Eq. (2.6) might return unrealistic values for stiffness modulus at temperature greater than softening point. Ullidtz and Larsen (1984), limited the applicability of Eq. (2.6) to the duration of loading 0.01 < t < 0.1, range of penetration index −1 < PI < 1, and temperature range 10 °C < $(T_{rb} - T)$ < 70 °C.

Nevertheless, even within the ranges, mentioned by them, it differs essentially from Van der Poel's data, on which it is based. Therefore, for asphalt with PI = −1, which has P = 54.5 dmm and T_{rb} = 50 °C, at T = 30 °C and t = 0.1 s formula (2.6) predicts S = 2.35 MPa while the Van der Poel's nomograph (or program BitProps) returns S = 1.27 MPa, i.e., modulus is two times higher. For asphalt with PI = + 1, which has P = 109.5 dmm and T_{rb} = 50 °C, at T = 30 °C and t = 0.01 s formula (2.6) predicts $S = 0.742$ MPa, while the program BitProps returns $S = 2.021$ MPa, i.e., modulus is three times lower. Calculations show that the coefficient of variation of stiffness modulus calculated using Eq. (2.6) from the Van der Poel's data even in that quite narrow range of input parameters, is 40%.

Nevertheless, the Eq. (2.6) was used in a number of works, for example in (Collop and Cebon 1995), but the authors of the last paper reduced the temperature interval of its applicability to 20 °C < $(T_{rb} - T)$ < 60 °C. For example, at T_{rb} = 50 °C it means applicable for using only at T from −10 to 30 °C, but it does not allow using formula (2.6) for analysis of pavement behavior neither at low winter, nor for high summer temperatures.

Two formulas like (2.6), but more cumbersome, were proposed by Shahin (1977), who worked in predicting of low temperature cracks formation in asphalt concrete pavements. Shahin used three operational variables in his equations: the logarithm of time $\lg(t)$, a penetration index PI, and the temperature difference $(T - T_{rb})$. Based on Van der Poel's nomograph and using two hundred points, by regression analysis he found stiffness as a function of $\lg(t)$, PI, and $(T - T_{rb})$; the square and the cube of these variables, as well as their products. That is why the Shahin's two equations for log(S) are so lengthy.

Shahin did not specify the range of their applicability. He only mentioned that his first formula was intended for small stiffness modulus $S < 1$ MPa [formula (2.2) in (Shahin 1977)], and the second formula—for $S > 1$ MPa [formula (2.3) in (Shahin 1977)].

Meanwhile, it is unknown beforehand what value of stiffness modulus will be and which of these formulas should be used. Moreover, for example for asphalt with PI = −1, which has P = 54.5 dmm and T_{rb} = 50 °C, at T = 23 °C and t = 1 s according to the formula for small moduli, the asphalt stiffness equals to $S = 0.826$ MPa while according to formula for large ones the asphalt stiffness equals $S = 13.34$ MPa, i.e., both formulas are applicable, but it is unknown which of the results is correct. The similar picture can be observed for the asphalt at all temperatures from 23 to 35 °C. Similar situation is valid for other asphalts as well.

Recently, Molenaar (2005) of Delft University, using Shahin's formulas (1977), tried to establish the range of their applicability for the binders of penetration index −2 < PI < 2. According to our evaluation, Shahin's equation for large stiffness is worse than for small stiffness. Particularly, for asphalt with PI = −1, at T = −15 °C and $t = 0.01$ s, Eq. (2.3) in (Shahin 1977) returns the bitumen stiffness of 16,000 MPa. This value is 6–7 times greater than asphalt stiffness modulus in glassy state and even exceeds asphalt concrete gassy modulus.

As one can see, many researches expressed their interest to the equations that relate the stiffness modulus of bitumen with its standard characteristics, time, and temperature. Attempts to find such relations in the papers (Ullidtz and Larsen 1984; Saal 1955 and Shahin 1977) were not successful. In our opinion, it can be explained by usage of purely statistical methods for their derivation without involving of the theoretical interrelationships describing the viscoelastic behavior of asphalt binder.

2.3 Development of Formula for Asphalt Stiffness

As viscoelastic material, asphalt combines the properties of perfectly elastic solid and perfectly viscous liquid. It is illustrated by Van der Poel's data on asphalt stiffness $S(t)$ (Fig. 2.1). Asphalt is characterized by penetration depth P = 80 dmm at 25 °C and softening point T_{rb} = 50 °C (penetration index PI = 0). Thirty-two values for stiffness modulus are taken with the help of program BitProps for temperature 15 °C and 32 values of load duration t within the range of 10^{-6} and 10^3s that are shown by points on the graph.

Under constant stress σ_c,at short loading time $t \to 0$ the asphalt deformation $\varepsilon(t)$ tends to its instantaneous elastic value $\varepsilon = \sigma_c/E_g$, and therefore, its stiffness modulus, according to Eq. (2.1), at $t \to 0$ tends to instantaneous longitudinal elastic modulus E_g(glassy modulus). Horizontal asymptote of the curve $S(t)$ corresponds to the value of E_g.

As load duration increases, input of viscous flow into deformation increases too. Deformation during tension of ideal viscous liquid, under Newtonian law Eq. (1.7), is equal to $\varepsilon(t) = \sigma_c t/3\eta$, and therefore, according to Eq. (2.1), at $t \to \infty$ stiffness modulus tends to $S(t) = 3\eta/t$. In logarithmic scale log(S) − log(t) last equation is shown by straight line with slope 45°.

Horizontal (elastic) and inclined (viscous) asymptotes of the curve $S(t)$ are shown in the Fig. 2.1. It is obvious that they are crossing in the point, where $E_g = 3\eta/t$, i.e., $t_0 = 3\eta/E_g$. For this example t_0 = 0.005 s. These two asymptotes limit the area of location for the curve $S(t)$. It is convenient to describe the curve of such geometry by the function like

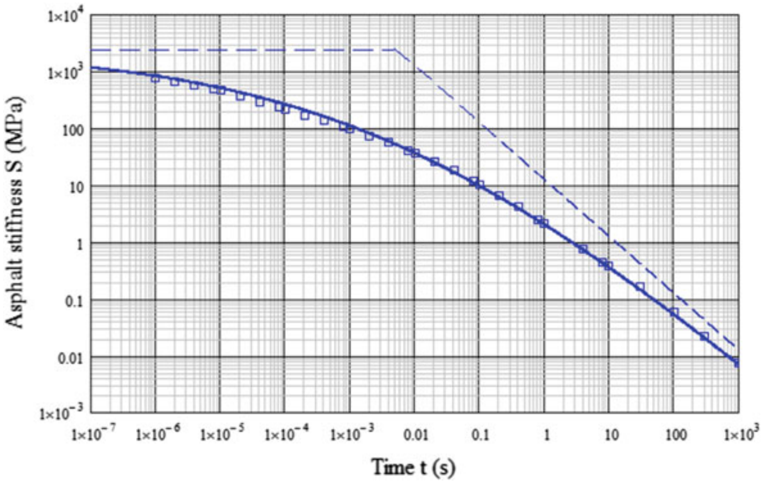

Fig. 2.1 Typical master curve of creep stiffness as a function of loading time at constant temperature

$$F(t) = E_g \left[1 + \left(\frac{t}{t_0} \right)^{v} \right]^{-1/w} \tag{2.7}$$

This expression was proposed by Havrilyak and Negami (1966) during their research of dielectric permittivity for polymers, which is mathematically similar to complex modulus of viscoelastic material. This function with the equal exponents v = w was adopted in 1992 for description of asphalt complex modulus $|G^*(\omega)|$ by Christensen and Anderson (1992), Christensen (1992), and in short it is called CA model (Christensen-Anderson model). Parameter t_0 of this model depends on temperature, but parameter v does not depend on it, and both parameters are determined from experimental data. Meanwhile, Christensen and Anderson did not find the relationship between parameter v and asphalt penetration index *PI* (Christensen and Anderson 1992). The authors of CA model limited its applicability by the values of asphalt complex modulus greater than 0.1 MPa (Christensen 1992, p. 198).

Later Anderson and Marasteanu considered the applicability of model (2.7) with unequal exponents of $v \neq w$ (Marasteanu and Anderson 1999) (CAM model) to asphalt. Lesuer with co-authors did the same (Lesuer et al. 1997), but they considered all three parameters t_0, w and v as the temperature dependent. It made very difficult to use the model, as it required calculating three parameters at each temperature of testing.

We assumed the following equation for asphalt stiffness modulus in this work

$$S = E_g \left[1 + \left(\frac{E_g t}{3\eta} \right)^{\beta} \right]^{-\frac{1}{\beta}} \tag{2.8}$$

where E_g is the uniaxial glassy modulus of binder (MPa); η is the steady-state Newtonian viscosity (MPa·s), which is dependent on asphalt type and its temperature; β is the exponent of power ($0 < \beta < 1$), which is supposed to be dependent only on asphalt type. Equation (2.8) is similar to CA model, which has been proposed in the paper Christensen and Anderson (1992) for complex modulus $|G^*(\omega)|$. Eq. (2.8) is shown by the curve in the Fig. 2.1.

Stiffness modulus (2.8) tends to instantaneous stiffness modulus E_g with the decrease of load duration. With unlimited increase of load duration, it tends to the equation describing viscous flow $S = 3\eta/t$. Exponent β is the shape parameter the curve $S(t)$. With the decrease of β at constant viscosity, the curve $S(t)$ becomes more flat, i.e., transformation from elastic to viscous asphalt behavior occurs more gradually.

We investigated the relations of E_g, η and β with the penetration P and the softening point T_{rb} for asphalt.

The instantaneous value for longitudinal modulus E_g was obtained by extrapolation of values for stiffness modulus $S(T, t)$ according to Van der Poel at low temperature T and small load duration t for $t \to 0$. With the purpose of extrapolation, we used the model, developed by Drozdov for description of viscoelastic properties of amorphous glass forming polymers (Drozdov 2001). For a set of eight loading times t = 0.00005, 0.0001, 0.0002, 0.0005, 0.001, 0.002, 0.005 and 0.010 s, we selected the lowest temperature for the asphalt with certain penetration index, for which, using a program BitProps the value of stiffness S can be obtained fort = 0.00005 s. This lowest temperature was from −42 °C for PI =+2 to 9 °C for PI = −3. We obtained S at a very low temperature for those eight periods of time. Then using the formulas (18) and (37) of (Drozdov 2001) we found value of instantaneous modulus in such a way so that influence curve of modulus from temperature could pass through the points, corresponding to those eight periods of time.

We concluded that for asphalts with *PI* from −2 to +3 the average value of instantaneous longitudinal modulus equals to E_g = 2460 MPa with variation coefficient of 7%. Meanwhile, no regular variation of E_g was observed with variation of penetration index (Fig. 2.2).

Therefore, glassy modulus can be assumed to be E_g = 2460 MPa in extension or flexure. For reference, Van der Poel assumed E_g = 2500 MPa for all asphalts (1954a, b). Dobson accepted G_g = 1000 MPa, which is similar to E_g = 3000 МПа (Dobson 1969). Christensen and Anderson, according to the test results for eight types of asphalts with various levels of ageing, selected values for instantaneous shear modulus within the range of G_g = 720 MPa and G_g = 1120 MPa, which corresponds to $E_g = 2160 - 3360$ MPa, but they recommended to assume E_g = 3000 MPa (Christensen and Anderson 1992).

Obviously, asphalt stiffness is a function of time and temperature (Fig. 2.3). The steady-state viscosity η reflects a temperature dependency of stiffness S in the Eq. (2.8). To obtain the viscosity as a function of temperature, the equation for a time–temperature shift factor $a_{T_r}(T)$ should be formulated.

As it was specified in Sect. 1.5, it is convenient to think of $a_{T_r}(T)$ as of the coefficient, into which you should divide the loading time t during the testing at temperature T to obtain the same strain value for a loading time t_r during the testing

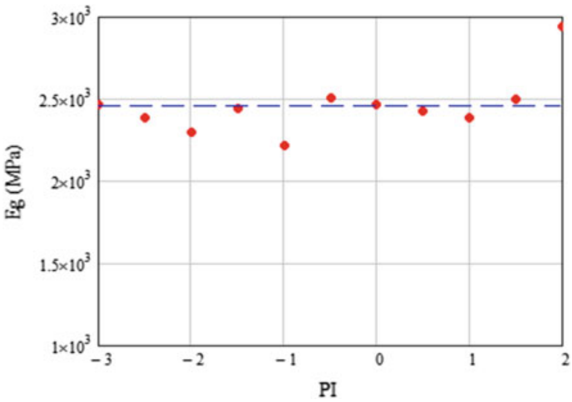

Fig. 2.2 Instantaneous glassy modulus of asphalt calculated by extrapolation of Van der Poel's data (horizontal line shows average value E_g = 2460 MPa)

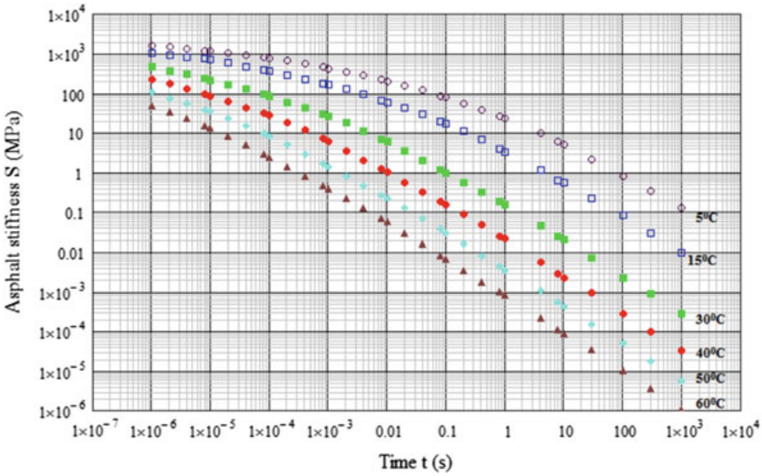

Fig. 2.3 Variation of asphalt stiffness with time and temperature: points correspond to Van der Poel's data for asphalts with PI = −0.5, T_{rb} = 50 °C

at reference temperature T_r, i.e., to perform time-temperature shift $t_r = t/a_T(T)$ for creep or relaxation curve along axis log(t) in order to join the curves, obtained at different temperatures. In other words,

$$a_{Tr}(T) = t/t_r, \qquad (2.9)$$

where t and t_r are such loading times at the temperatures T и T_r that the moduli (or compliances) are equal.

For instance, for asphalt with PI = −0.5 and T_{rb} = 50 °C (Fig. 2.3) we will find a value of time-temperature shift function for test results at the temperature T = 15 °C to the reference temperature T_r = 40 °C. Using the program BitPros, we can find at T = 15 °C and t = 100 s the stiffness S = 0.0807 MPa. The same value of stiffness modulus for this type of asphalt is obtained at temperature T_r = 40 °C for the loading time t_r = 0.2205 s. Now from Eq. (2.9) one can obtain for this type of asphalt $a_{T_r}(T)$ = 100/0.2205 = 454. In other words, difference of temperatures 40 − 15 = 25 °C for this type of asphalt is equivalent to variation of t in 454 times. Let us check that this value does not depend on load duration. Let us take for T = 15 °C loading time t = 10 s (instead of 100 s as before). Then stiffness modulus for this type of asphalt is S = 0.5395 MPa. The same value of stiffness modulus for this type of asphalt at temperature T_r = 40 °C will be at the loading time t_r = 0.02153c, from which $a_{T_r}(T)$ = 10/0.02153 = 464. The difference between obtained values is only 2%. Thus, the curves S(t, T = 15 °C) and S(t, T = 40 °C) for this type of asphalt in Fig. 2.3 can be joined by displacement along the axis log(t) on a segment around log(460) = 2.663 in logarithmic scale.

In the same manner using Eq. (2.9), we found the values of $a_{T_r}(T)$ for asphalts with the various PI. Parameters of function $a_{T_r}(T)$ were fitted to those values.

Function $a_{T_r}(T)$ was composed from equations of Arrhenius and M. Williams, R. Landel, J. Ferry (WLF) (Ferry 1963), described above in Sect. 1.5:

$$a_{Tr\,Ahrr}(T) = \exp\left[\frac{E_a}{R}\left(\frac{1}{(273+T)} - \frac{1}{(273+T_r)}\right)\right] \tag{2.10}$$

$$a_{Tr\,WLF}(T) = \exp\left[\frac{In(10)C_1(T-T_r)}{C_2+(T-T_r)}\right] \tag{2.11}$$

Arrhenius's equation was chosen for temperatures lower than the reference temperature T < T_r while the WLF equation was selected for T > T_r. As a reference temperature, we selected a temperature 10 °C lower than the softening point, determined by "ring and ball" method.

Assuming the reference temperature T_r = (T_{rb} − 10), we took into consideration that the glass transition temperature for asphalt T_g is usually about 50–70 °C lower than the softening point T_{rb} (Schmidt and Santucci 1965; Bahia and Anderson 1993), and the WLF equation is intended for temperatures higher than the glass transition temperature. Moreover, the "standard values" of its constants C_1 = 8.86 and C_2 = 101.6 were determined for the reference temperature T_r = (T_g + 50). Then the WLF equation is valid for the temperatures (T_g + 100) and higher. Therefore, having selected T_r = (T_{rb} − 10), we can with the help of the WLF equation cover the range of summer temperatures for asphalt concrete pavement. On the other hand, Arrhenius's equation should cover the range from (T_{rb} − 10) to low winter temperatures. For penetration indexes from PI = −3 to PI = + 2 with values of difference (T − T_r) from −100 to +140 and a step 10 °C we determined values $a_{T_r}(T)$ based on Van der Poel's data using Eq. (2.9) and found the relationships $E_a(PI)$ and $C_1(PI)$, $C_2(PI)$.

The energy of activation in Arrhenius's equation was expressed as

$$E_a = 9.745 \times 10^4 \frac{3(30+PI)}{5(10+PI)}$$

It has appeared that energy of activation varies within E_a = 1.56 × 10^5 J/mol for PI = +2 and E_a = 2.25 × 10^5 J/mol for PI = −3. As a comparison, Van der Poel (1954a, b) accepted for asphalts at average E_a = 50 kcal/mol = 2.09 × 10^5 J/mol, having selected as a reference temperature T_r = T_{rb}. Christensen and Anderson assumed E_a = 2.5 × 10^5 J/mol (Christensen 1992) or E_a = 2.61 × 10^5 J/mol (Christensen and Anderson 1992) at a reference temperature much lower than T_{rb}.

We obtained the following coefficients in WLF equation:

$$C_1 = \frac{1}{0.11+0.0077PI}, C_2 = 104.5$$

The first one varies from C_1 = 7.97 for PI = +2 to C_1 = 11.5 for PI = −3, and the second one was found to be independent on PI. As a comparison, standard values, obtained WLF (Ferry 1963), C_1 = 8.86, C_2 = 101.6. Therefore, we obtained such equation for time–temperature shift factor $a_{T_r}(T)$:

– at T $\leq (T_{rb} - 10)$:

$$a_{Tr\,Ahrr}(T) = \exp\left[11720 \cdot \frac{3(30+PI)}{5(10+PI)}\left[\frac{1}{(T+273)} - \frac{1}{(T_{rb}+263)}\right]\right] \quad (2.12a)$$

– at T > $(T_{rb} - 10)$:

$$a_{Tr\,WLF}(T) = \exp\left[-\frac{2.303(T - T_{rb}+10)}{(0.11+0.0077PI)(114.5+T-T_{rb})}\right] \quad (2.12b)$$

Now the asphalt stiffness modulus as a function of temperature and loading time can be shown as a single curve, if instead of time t the stiffness is plotted versus the reduced time $t_r = t/a_{T_r}(T)$ in X-coordinate. As an example in Fig. 2.3, the points of six curves $S(t)$ for various temperatures T grouped together forming one master curve at reference temperature in Fig. 2.4. This curve coincides with the constructed one according to (2.12a, b). As the result, the time range that is for Fig. 2.3 from 10^{-6} to 10^{3}s, extended from 10^{-10} to 10^{5} s, i.e., it is increased for six decimal orders.

Using Eq. (2.12a, b) for $a_{T_r}(T)$, one can obtain asphalt viscosity at any temperature $\eta(T)$ if it is known at a reference temperature $\eta(T_r)$:

$$\eta(T) = \eta(T_r) \cdot a_{Tr}(T) \quad (2.13)$$

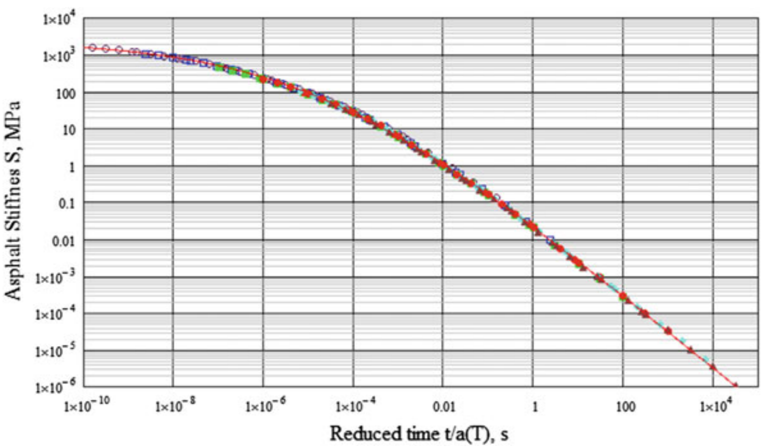

Fig. 2.4 Stiffness modulus of asphalt as a function of time and temperature after reducing to the reference temperature T = T_r: points Van der Poel's data (from Fig. 2.3), continuous curve–calculated from Eq. (2.12a, b) with T_r = 40 °C

Based on Van der Poel's data, we found the following approximate expression for asphalt viscosity at reference temperature

$$\eta(T_r) = 0.00124\left[1 + 71\exp\left[-\frac{12(20 - PI)}{5(10 + PI)}\right]\right] \cdot \exp\left(\frac{0.2011}{0.11 + 0.0077PI}\right) \quad (2.14)$$

Equations (2.13) and (2.12a, b) lead to the following formula for asphalt viscosity as a function of temperature:

$$\eta = a_{Tr\,Ahrr}(T) \cdot \eta(T_r)\ (T \le T_{rb} - 10);\ \eta = a_{Tr\,WLF}(T) \cdot \eta(T_r)\ (T > T_{rb} - 10) \quad (2.15)$$

where viscosity η is in MPa $\cdot$ s, and $a_{T_r\,Ahrr}(T)$, $a_{T_r\,WLF}(T)$, $\eta(T_r)$ are expressed by Eqs. (2.12a, b) and (2.14), respectively.

Asphalt viscosity calculated from Eq. (2.15) as a function of the penetration index and temperature is shown in Fig. 2.5. These curves are in agreement with the curves of SHELL (Molenaar 2005, p. 70) and (Claessen et al. 1977, p. 68).

In Eq. (2.8) for asphalt stiffness, the viscosity η as a function of time and temperature and the instantaneous longitudinal modulus E_g are already known. To obtain the exponent of power β, its value was considered as independent on temperature. In accordance with our assumption regarding applicability of time-temperature superposition principle, time and temperature should be included into formulas for mechanical characteristics of viscoelastic materials only using the ratio $(t/a_{T_r}(T))$. Equation for $\beta(PI)$ was fitted for each value of penetration index ranging from $PI = -3$ to $PI = +2$ within the temperature range $(T - T_{rb})$ from −45 to 10 °C to minimize the quadratic deviation of values $S(t)$, calculated using Eq. (2.8) from Van der Poel's data. The following approximate formula was obtained for the exponent of power as a function of penetration index:

$$\beta = \frac{0.1794}{1 + 0.2084PI - 0.00524PI^2} \quad (2.16)$$

It varies from $\beta = 0.1285$ for $PI = +2$ to $\beta = 0.5476$ for $PI = -3$.

Therefore, a stiffness modulus of asphalt S can be calculated as a function of penetration index, softening point, loading time and temperature using formula (2.8) where $E_g = 2460$ MPa, viscosity η is determined by formula (2.15), and β—by formula (2.16).

For example, for asphalt with the penetration depth $P = 80$ dmm at 25 °C and the softening point $T_{rb} = 50$ °C, penetration index PI = 0 we will obtain S at the temperature 15 °C and load duration 1 s. Using Eq. (2.12a), we calculate value of time–temperature shift factor $a_{T_r}(T)$ from the testing temperature T = 15 °C to the reference temperature $T_r = 40$ °C and obtain $a_{T_r\,Ahrr}(T) = 347.5$. Viscosity at the reference temperature, according to Eq. (2.14) is equal to $\eta(T_r) = 0.01222$ MPa $\cdot$ s. according to Eq. (2.15), viscosity at 15 °C is equal to their product $\eta = 4.25$ MPa $\cdot$ s. The exponent of power from formula (2.16)

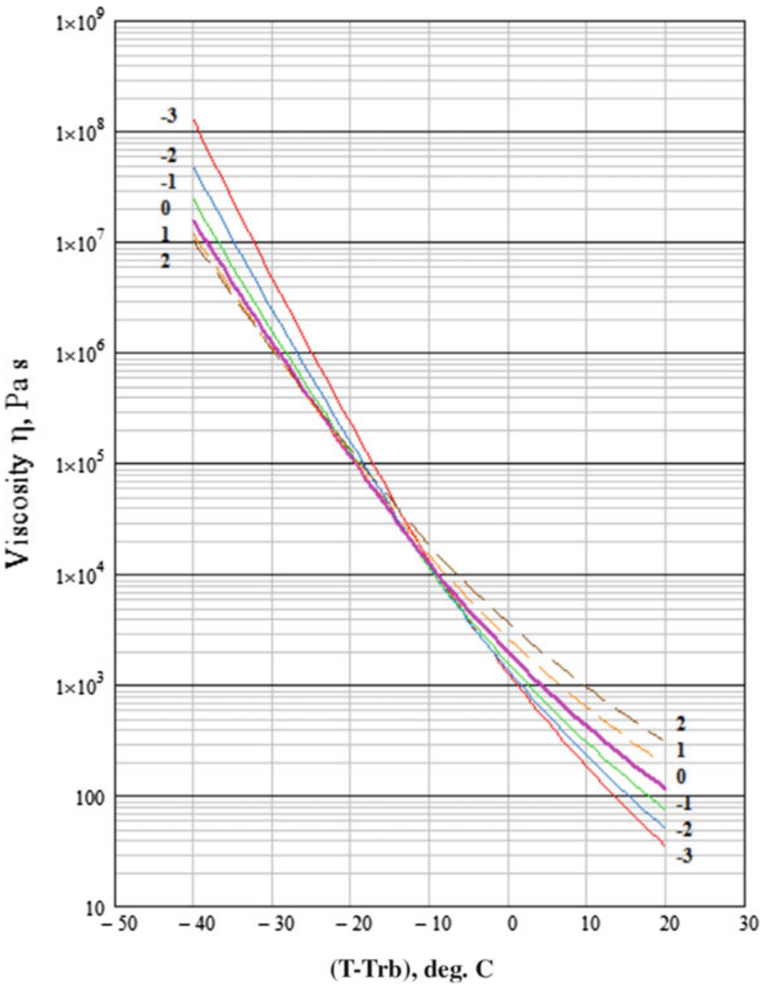

Fig. 2.5 Asphalt viscosity as a function of temperature and penetration index

$\beta = 0.1794$. Finally, the stiffness modulus of asphalt can be calculated from Eq. (2.8):

$$S = 2460\left[1 + \left(\frac{2460 \cdot 1}{3 \cdot 4.25}\right)^{0.1794}\right]^{-\frac{1}{0.1794}} = 2.042\ M\Pi a$$

Program BitProps gives the value $S = 2.107$ MPA, i.e., inaccuracy of our approximate formula is 3% in this example. Calculation results of stiffness modulus for this equation within large range of loading time are shown in Fig. 2.1 with continuous curve. As opposed to the nomograph or to the program BitProps, the operation with obtained equation using such standard programs as Mathcad or

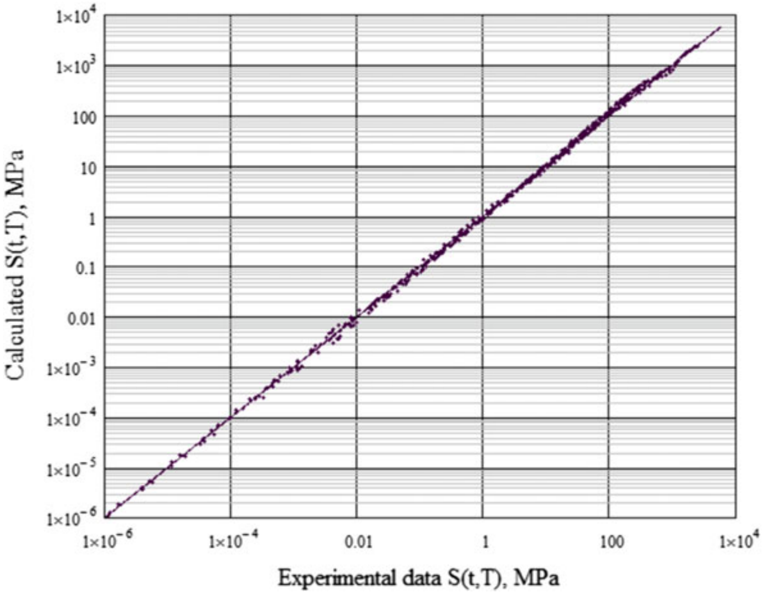

Fig. 2.6 Comparison of stiffness moduli from data of Van der Poel with stiffness moduli calculated using Eq. (2.8): *PI* = −3, 0 and *PI* = 1; T = 5, 15, 30, 40, 50, 60 °C; *t* = from 10^{-6} to 10^4 c

Excel, the construction of graph like in the Fig. 2.1takes just a few cent seconds. For 1910 points in the range of *PI* from −3 to +2,$(T - T_{rb})$ from −45 to 10 °C, and t from 10^{-4} to 10^4 s, the coefficient of variation of stiffness calculated using Eq. (2.8) from the Van der Poel's data (program BitPtops) was 14.6% and the coefficient of determination R^2 = 0.978. The comparison of stiffness moduli for 540 points calculated using Eq. (2.8) and those taken from Van der Poel's data is shown in Fig. 2.6.

One can analyze a number of practical problems using Eq. (2.8) for stiffness of asphalt. For instance, we can estimate the delayed elastic part of deformation for three types of bitumen having the same softening point T_{rb} = 50 °C, but different penetration: P = 54.5, 80 and 109.5 dmm (penetration indexes *PI* = −1, 0 and 1, respectively).

It is clear that total creep deformation under the constant stress σ at loading time t is $\varepsilon = \sigma/S(t)$. It consists of instantaneous elastic part $\varepsilon_g = \sigma/E_g$, deformation of viscous flow $\varepsilon_{visc} = \sigma t/3\eta$, and delayed elastic deformation ε_{del}. The latter occurs in time and is recoverable. It is not difficult to see that proportion of delayed elastic deformation is equal to

$$c_{del} = 1 - \frac{S(t)}{E_g} - \frac{S(t)t}{3\eta} \tag{2.17}$$

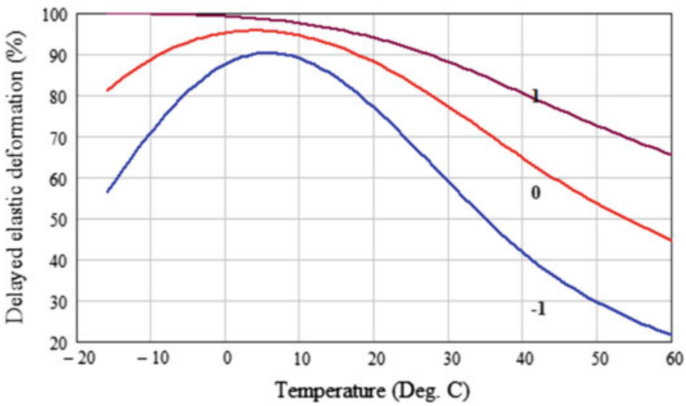

Fig. 2.7 Delayed elastic part of deformation for asphalt with various penetration indexes as the function of temperature: softening point T_{rb} = 50 °C, penetration index is shown on the curves, load duration t = 0.1 s

The results of calculation (Fig. 2.7) show that at loading time t = 0.1 s that is typical for asphalt concrete pavements under a moving load during passage of trucks, asphalts with penetration index −1 and 0 reveal maximum delayed elasticity at temperature about 5 °C—around 45 °C lower than softening point. Such temperature is typical for pavements in spring or late autumn during the periods of high water content in subgrade soil. Portion of delayed elastic deformation reduce with temperature decrease or increase, moreover, very dramatically for asphalt with $PI = -1$. As for asphalt of lower temperature susceptibility ($PI = +1$), the portion of its delayed elastic deformation slowly reduces with the increase of temperature and does not have maximum within service temperatures of pavement from −40 to 60 °C. It means that asphalt with high softening point T_{rb} = 50 °C and penetration 109.5 dmm (at 25 °C) keeps an ability to recover deformation at low winter temperatures, which might be important for resistance of asphalt concrete pavement to low temperature cracking.

Equation for stiffness modulus, obtained here will be used in final section of our work for calculation of relaxation modulus of asphalt, its dynamic modulus and asphalt concrete moduli depending on temperature and load duration.

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